Estimating the Probability of a Conjecture to be a Theorem in PLN for Inference Control

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SingularityNET

Artificial Intelligence and Theorem Proving 2025 (AITP-25)

Outline

1. Probability of Conjecture to be Theorem (in PLN)
2. Use such Estimates to Guide Reasoning

State of the Art (notable papers):

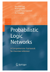
- *Logical Prior Probability*, Abram Demski (2016)
- *Uniform Coherence*, Scott Garrabrant et al (2016)
- *Logical Induction*, Scott Garrabrant et al (2016)

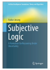
Ternary predicate relating theories, proofs and propositions

$$\Theta : \texttt{Theory} \times \texttt{Proof} \times \texttt{Proposition} \rightarrow \texttt{Bool}$$

PLN Recall

- 

Non-Axiomatic Logic (NAL), *Pei Wang, 2013*
- 

Probabilistic Logic Networks (PLN), *Ben Goertzel et al, 2008*
- 

Subjective Logic, *Audun Jøsang, 2016*

PLN Call

Traditional Logic:

$$\Gamma \vdash \mathrm{T}$$

$$\rightarrow$$

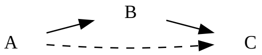
PLN:

$$\Gamma \vdash \mathrm{T}$$

$$\rightleftarrows$$

PLN Recall

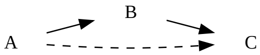
Deduction:



$$\frac{B \Rightarrow C \qquad A \Rightarrow B}{A \Rightarrow C}$$

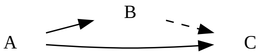
PLN Recall

Deduction:



$$\frac{B \Rightarrow C \qquad A \Rightarrow B}{A \Rightarrow C}$$

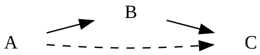
Induction:



$$\frac{A \Rightarrow C \qquad A \Rightarrow B}{B \Rightarrow C}$$

PLN Recall

Deduction:



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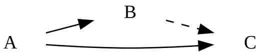
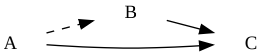
Induction:



$$\frac{A \Rightarrow C \qquad A \Rightarrow B}{B \Rightarrow C}$$

Abduction:



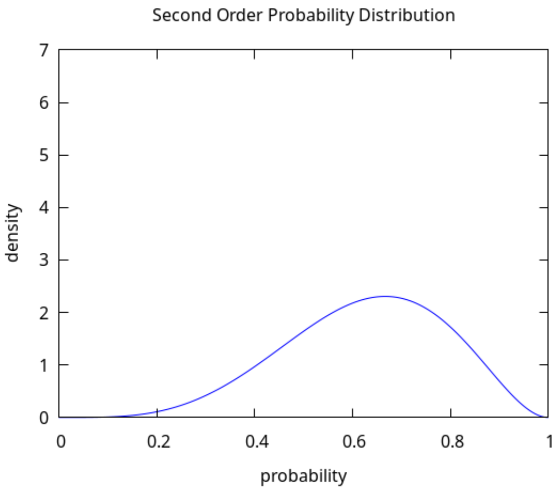
$$\frac{A \Rightarrow C \qquad B \Rightarrow C}{A \Rightarrow B}$$

PLN Recall

Truth Value:

$$A \Rightarrow B \stackrel{\mathrm{m}}{=} \mathrm{TV}$$

$$\mathrm{TV}$$
$$=$$
Second Order Probability Distribution
$$\approx$$
$$P(B|A)$$

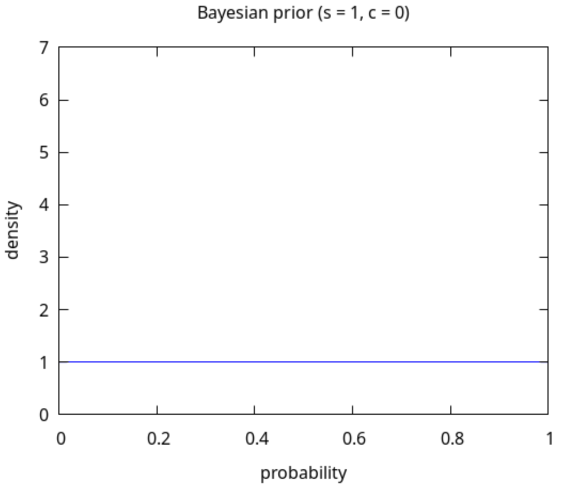


PLN Recall

Simple Truth Value:

$$A \Rightarrow B \stackrel{\text{m}}{=} <s, c>$$

- s = *strength*
- c = *confidence*
- Beta Distribution



PLN Recall

<u>*Simple Truth Value:*</u>

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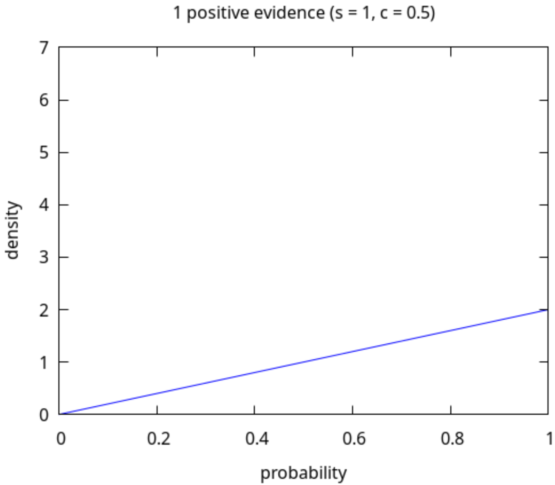

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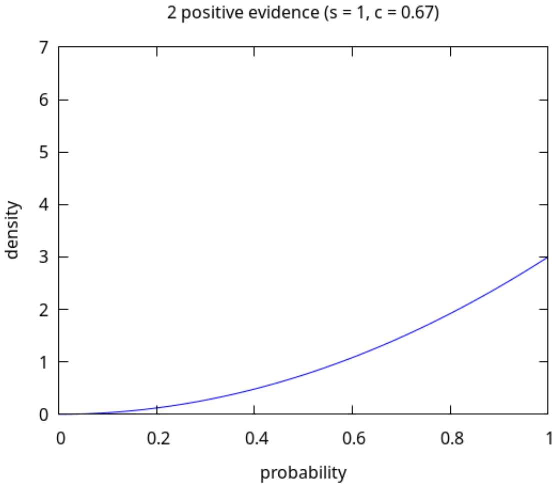

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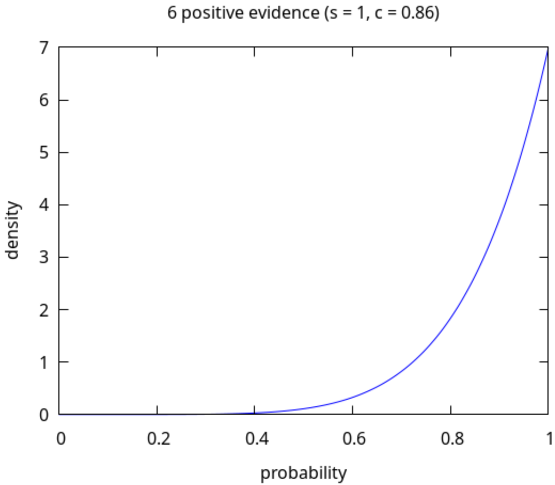

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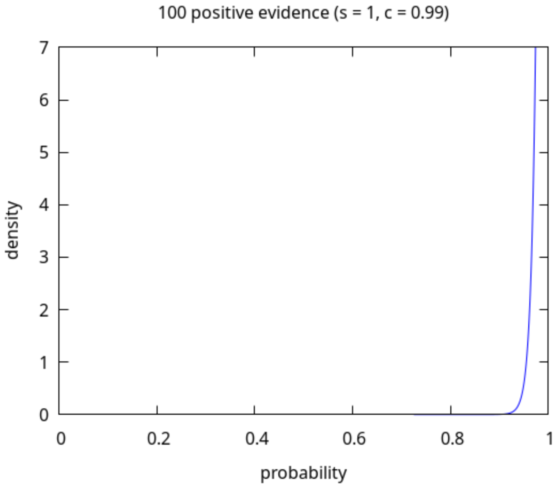

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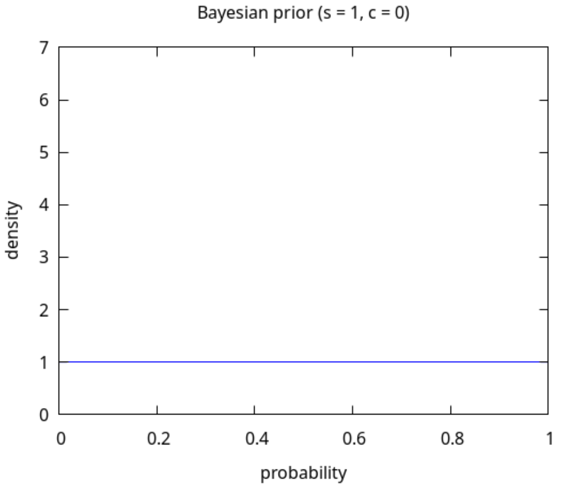

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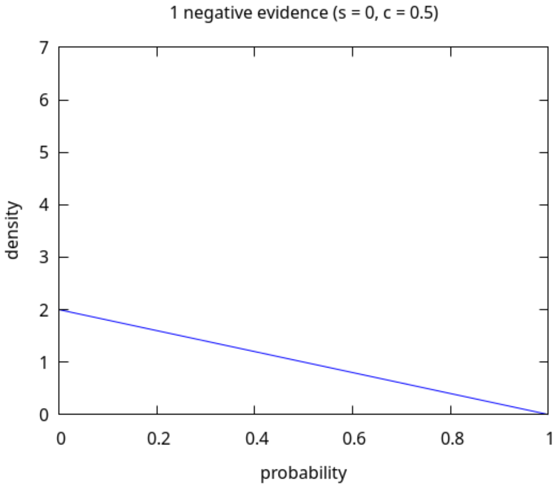

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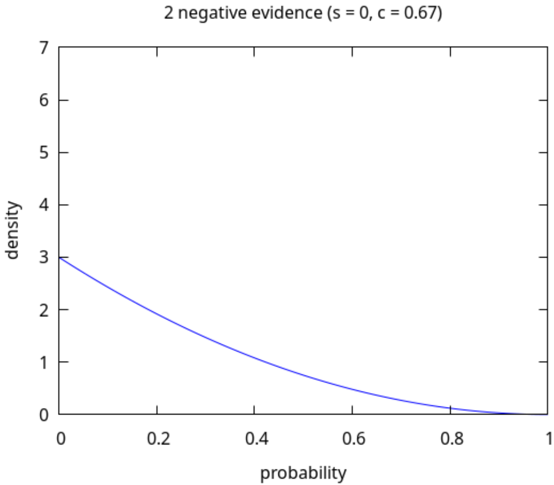

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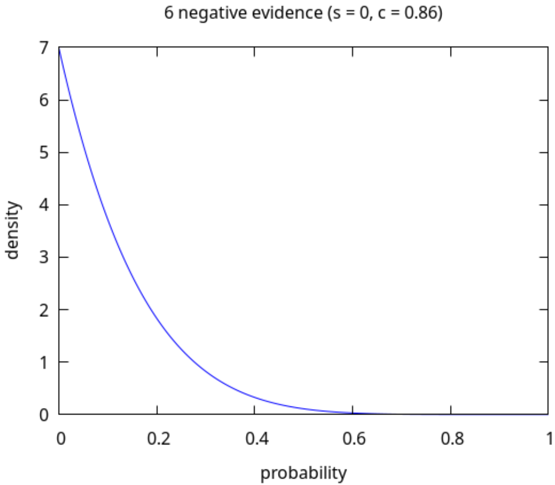

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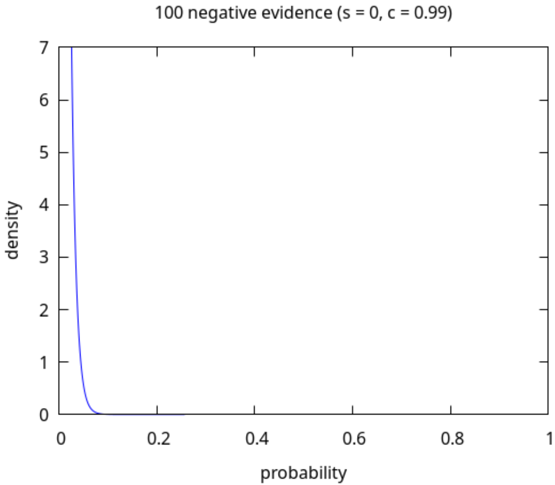

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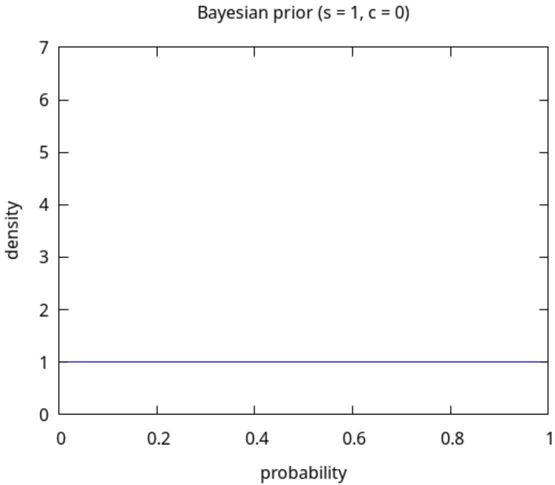

PLN Recall

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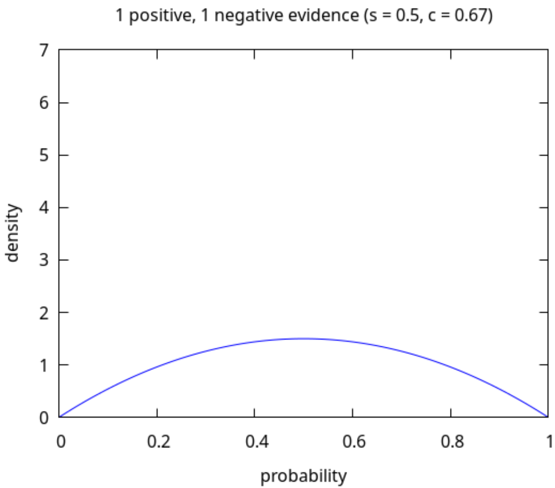

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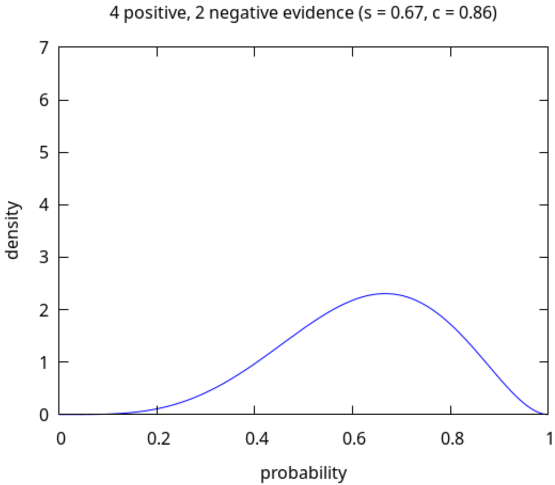

PLN Recall

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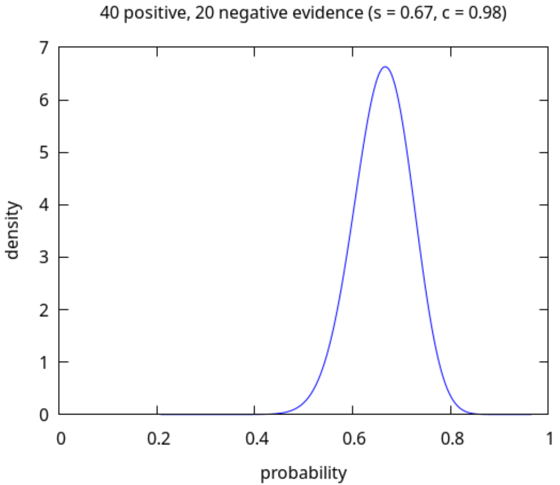
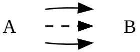
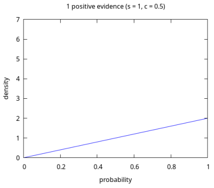


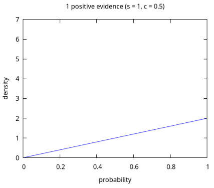
PLN Recall

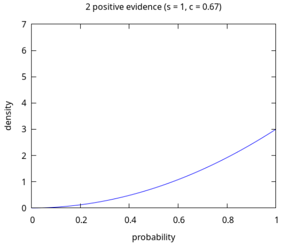
Revision:



$$\frac{\overline{A \Rightarrow B}\,(e) \qquad \overline{A \Rightarrow B}\,(f) \qquad e \perp f}{A \Rightarrow B}$$

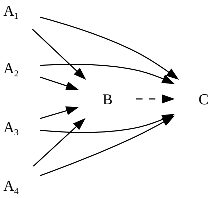


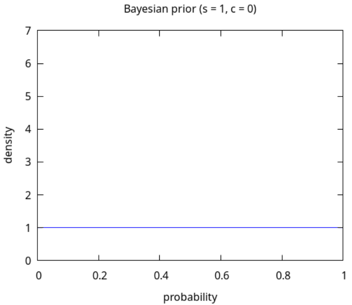




PLN Recall

Induction + Revision:

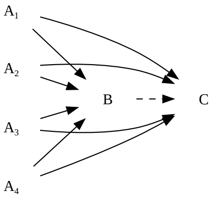


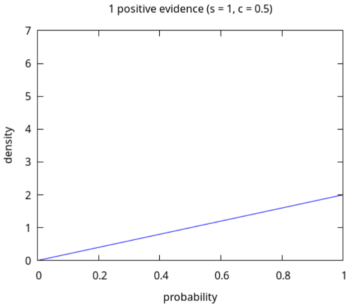


$$\cfrac{\cfrac{\cfrac{\cfrac{A_1 \Rightarrow C \quad A_1 \Rightarrow B}{B \Rightarrow C}\,(\text{Ind}) \quad \cfrac{A_2 \Rightarrow C \quad A_2 \Rightarrow B}{B \Rightarrow C}\,(\text{Ind})}{B \Rightarrow C}\,(\text{Rev}) \quad \cfrac{A_3 \Rightarrow C \quad A_3 \Rightarrow B}{B \Rightarrow C}\,(\text{Ind})}{B \Rightarrow C}\,(\text{Rev}) \quad \cfrac{A_4 \Rightarrow C \quad A_4 \Rightarrow B}{B \Rightarrow C}\,(\text{Ind})}{B \Rightarrow C}\,(\text{Rev})$$

PLN Recall

Induction + Revision:

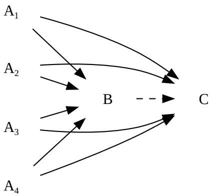


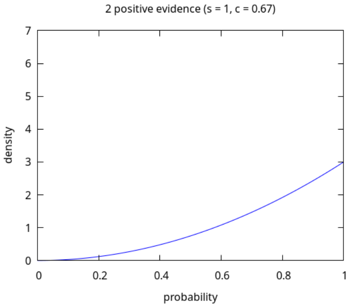


$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{A_1 \Rightarrow C \quad A_1 \Rightarrow B}{B \Rightarrow C}\text{(Ind)} \quad \dfrac{A_2 \Rightarrow C \quad A_2 \Rightarrow B}{B \Rightarrow C}\text{(Ind)}}{B \Rightarrow C}\text{(Rev)} \quad \dfrac{A_3 \Rightarrow C \quad A_3 \Rightarrow B}{B \Rightarrow C}\text{(Ind)}}{B \Rightarrow C}\text{(Rev)} \quad \dfrac{A_4 \Rightarrow C \quad A_4 \Rightarrow B}{B \Rightarrow C}\text{(Ind)}}{B \Rightarrow C}\text{(Rev)}$$

PLN Recall

Induction + Revision:

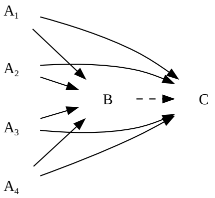


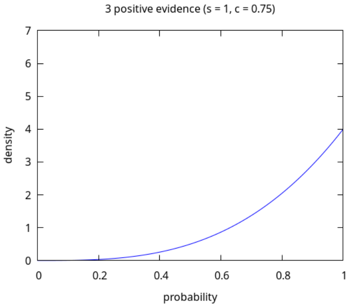


$$\dfrac{\dfrac{\dfrac{\dfrac{A_1 \Rightarrow C \qquad A_1 \Rightarrow B}{B \Rightarrow C}\text{(Ind)} \qquad \dfrac{A_2 \Rightarrow C \qquad A_2 \Rightarrow B}{B \Rightarrow C}\text{(Ind)}}{B \Rightarrow C}\text{(Rev)} \qquad \dfrac{A_3 \Rightarrow C \qquad A_3 \Rightarrow B}{B \Rightarrow C}\text{(Ind)}}{B \Rightarrow C}\text{(Rev)} \qquad \dfrac{A_4 \Rightarrow C \qquad A_4 \Rightarrow B}{B \Rightarrow C}\text{(Ind)}}{B \Rightarrow C}\text{(Rev)}$$

PLN Recall

Induction + Revision:

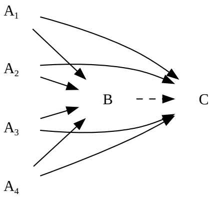


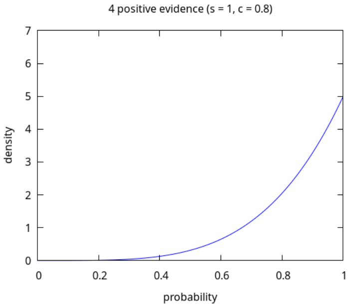


$$\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{A_1 \Rightarrow C \quad A_1 \Rightarrow B}{B \Rightarrow C}\text{(Ind)} \quad \cfrac{A_2 \Rightarrow C \quad A_2 \Rightarrow B}{B \Rightarrow C}\text{(Ind)}}{B \Rightarrow C}\text{(Rev)} \quad \cfrac{A_3 \Rightarrow C \quad A_3 \Rightarrow B}{B \Rightarrow C}\text{(Ind)}}{B \Rightarrow C}\text{(Rev)} \quad \cfrac{A_4 \Rightarrow C \quad A_4 \Rightarrow B}{B \Rightarrow C}\text{(Ind)}}{B \Rightarrow C}\text{(Rev)}}{}$$

PLN Recall

Induction + Revision:

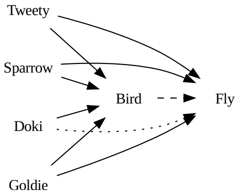


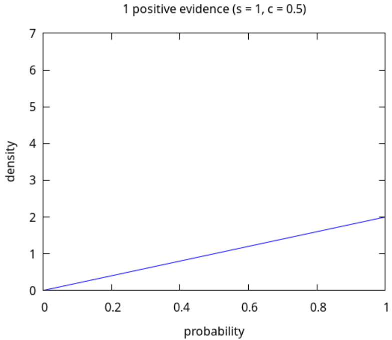


$$\dfrac{\dfrac{\dfrac{\dfrac{A_1 \Rightarrow C \qquad A_1 \Rightarrow B}{B \Rightarrow C}\text{(Ind)} \qquad \dfrac{A_2 \Rightarrow C \qquad A_2 \Rightarrow B}{B \Rightarrow C}\text{(Ind)}}{B \Rightarrow C}\text{(Rev)} \qquad \dfrac{A_3 \Rightarrow C \qquad A_3 \Rightarrow B}{B \Rightarrow C}\text{(Ind)}}{B \Rightarrow C}\text{(Rev)} \qquad \dfrac{A_4 \Rightarrow C \qquad A_4 \Rightarrow B}{B \Rightarrow C}\text{(Ind)}}{B \Rightarrow C}\text{(Rev)}$$

PLN Recall

Induction + Revision example:

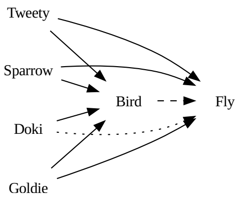


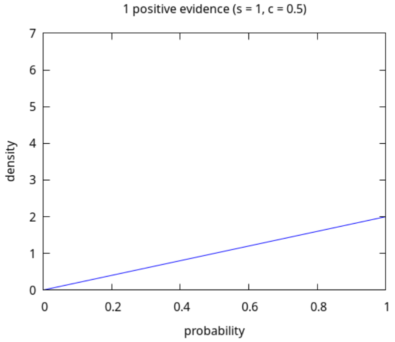


$$\frac{\text{Tweety} \Rightarrow \text{Fly} \stackrel{m}{=} <1, 1> \qquad \text{Tweety} \Rightarrow \text{Bird} \stackrel{m}{=} <1, 1>}{\text{Bird} \Rightarrow \text{Fly} \stackrel{m}{=} <1, 0.5>} (\text{Ind}, t)$$

PLN Recall

Induction + Revision example:

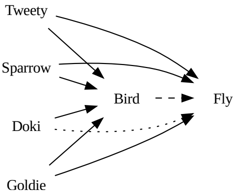


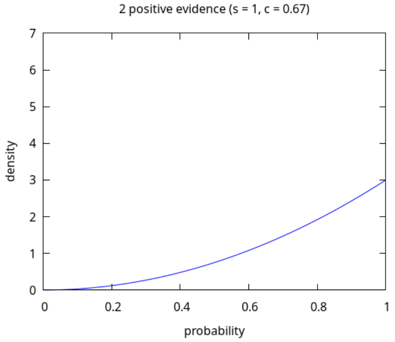


$$\frac{\text{Sparrow} \Rightarrow \text{Fly} \overset{\text{m}}{=} <1,1> \qquad \text{Sparrow} \Rightarrow \text{Bird} \overset{\text{m}}{=} <1,1>}{\text{Bird} \Rightarrow \text{Fly} \overset{\text{m}}{=} <1,0.5>} (\text{Ind}, s)$$

PLN Recall

Induction + Revision example:

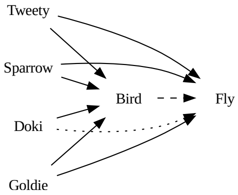


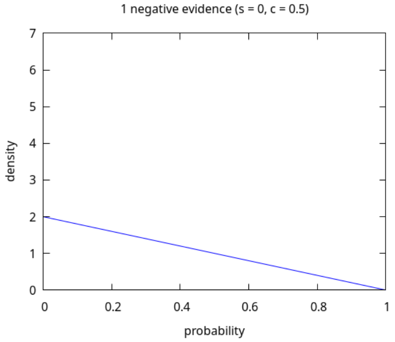


$$\dfrac{\dfrac{}{\text{Bird} \Rightarrow \text{Fly} \stackrel{m}{=} <1, 0.5>}(t) \quad \dfrac{}{\text{Bird} \Rightarrow \text{Fly} \stackrel{m}{=} <1, 0.5>}(s) \quad t \perp s}{\text{Bird} \Rightarrow \text{Fly} \stackrel{m}{=} <1, 0.67>}(\text{Rev}, t, s)$$

PLN Recall

Induction + Revision example:

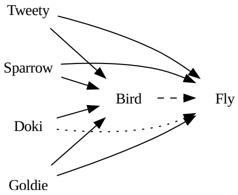


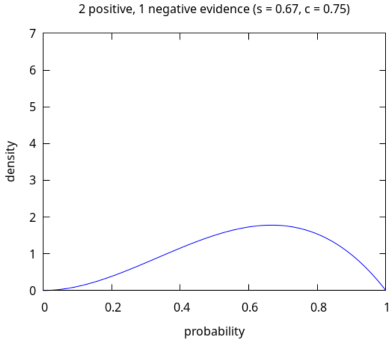


$$\frac{\text{Doki} \Rightarrow \text{Fly} \overset{m}{=} <0,1> \qquad \text{Doki} \Rightarrow \text{Bird} \overset{m}{=} <1,1>}{\text{Bird} \Rightarrow \text{Fly} \overset{m}{=} <0,0.5>} (\text{Ind}, d)$$

PLN Recall

Induction + Revision example:

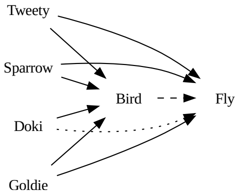


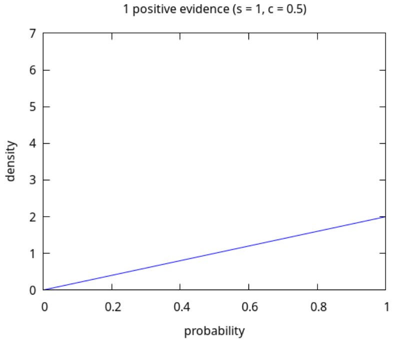


$$\dfrac{\dfrac{}{\text{Bird} \Rightarrow \text{Fly} \ \stackrel{m}{=} <1, 0.67>}(t,s) \quad \dfrac{}{\text{Bird} \Rightarrow \text{Fly} \ \stackrel{m}{=} <0, 0.5>}(d) \quad t, s \perp d}{\text{Bird} \Rightarrow \text{Fly} \ \stackrel{m}{=} <0.67, 0.75>}(\text{Rev}, t, s, d)$$

PLN Recall

Induction + Revision example:

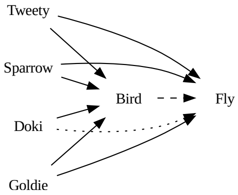


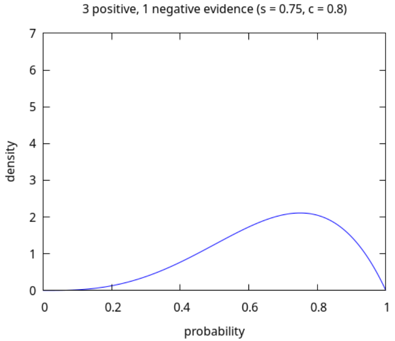


$$\frac{\text{Goldie} \Rightarrow \text{Fly} \overset{\text{m}}{=} <1,1> \qquad \text{Goldie} \Rightarrow \text{Bird} \overset{\text{m}}{=} <1,1>}{\text{Bird} \Rightarrow \text{Fly} \overset{\text{m}}{=} <1,0.5>} (\text{Ind}, g)$$

PLN Recall

Induction + Revision example:

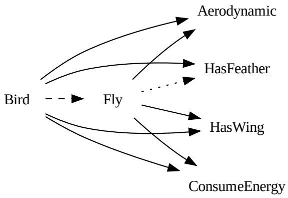




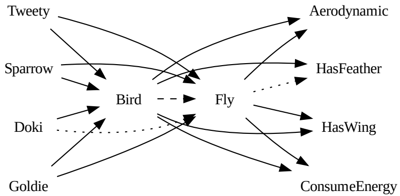
$$\dfrac{\dfrac{}{\text{Bird} \Rightarrow \text{Fly} \overset{m}{=} <0.67, 0.75>}\text{(t,s,d)} \quad \dfrac{}{\text{Bird} \Rightarrow \text{Fly} \overset{m}{=} <1, 0.5>}\text{(g)} \quad t, s, d \perp g}{\text{Bird} \Rightarrow \text{Fly} \overset{m}{=} <0.75, 0.8>}\,(\text{Rev}, t, s, d, g)$$

PLN Recall

Abduction + Revision:



Induction + Abduction + Revision:



PLN also has:

- Quantifiers ∃, ∀
- Traditional Connectors ∧, ∨, ¬
- Composite Predicates: Bird ∧ ¬Penguin ⇒ Fly
- Probabilistic Computational Model

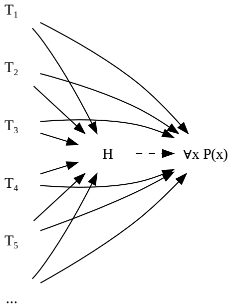
PLN and Theorem Proving

Uncertain Reasoning: $H \Rightarrow \forall x \; P(x) \;\; ?$

PLN and Theorem Proving

Uncertain Reasoning: $H \Rightarrow \forall x\ P(x)$?

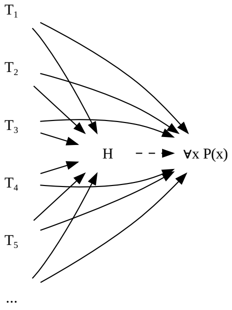
Induction + Revision:



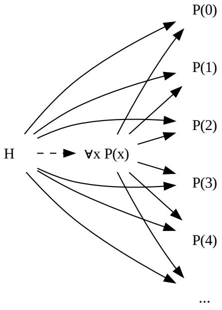
PLN and Theorem Proving

Uncertain Reasoning: $H \Rightarrow \forall x\, P(x)$?

Induction + Revision:



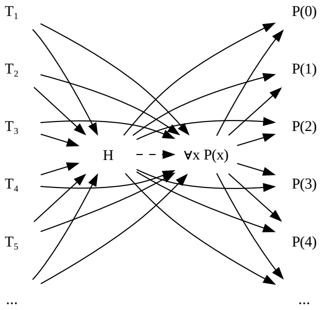
Abduction + Revision:



PLN and Theorem Proving

Uncertain Reasoning: $H \Rightarrow \forall x\ P(x)$?

Induction + Abduction + Revision:



Ternary predicate relating theories, proofs and propositions:

$$\Theta : \texttt{Theory} \times \texttt{Proof} \times \texttt{Proposition} \rightarrow \texttt{Bool}$$

- `Theory`: *Typing relationships* encoding axioms and inference rules.

  `{Z : Nat, S : Nat->Nat}`

- `Proof`: *Inhabitant* of a type.

  `(S (S (S Z)))`

- `Proposition`: *Type*.

  `Nat`

$$\Theta(\texttt{\{Z : Nat, S : Nat->Nat\}}, \texttt{(S (S (S Z)))}, \texttt{Nat}) \stackrel{m}{=} <1, 1>$$

Example (Propositional Calculus):

Instances:

$$
\Theta\ \Big(\ \{
$$

```
ax-1  : (ϕ → (ψ → ϕ)),
ax-2  : ((ϕ → (ψ → χ)) → ((ϕ → ψ) → (ϕ → χ))),
ax-3  : (((¬ ϕ) → (¬ ψ)) → (ψ → ϕ)),
ax-mp : ϕ -> (ϕ → ψ) -> ψ
},
(λ mp2.1 mp2.3 (ax-mp mp2.1 mp2.3)),
ϕ -> (ϕ → (ψ → χ)) -> (ψ → χ)
```

$$
\Big)\ \stackrel{m}{=} <1,1>
$$

Example (Propositional Calculus):

Instances:

$$
\Theta\ (\ \{
$$

$$
\begin{aligned}
&\texttt{ax-1} : (\phi \rightarrow (\psi \rightarrow \phi)),\\
&\texttt{ax-2} : ((\phi \rightarrow (\psi \rightarrow \chi)) \rightarrow ((\phi \rightarrow \psi) \rightarrow (\phi \rightarrow \chi))),\\
&\texttt{ax-3} : (((\neg \phi) \rightarrow (\neg \psi)) \rightarrow (\psi \rightarrow \phi)),\\
&\texttt{ax-mp} : \phi \texttt{ -> } (\phi \rightarrow \psi) \texttt{ -> } \psi
\end{aligned}
$$

$$
\},
$$

$$
(\lambda\ \texttt{mp2.1 mp2.3 (ax-mp mp2.3 mp2.1)}),
$$

$$
\phi \texttt{ -> } (\phi \rightarrow (\psi \rightarrow \chi)) \texttt{ -> } (\psi \rightarrow \chi)
$$

$$
)\ \stackrel{m}{=} <0,1>
$$

PLN and Theorem Proving

Crisp Patterns:

- Modus ponens:

$$\Theta(\Gamma, f, a \to b) \wedge \Theta(\Gamma, x, a) \Rightarrow \Theta(\Gamma, f(x), b)$$

$$\stackrel{\mathrm{m}}{=}$$

$$<1, 1>$$

- Existential Quantification Introduction:

$$\Theta(\Gamma, \Pi, \mathrm{T}) \wedge (\mathrm{cl}\ \Gamma) \wedge (\mathrm{cl}\ \Pi) \wedge (\mathrm{cl}\ \mathrm{T}) \Rightarrow \exists \pi\ \Theta(\Gamma, \pi, \mathrm{T})$$

$$\stackrel{\mathrm{m}}{=}$$

$$<1, 1>$$

PLN and Theorem Proving

Uncertain Patterns:

- Marginal estimate:

$$\exists \pi \; \Theta(\mathsf{PC}, \pi, \tau) \stackrel{\text{m}}{=} \; <0.001, 0.8>$$

- Conditional estimate:

$$P(\tau) \Rightarrow \exists \pi \; \Theta(\mathsf{PC}, \pi, \tau) \stackrel{\text{m}}{=} \; <0.2, 0.7>$$

Θ, Probabilistic Logic Networks (PLN)

- How likely is there a *proof* of T in Γ:

$$\exists \pi \; \Theta(\Gamma, \pi, \mathrm{T}) \; \stackrel{\mathrm{m}}{=} \; \$\mathtt{TV}$$

Θ, Probabilistic Logic Networks (PLN)

- How likely is there a *proof* of T in Γ:

$$\exists\pi\ \Theta(\Gamma, \pi, \mathrm{T}) \stackrel{\mathrm{m}}{=} \$\mathrm{TV}$$

- How likely is Π proving a *theorem* in Γ:

$$\exists\tau\ \Theta(\Gamma, \Pi, \tau) \stackrel{\mathrm{m}}{=} \$\mathrm{TV}$$

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- How likely is there a *proof* of T in Γ:

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- How likely is Π proving a *theorem* in Γ:

$$\exists \tau \ \Theta(\Gamma, \Pi, \tau) \ \stackrel{\mathrm{m}}{=} \ \$\mathrm{TV}$$

- How likely is there a *theory* in which Π proves T:

$$\exists \gamma \ \Theta(\gamma, \Pi, \mathrm{T}) \ \stackrel{\mathrm{m}}{=} \ \$\mathrm{TV}$$

Θ, Probabilistic Logic Networks (PLN)

- How likely is there a *proof* of T in Γ:

$$\exists\pi\ \Theta(\Gamma, \pi, \mathrm{T}) \ \stackrel{\mathrm{m}}{=}\ \$\mathrm{TV}$$

- How likely is Π proving a *theorem* in Γ:

$$\exists\tau\ \Theta(\Gamma, \Pi, \tau) \ \stackrel{\mathrm{m}}{=}\ \$\mathrm{TV}$$

- How likely is there a *theory* in which Π proves T:

$$\exists\gamma\ \Theta(\gamma, \Pi, \mathrm{T}) \ \stackrel{\mathrm{m}}{=}\ \$\mathrm{TV}$$

- How likely is there a *proof* of a *theorem* in a *theory* with certain *properties*:

$$\exists\gamma, \pi, \tau\ \Theta(\gamma, \pi, \tau) \land P(\gamma) \land Q(\pi) \land R(\tau) \land S(\gamma, \pi, \tau) \ \stackrel{\mathrm{m}}{=}\ \$\mathrm{TV}$$

Estimate Probability of Conjecture to be Theorem in Practice

```
(bc PARAMS THEORY (: $proof PROP))
```

Estimate Probability of Conjecture to be Theorem in Practice

```
(bc PARAMS THEORY (: $proof PROP))
```

$\downarrow$ `[$proof]` $=$ `z`

```
(bc PLN_PARAMS PLN_THEORY (: $pln_proof (≝ (∃ z (Θ [THEORY] z [PROP])) $TV))
```

Estimate Probability of Conjecture to be Theorem in Practice

```
(bc PARAMS THEORY (: $proof PROP))
```

↓ [$proof] = z

```
(bc PLN_PARAMS PLN_THEORY (: $pln_proof (≝ (∃ z (Θ [THEORY] z [PROP])) $TV))
```

↓

$TV = ?

Estimate Probability of Conjecture to be Theorem in Practice

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(bc PARAMS THEORY (: $proof PROP))
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$\downarrow$ `[$proof]` $=$ `z`

```
(bc PLN_PARAMS PLN_THEORY (: $pln_proof (≞ (∃ z (Θ [THEORY] z [PROP])) $TV))
```

$\downarrow$

`$TV` = ?

- Complete ignorance: `$TV` = $<1, 0>$

Estimate Probability of Conjecture to be Theorem in Practice

```
(bc PARAMS THEORY (: $proof PROP))
```

$\downarrow$ `[$proof]` $=$ `z`

```
(bc PLN_PARAMS PLN_THEORY (: $pln_proof (≝ (∃ z (Θ [THEORY] z [PROP])) $TV))
```

$\downarrow$

`$TV` = ?

- Complete ignorance: `$TV` $= <1, 0>$
- Complete certainty: `$TV` $= <1, 1>$

Estimate Probability of Conjecture to be Theorem in Practice

```
(bc PARAMS THEORY (: $proof PROP))
```

↓ [$proof] = z

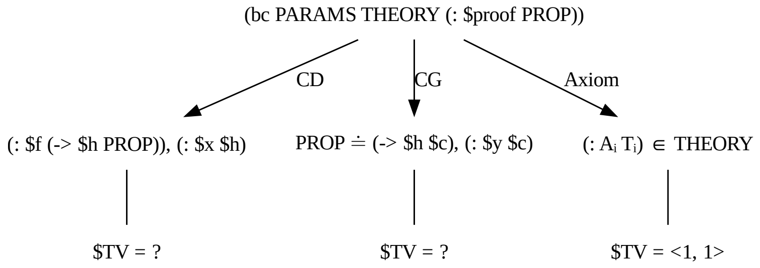
```
(bc PLN_PARAMS PLN_THEORY (: $pln_proof (≝ (∃ z (Θ [THEORY] z [PROP])) $TV))
```

↓

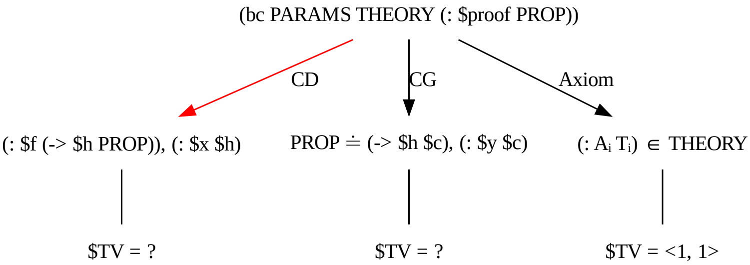
$TV = ?

- Complete ignorance: $TV $= <1, 0>$
- Complete certainty: $TV $= <1, 1>$
- Partial certainty: $TV $= <0.7, 0.8>$

Estimate Probability of Conjecture to be Theorem as Guide



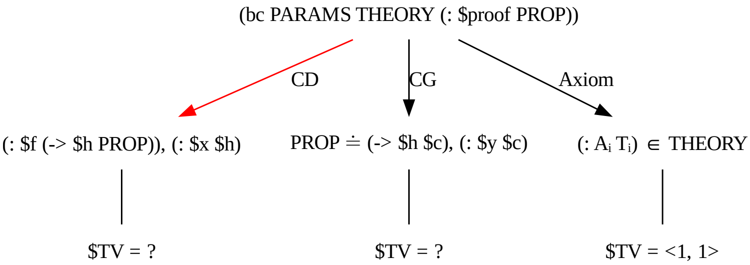
Estimate Probability of Conjecture to be Theorem as Guide



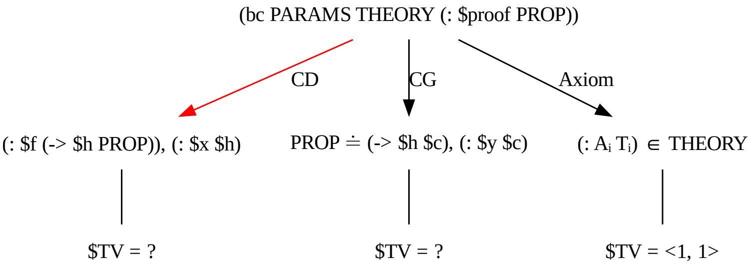
Estimate Probability of Conjecture to be Theorem as Guide

```
;; Backward Chainer
(: bc (-> $a                            ; Knowledge base space
          Nat                           ; Maximum depth
          $b                            ; Query
          $b))                          ; Result
;; Base case
(= (bc $kb $_ (: $prf $ccln)) (match $kb (: $prf $ccln) (: $prf $ccln)))
;; Recusive step (Condensed Detachment)
(= (bc $kb (S $k) (: ($prfabs $prfarg) $ccln))
   (let* (((: $prfabs (-> $prms $ccln)) (bc $kb $k (: $prfabs (-> $prms $ccln))))
          ((: $prfarg $prms) (bc $kb $k (: $prfarg $prms))))
     (: ($prfabs $prfarg) $ccln)))
```

Estimate Probability of Conjecture to be Theorem as Guide

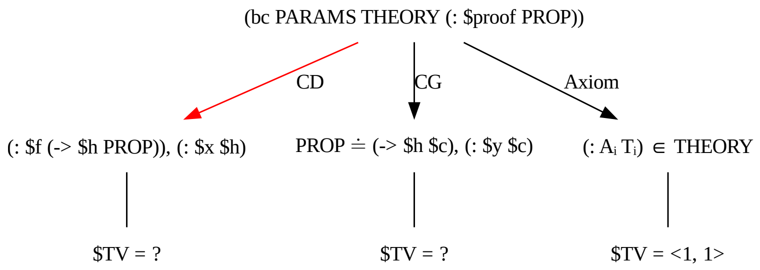


Estimate Probability of Conjecture to be Theorem as Guide




```
(bc  PLN_PARAMS PLN_THEORY
     (: $pln_proof (≞ (∃ f h x (∧  (Θ [THEORY] f (-> h [PROP]))
                                   (Θ [THEORY] x h)) $TV)))
```

Estimate Probability of Conjecture to be Theorem as Guide




```
(bc PLN_PARAMS PLN_THEORY (: $pln_proof (≞ (∃x (Θ [THEORY] x H)) $TV)))
```

Conclusion

- Early experiment (https://github.com/trueagi-io/chaining/tree/main/experimental/pln-inf-ctl)
  - Small MetaMath corpus
  - 1^{th} run: exhaustive search, *populate* Θ

```
(≞ (Θ (Cons ([:] [a1i.1] [φ]) [PC]) [a1i.1] [φ]) (STV 1 1))
(≞ (Θ [PC] [ax-3] ([→] ([→] ([¬] [φ]) ([¬] [ψ])) ([→] [ψ] [φ]))) (TV 1 1))
(≞ (Θ (Cons ([:] [a1i.1] [φ]) [PC]) [ax-1] ([→] [φ] ([→] [ψ] [φ]))) (STV 1 1))
(≞ (Θ (Cons ([:] [a1i.1] [φ]) [PC]) ([ax-mp] [a1i.1] [ax-1]) ([→] [ψ] [φ])) (STV 1 1))
...
```

  - 2^{nd} run: speed-up via *PLN existential reasoning*
  - Glorified Memoizer
- Future work
  - Large MetaMath corpus
  - Induction, abduction and more