Learning Efficient Recursive Numeral Systems via Reinforcement Learning

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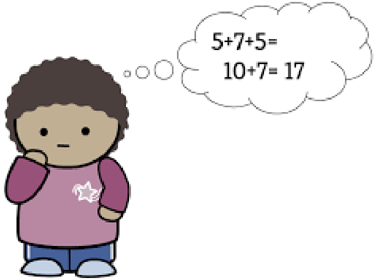
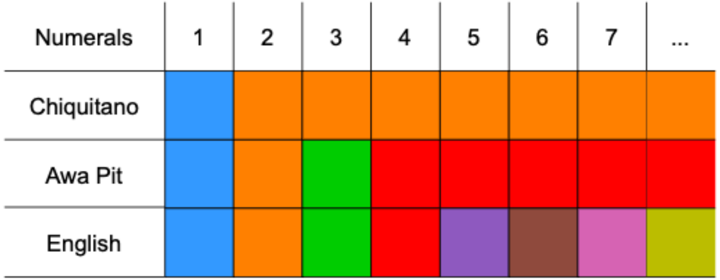


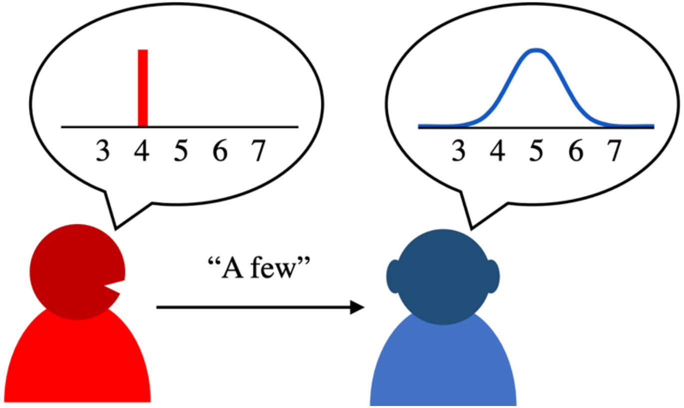
Motivation

- Animals and infants have notion of (approximate) numerosity.
- Only humans have capacity to generate **infinite set of numbers – central to human cognition.**
  - Necessary for development of sophisticated mathematics.
- How can we explain the origin and development of various kinds of number systems?

Numeral Systems

From approximate, to exact-restricted to recursive

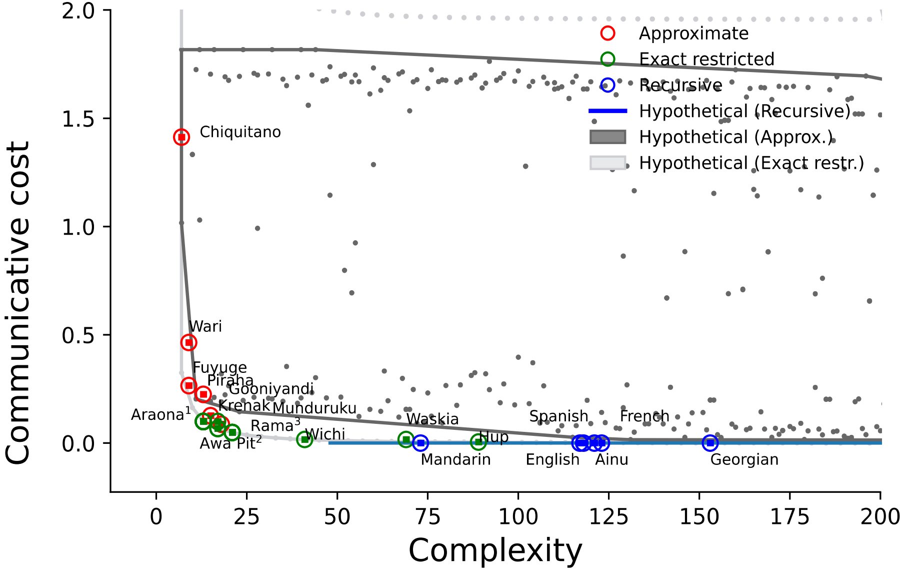




Carlsson et al., 2021

Efficient Communication

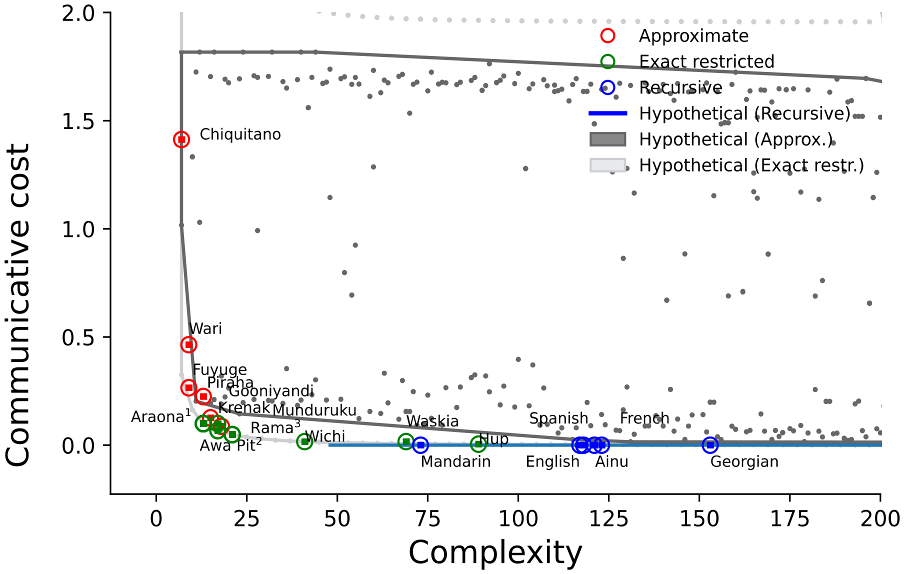
- Languages are under pressure to be **informative** and to **minimise cognitive load**.
  - Allow **precise** communication while not making the language hard to **learn and remember**.
- Multiple semantic domains show this:
  - Colour systems (Regier et al.),
  - Kinship (Kemp & Regier),
  - Sign language shapes (Yin et al.), and,
  - Numeral systems (Xu et al., Denic et al. 2024)
- Example: Eng: *Aunt* vs Swe: *Faster/Moster*



Xu et al., 2022

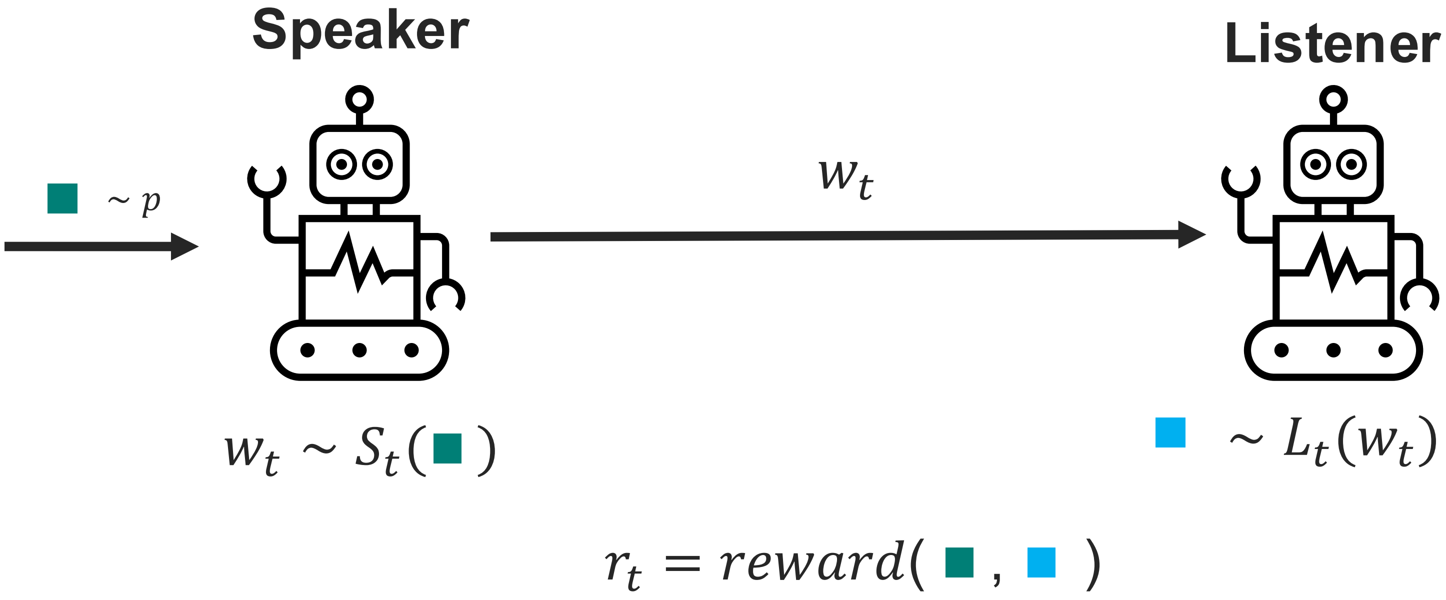
Efficient Communication

- Human number systems are efficient according to this trade-off.
- Can a model based of **reinforcement learning** and **language use in signalling games** explain this?



Xu et al., 2022

The Signaling Game



E. Carlsson, F. Johansson, D. Dubhashi, CogSci2021

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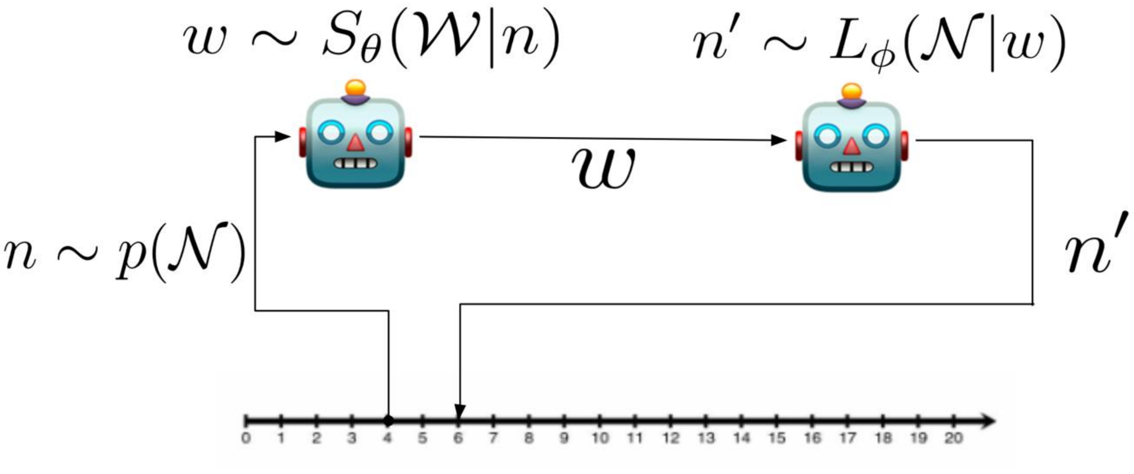
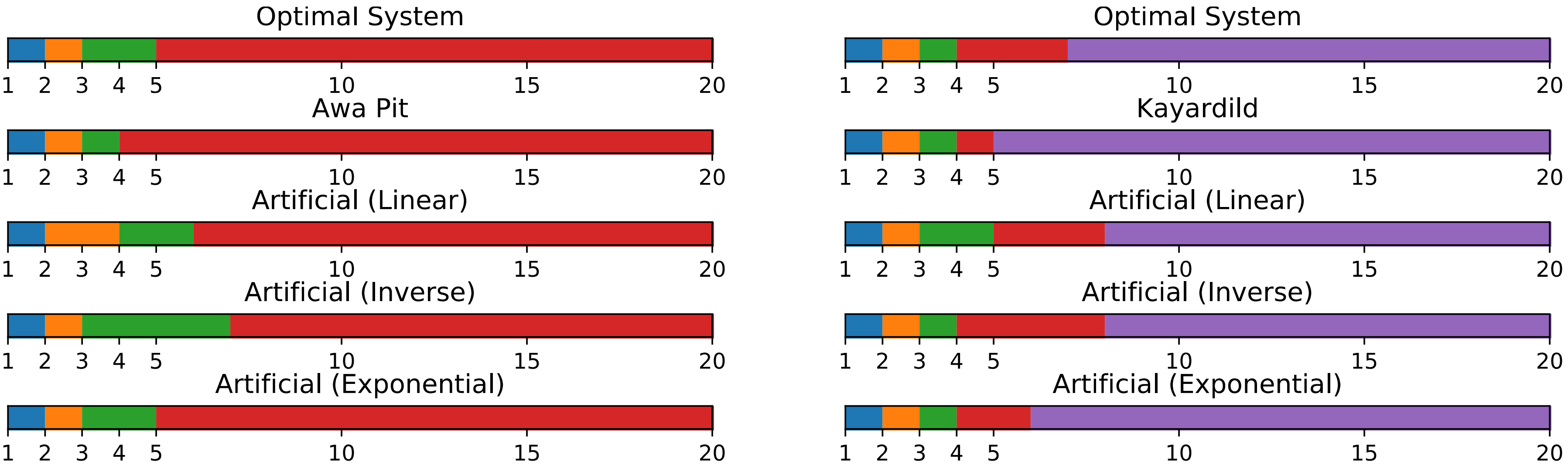
Number Game: learning approximate numbers

- Vocabulary size 3-10
- Need-probability derived from human data
  - how often a number is chosen in game
- Various reward functions



$$reward(n, n') = 1 - \frac{|n - n'|}{N}$$

Comparison to human systems



Learning Recursive Number Systems

How to explain development of recursive numbers expressing arbitrarily large numbers?

- Chomsky: innate ”merge” giving rise to successor function
  - But no traces in natural language number systems!
  - Other approach: merger between exact restricted and approximate systems?
- Denic and Szymanik (2024): Not just simplicity and informativeness!
  - Informativeness in recursive number systems is perfect – unlike approximate ones.
- Appropriate trade-off is between: **morphosyntactic complexity** and **number of lexicalised terms.**
  - **How complicated is it (on average) to construct any number using an appropriate grammar?**

Learning Recursive Number Systems

Hurford Grammar (1975)

$$\begin{aligned} \textbf{Num} &= D \mid Phrase \mid Phrase + Num \mid Phrase - Num \\ \textbf{Phrase} &= Num * M \end{aligned} \quad (1)$$

Language	D	M
English	[1, 2, 3, 4, 5, 6, 7, 8, 9, 11]	[10]
French	[1, 2, 3, 4, 5, 6, 7, 8, 9]	[10, 20]
Kunama	[1, 2, 3, 4]	[5, 10]

Example: 23 == (2 * 10) + 3

- **But what if we start from a non-efficient D, M pair?**
- **Can RL + communication lead agents to move towards an efficient recursive system?**

Morphosyntactic Complexity

- Denic et al (2024): recursive number systems in human languages optimise between **morphosyntactic complexity** and **lexicon size.**
- Want RL to find D, M pairs with **low average morphosyntactic complexity** i.e. **short expressions** in Hurford meta-grammar for most numbers.
- (2 * 10) + 3 has ms_complexity 5 in English --- simply **number of symbols in grammar needed to express** it.

$$\begin{aligned}\textbf{Num} &= D \mid \mathit{Phrase} \mid \mathit{Phrase} + \mathit{Num} \mid \mathit{Phrase} - \mathit{Num} \\ \textbf{Phrase} &= \mathit{Num} * M\end{aligned} \qquad (1)$$

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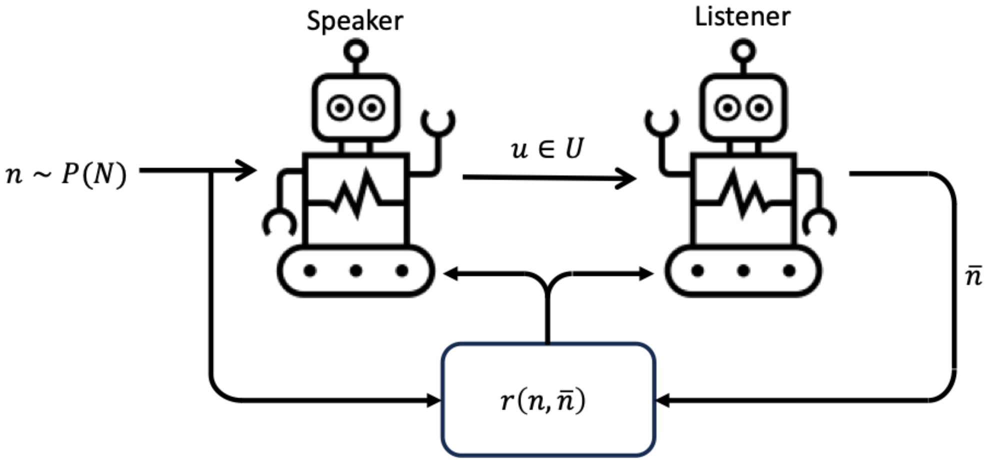
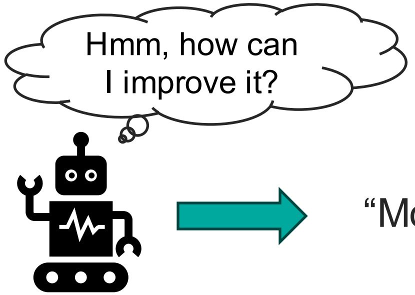
- NB: 5 has complexity 2 in Kunama (1 * 5), but 1 in English and French (5).

Signaling game with grammar modifications



Observation

$D = [1, 2, 3, 4]$
$M = [5, 6]$



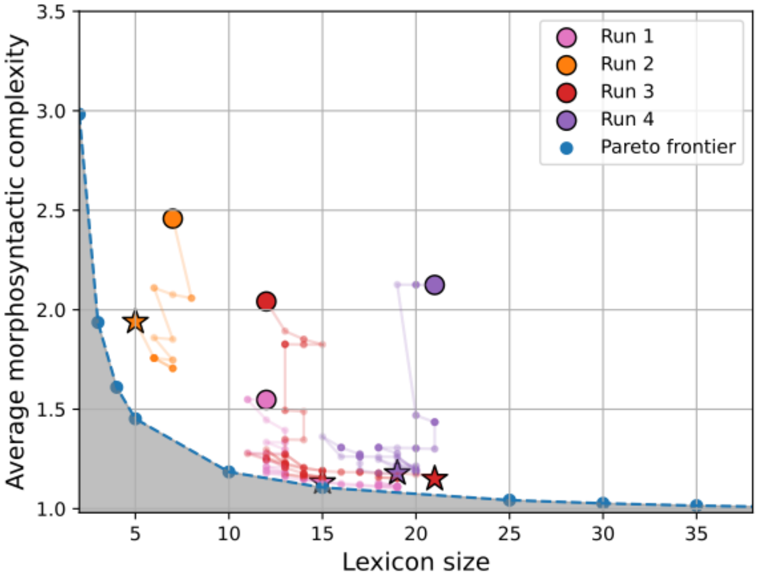
"Move 5 to *D*"

Bandits and a game between grammars

- After modification: Speaker can **explore or exploit** in choice between **old and new grammar**.
- **Bandit algorithm:** choose which arm (grammar) believed to give highest reward.
  - Reward: listener produce exactly right number.
  - Q-values updated through communication.
- Eventually, the language which is **easier for the listener to learn** will get **highest value.**
  - Keep this for next modification round.

Results: optimising alien number systems

#	*D*	*M*
1	[1, 2, 3, 20, 35, 37, 40, 47, 49]	[4, 25, 45]
2	[1, 4, 7, 8, 15]	[10, 33]
3	[1, 20, 25, 28, 31, 39, 41, 45]	[2,3,4,15]
4	[1, 4, 19, 21, 39, 40, 45, 47, 49]	[3,5,8,10,18,23,28,30,37,42,43,48]
5	[1, 2, 3, 4, 5, 10, 20, 30, 35, 40, 47, 49]	[6, 11, 13, 15, 45, 50]
6	[1, 2, 35, 37, 40, 47, 49]	[3, 5, 10, 20, 30, 40]
7	[1, 4, 12]	[9, 25]
8	[1, 4, 17, 22, 49]	[9, 10, 25, 28, 31, 41, 45]



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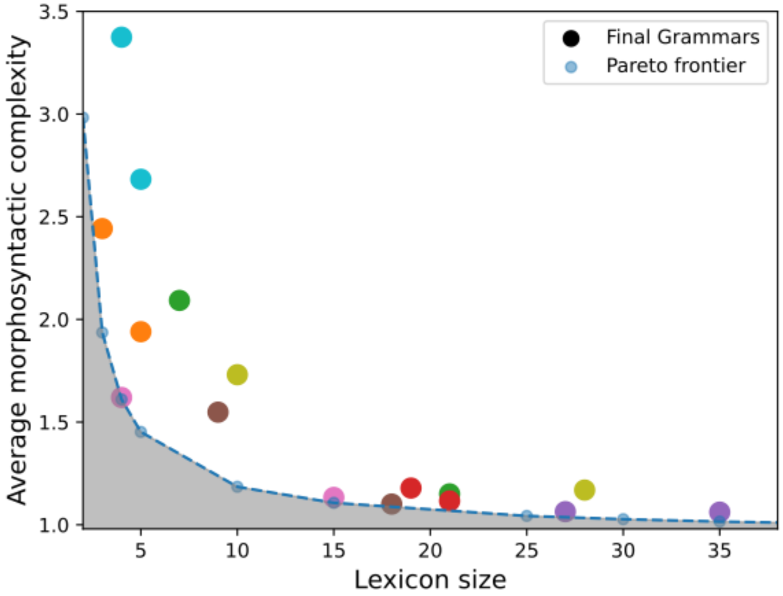

Ongoing work

- Is (average) morphosyntactic complexity enough?
- What is we also consider the “regularity” of the number line?
  - How often do we witch to construct numbers using a different grammar rule?
    - **French French vs Swiss French**
    - soixante-sept soixante-sept (67)
    - soixante-dix-sept septante-sept (77)
    - quatre-vingt-dix-sept nonante-sept (97)
- Ongoing work by in collaboration with researcher in Edinburgh.

Conclusion

- Communication Signalling Game (neural)
- + grammar modifications (symbolic)
- + RL (bandit algorithm)
- … lead to efficient recursive number systems!

Learning Efficient Recursive Numeral Systems via Reinforcement Learning. Proceedings of the Annual Meeting of the Cognitive Science Society 2025

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