10th Conference on Artificial Intelligence and Theorem Proving, 2nd September 2025, Aussois, France

Le Miz s'approche: auto-informalization and autoformalization combining Mizar and Naproche

Mario Carneiro [2] Adrian De Lon [1, 3] Atle Hahn [3] Peter Koepke [1] Josef Urban [3]

[1] Mathematical Logic Group, University of Bonn
[2] Chalmers, Gothenburg
[3] CIIRC CTU, Prague
Supported by GAČR Grant 25-17929X *NextReason*

CZECH INSTITUTE OF INFORMATICS ROBOTICS AND CYBERNETICS CTU IN PRAGUE

Example: proof task from a proof step in Naproche-ZF

Theorem (Cantor). There exists no surjection from A to 2^A.

Proof. Suppose not. Consider a surjection f from A to 2^A. Let $B = \{a \in A \mid a \notin f(a)\}$. Then $B \in 2^A$. There exists $a' \in A$ such that $f(a') = B$. Now $a' \in B$ iff $a' \notin f(a') = B$. Contradiction.

Negated conjecture from « Then $B \in 2^A$ »	$B \notin 2^A$
Global premise from a lemma	$\forall X.\, \forall Y.\, X \subseteq Y \implies X \in 2^Y$
Global premise from a definition	$\forall X. \forall X.\, X \subseteq Y \iff (\forall x.\, x \in X \implies x \in Y)$
$\vdots$	$\vdots$
Local premise from « Let $B = \cdots$ »	$\forall a.\, a \in B \iff (a \in A \wedge a \notin f(a))$
Local premise from « Consider ... »	$f \in \mathsf{Surj}(A, 2^A)$
Local premise from « Suppose not »	$\exists g.\, g \in \mathsf{Surj}(A, 2^A)$

```
definition let X,Y;
    pred X c= Y means for x being object holds x in X implies x in Y;
```

Definition. Let X, Y be sets. $X \subseteq Y$ iff for all objects x such that $x \in X$ we have $x \in Y$.

```
theorem for C being countable Language, phi wff string of C, X being set st
    X c= AllFormulasOf C & phi is X-implied holds phi is X-provable
```

Theorem (Completeness Theorem). Let L be a countable language, φ a wellformed L-formula, and Γ a set of L-formulas such that $\Gamma \vDash \varphi$. Then $\Gamma \vdash \varphi$.

```
theorem Th19:
  for T being non empty normal TopSpace, A,B being closed Subset of T st
  A <> {} & A misses B holds ex F being Function of T,R^1 st
  F is continuous & for x being Point of T holds 0 <= F.x & F.x <= 1 &
  (x in A implies F.x = 0) & (x in B implies F.x = 1)
```

Theorem (Urysohn). Let T be a non-empty normal space. Let A, B be closed subsets of T such that $A \neq \emptyset$ and $A \cap B = \emptyset$. Then there exists a continuous function f from T to $\mathbb{R}$ such that for all points x of T we have $0 \leq f(x) \leq 1$ and $x \in A \implies f(x) = 0$ and $x \in B \implies f(x) = 1$.

Why Mizar?

Large: MML is the largest quasinatural formal library with 1473 articles by over 260 authors, containing 3.6 M lines, 74 k theorems, and 15 k definitions.

Automatable: over 80% of problems from the MML can be solved by ATPs.

Set-theoretic: Mizar's Tarski–Grothendieck foundations are a stronger version of Zermelo–Fraenkel set theory which is the standard foundation of mathematics.

Pre-mapped: Journal of Formalized Mathematics publishing pipeline already includes a vocabulary mapping from Mizar identifiers to LaTeX-markup.

mizar-rs: a modern and performant reimplementation of the Mizar system by Mario

Our evil scheme (Part I): auto-informalization combining mizar-rs and Naproche-ZF

Obtain a vocabulary mapping from Mizar identifiers to patterns of LaTeX markup.

Implement a bidirectional verifiability-preserving syntactic translation from the Mizar language to controlled natural language, adapted from the front-end of Naproche-ZF.

Make the translated result more readable (with simple heuristics, LLMs, manual editing, *&c.*).

Give the theorems useful names and labels (instead of `Th01`,`Th02`,...).

⋮

The result should be understandable to mathematicians unfamiliar with Mizar or other proof assistants.

Vocabulary mapping (11k+) derived from the publishing process for Formalized Mathematics

(functor)	`[: A,B :]`	$A \times B$	(mixfix)
(functor)	`X "/\" Y`	$X \sqcap Y$	(mixfix)
(relation)	`A c= B`	$A \subseteq B$	(relation)
(relation)	`a,b equiv c,d`	$\overline{ab} \cong \overline{cd}$	(predicate)
(relation)	`f unifies t1,t2`	f unifies t_1 with t_2	(verb)
(relation)	`x is_/\-reducible_in X`	x is $\cap$-reducible in X	(adjective)
(mode)	`language of Y,S`	language over Y and S	(noun)
(mode)	`Homomorphism of G,H`	homomorphism from G to H	(noun)
(attribute)	`subst-forex`	$\forall$-$\exists$-substituting	(adjective)
(attribute)	`k-halting`	k-halting	(adjective)

Our evil scheme (Part II): proof automation and compression

Integrate ATPs into the Mizar checker.

Semi-automatically compress existing Mizar proofs to make their level of detail more natural.

Can we develop systematic criteria for *naturalness* of proofs?

Mario's mizar-rs can check the MML in under 3 minutes on a 128-thread CPU (most of the library finishes under a minute, but there are a couple of articles that are stragglers).

Grammar-based parser for the controlled language adds a bit of overhead compared to an optimized parser for the simpler quasinatural Mizar language.

We have to be smart about using ATPs and cache their results.

Our evil scheme (Part III): LLM-based autoformalization in controlled natural language

Controlled natural language is a promising target for autoformalization, since LLMs have seen much more natural language mathematics in their training data than formal mathematics.

Restricting to controlled language via prompting works quite well.

Non-controlled sentences generated by the LLM can indicate where to extend the grammar.

One could use grammar augmentation (syntactically constrained sampling) to force controlled natural language output.

Other difficulties of autoformalization remain: global coherence, implicitness, high-level informal reasoning, *&c.*

More on Thursday in Atle's talk!

Thanks!