Le Miz s'approche: Informalization and Autoformalization with Mizar and Naproche

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1 Introduction

Mizar is an influential interactive theorem proving system with a rich history dating back to 1973 [18, 16]. Its main library, the Mizar Mathematical Library (MML) [11], is one of the largest repositories of formalized mathematics. Mizar uses a quasinatural input language, familiar foundations in set theory and classical logic, and a declarative proof format based on natural deduction. Mizar's emphasis on naturalness, which allows its users to approximate informal mathematical writing to a certain extent, along with the large scale of the MML, makes it a compelling subject for research into proof automation, informalization, and autoformalization.

Naproche is an experimental interactive theorem prover that places even greater emphasis on naturalness [2, 9]. Naproche uses a controlled natural language (a sublanguage of ordinary mathematical English, using LaTeX syntax) as its input format and uses automated provers (such as E [10] and Vampire [15]) as *hammers* [13] to verify proof steps. This makes it possible to write formalizations that are both machine-verifiable and human-readable. However, the scale of Naproche's library of formalizations is still far from the large libraries of formalized mathematics of other systems.

We are initiating a project to extend the existing toolbox around the MML by incorporating ideas from Naproche. Our goal is to develop new tools focused on informalization to controlled natural language and autoformalization using large language models. This project will explore whether the Naproche approach can scale to large libraries.

2 Project Description

2.1 Controlled Informalization

MML articles are published in the journal *Formalized Mathematics* [1]. During the publishing process, articles are syntactically transformed to documents resembling typical mathematical articles, using a vocabulary mapping from Mizar identifiers to LaTeX patterns [17, 12]. Because many MML articles use the quasinatural features of the Mizar language to model informal mathematical idioms, the result is often reasonably legible to non-Mizar users. However, this translation is lossy; it omits many parts of the original article, including proof steps that would be important to a human reader.

We are building a new semantically lossless (and hence reversible) informalization tool that will target a Naproche-like controlled natural language embedded into LaTeX. This *controlled informalization* of the MML will be verifiability-preserving, since the target language is a formal controlled language and the translation is reversible; the result should be readable to humans while remaining verifiable by an extended Mizar `verifier`.

This informalization of the MML will be a large-scale project, resulting in a parallel corpus of Mizar formalizations and their controlled natural language equivalents. In cases where Mizar articles closely follow existing mathematical writing, we will even obtain a triform corpus. We will also experiment with informalizing ATP proofs using this new controlled natural language.

The initial version of the tool will use classical methods and simple heuristics. We are directly building on the existing vocabulary mapping used in *Formalized Mathematics*. For our initial implementation, we are reusing the natural language processing facilities of Naproche-ZF [4], adapted to use a JSON representation of Mizar's abstract syntax to interface with `mizar-rs`[6]. We have completed a mapping of all Mizar identifiers used in the MML to Naproche-compatible mixfix patterns in LaTeX. This lexicon contains over 10 k symbols and accounts for cases where Mizar symbols have different meanings based on their arity or fixity. To obtain this mapping, we used Mistral Large and in-context learning (with a handful of translation examples in the prompt) to create suitable Naproche representations from the Mizar identifiers and their LaTeX representation used in the publication process of *Formalized Mathematics*. After some additional post-processing and manual editing, the resulting large lexicon is now fully parsable by our informalization tool. Our bidirectional translation tool is currently in an early stage and does not yet cover the full Mizar language.

2.2 Proof Summarization

Since our initial controlled informalization will be semantically lossless, the informalized proofs will typically contain many details irrelevant to a human reader. To address this, we will develop *proof summarization* tools.

Mizar proofs consist of small logical steps corresponding to *obvious inferences* [20] acceptable to the Mizar `verifier`. With modern automated theorem proving, much larger proof steps are now verifiable [3, 7, 5, 8].

A representation of MML proofs in controlled language will allow us to use both direct syntactic transformations (e.g. deletion of justifications, proof steps, or entire subproofs) and LLM-based rephrasing and summarization to search for shorter proofs that can still be verified by ATPs. Direct transformations have already been used successfully by MML maintainers to refactor proofs automatically [14], so we expect that strengthening the `verifier` with ATPs will allow us to further lower the de Bruijn factor [19] of MML formalizations.

We will also need to develop further criteria and metrics to evaluate the naturalness of proofs, since verifiability by an ATP does not necessarily correspond to comprehensibility or acceptability to a human reader.

2.3 Autoformalization

As a sublanguage of ordinary mathematical English, the controlled natural language will make for a promising target format for autoformalization. We will also explore direct autoformalization with the original Mizar language to have a point of comparison.

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