On Lemma Conjecturing using Neural, Symbolic and Neuro-symbolic approaches

Sólrún Halla Einarsdóttir, Yousef Alhessi,
Emily First, Moa Johansson

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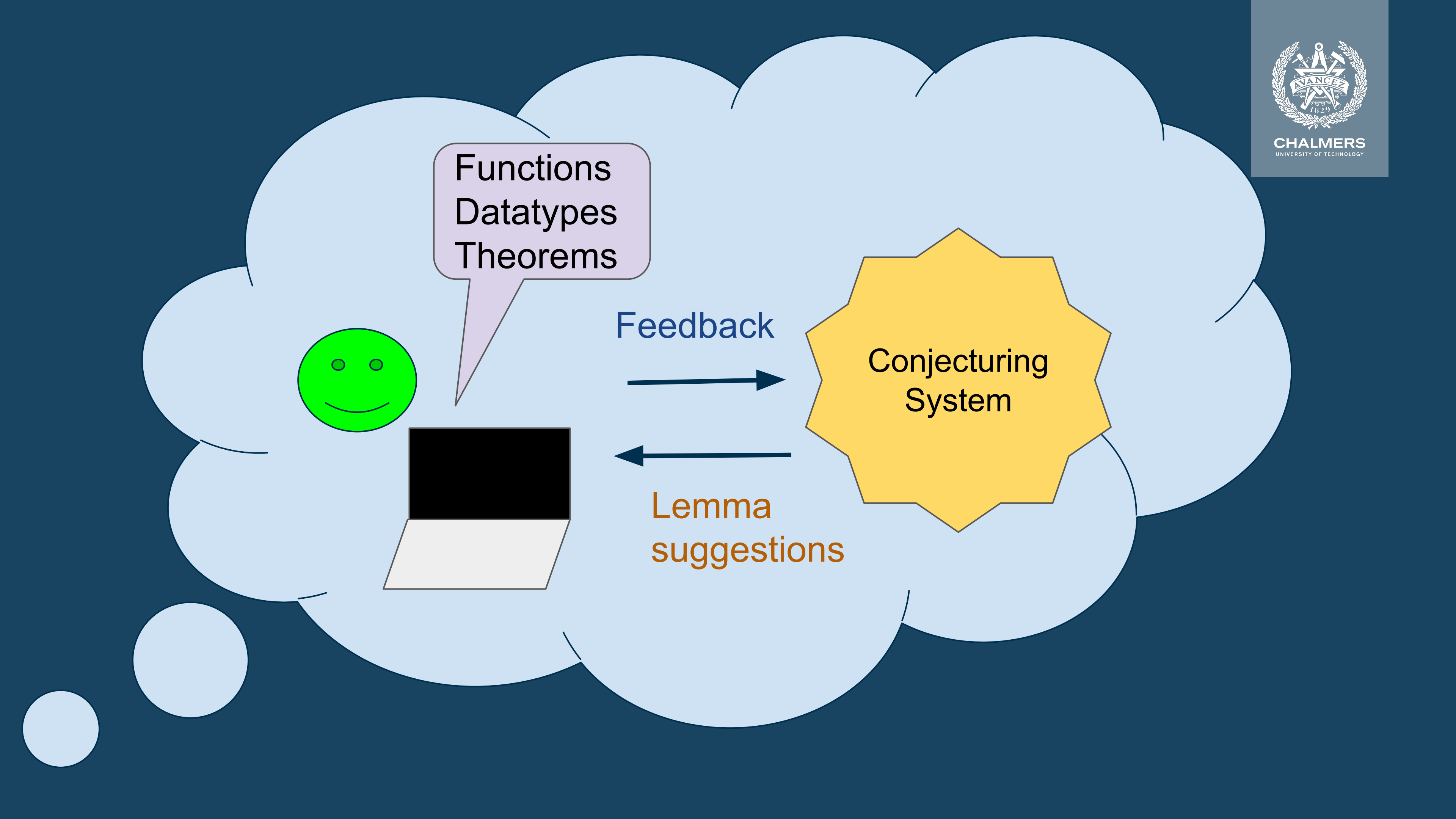
Functions
Datatypes
Theorems
Feedback
Conjecturing System
Lemma suggestions
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JUNE 8, 2024 | 12 MIN READ

AI Will Become Mathematicians' 'Co-Pilot'

Fields Medalist Terence Tao explains how proof checkers and AI programs are dramatically changing mathematics

BY CHRISTOPH DRÖSSER

```
rev [] = []
rev (x : xs) = (rev xs) ++ (x :
[])

[] ++ xs = xs
(x : xs) ++ ys = x : (xs ++ ys)
```

QuickSpec

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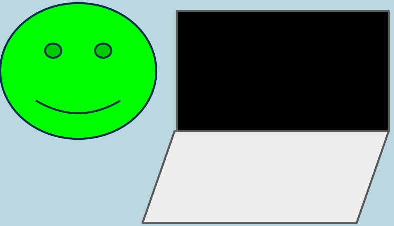
```
1. rev [] = []
2. x ++ [] = x
3. [] ++ x = x
4. rev (rev x) = x
5. rev (x : []) = x : []
6. (x ++ y) ++ z =
   x ++ (y ++ z)
7. x : (y ++ z) = (x : y) ++ z
8. rev x ++ rev y =
   rev (y ++ x)
9. (xs ++ (y : (z : []))) =
   rev (z : (y : (rev xs)))
```



QuickSpec

- Leverages property-based testing (QuickCheck) and equational reasoning (Twee) to generate equational properties.

- Has been used to generate lemmas to automate inductive proofs.
  (most recently with Vampire, see IJCAR ‘24 paper)

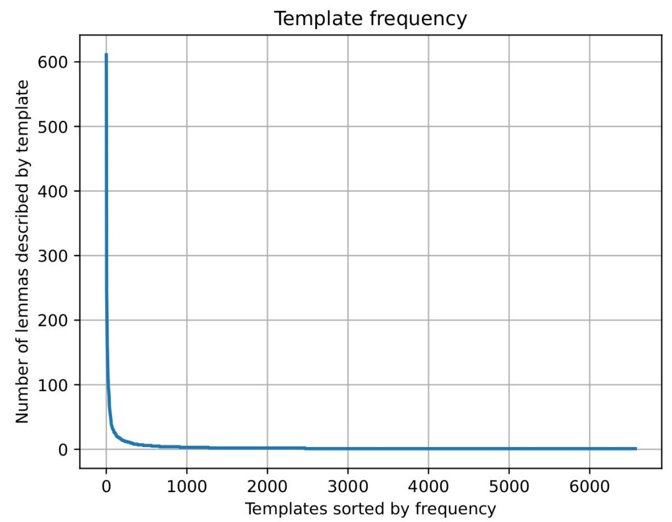
- Hipster: Isabelle tool for automated inductive proof using QuickSpec.

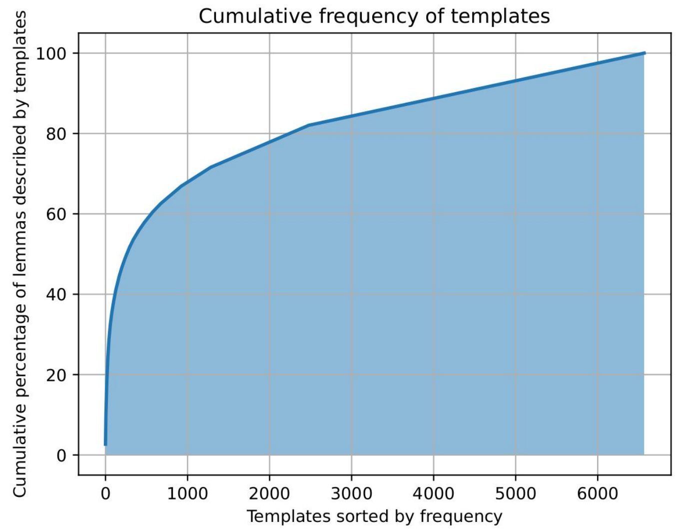
- Downsides: Becomes less efficient as the number of functions in scope grows.

RoughSpec

- Searches for conjectures that match a given template, e.g.
  - `?F(?F(x,y),z) = ?F(x,?F(y,z))`
  - Given `++`,`rev` and this template we'd find

    `(x ++ y) ++ z = x ++ (y ++ z)`
- A small number of templates can provide many lemmas
- How can we automatically come up with good templates to use for a new theory?





Symbolic vs. Neural conjecturing

Neural conjecturing:

Strengths

- Unrestricted in shape and size of conjectures.
- Can use information from names.

Weaknesses

- Prone to repetition.
- Generate false conjectures.
- Needs computational resources.

Symbolic conjecturing:

Strengths

- (More-or-less) true conjectures.
- Avoid repetition.
- Runs locally on a laptop.

Weaknesses

- Restricted in size and shape.
- Can't use name information.

Neuro-Symbolic Conjecturing: using RoughSpec!

- What if we get an LLM to generate templates for conjectures, which can then be filled in by RoughSpec?
- LLMs are good at capturing patterns/intuition.
- Compare to Neural-only approach: train a model to generate conjectures.
- Compare to symbolic-only approach:
  - QuickSpec
  - RoughSpec using simple heuristics/statistical analysis to choose templates.

Ongoing experiments: Neural Only Conjecturing

Fine-tuning Facebook OPT 1.3B parameter model

Data: Isabelle-HOL Library (around 30k examples)

Input: Symbols, Definitions	Target: Lemma Statement

(Near) Future: Bigger model, more data (AFP)

Evaluation

- Syntax-checking
- Counterexample-checking

(Near) Future:

- Provable?
- Trivial?
- Coverage?
- Usefulness in proof automation?

# Generated conjectures	3062
Pass syntax check	375
Pass counterexample check	163

```
"x \<and> y => x \<and> y"
```

```
"sq x * sq y = sq (x + y)"
```

Challenges/ Future Work

- Workflow: How do we interact with the LLM?
  - Sample once or many times, interleave/iterate back and forth?
  - Predict lemmas independently or many at a time?
  - Conditioned prediction?
  - What context to provide?
- How to evaluate lemma quality/interestingness?
- Training data leakage?
- Could this be extended to support other languages (Lean, Coq)?