Proof Recommendation System for HOL4 Theorem Prover

Nour Dekhil, Adnan Rashid, and Sofiène Tahar

Department of Electrical and Computer Engineering
Concordia University, Montreal, QC, Canada

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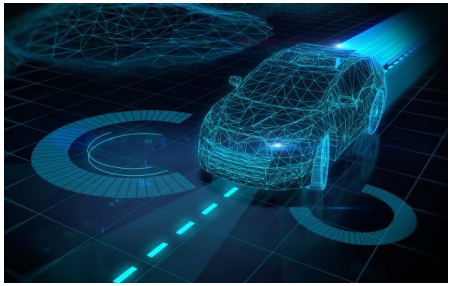
Why formal verification is so important?

> Ensuring the correct functionality of complex software systems or critical hardware components is crucial.

Automotive

Healthcare



Aerospace

> **Interactive Theorem Provers** (ITPs) help apply mathematical logic to verify these systems.

Challenges of ITPs

Challenging and labor-intensive !

Smart copilot that guides us through theorem proving.

Our Objective

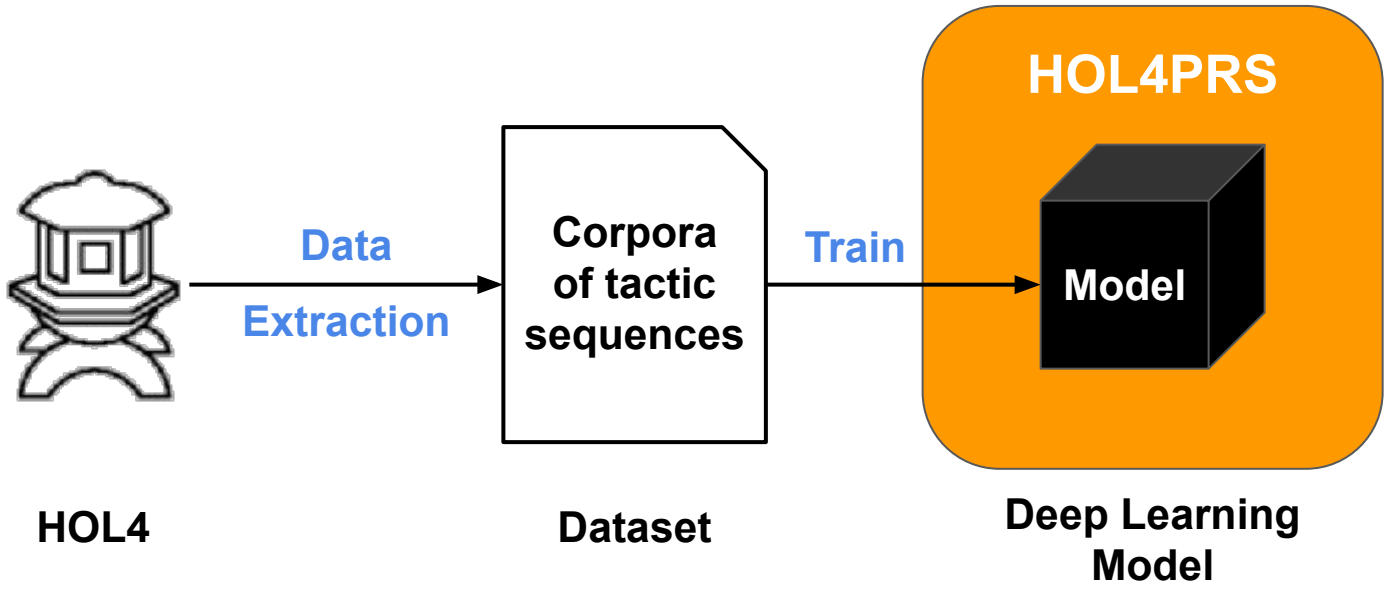
- Leverage Large Language Models (LLMs) capabilities to make proof writing more efficient.

- Recommend the best HOL4 proof step to use.

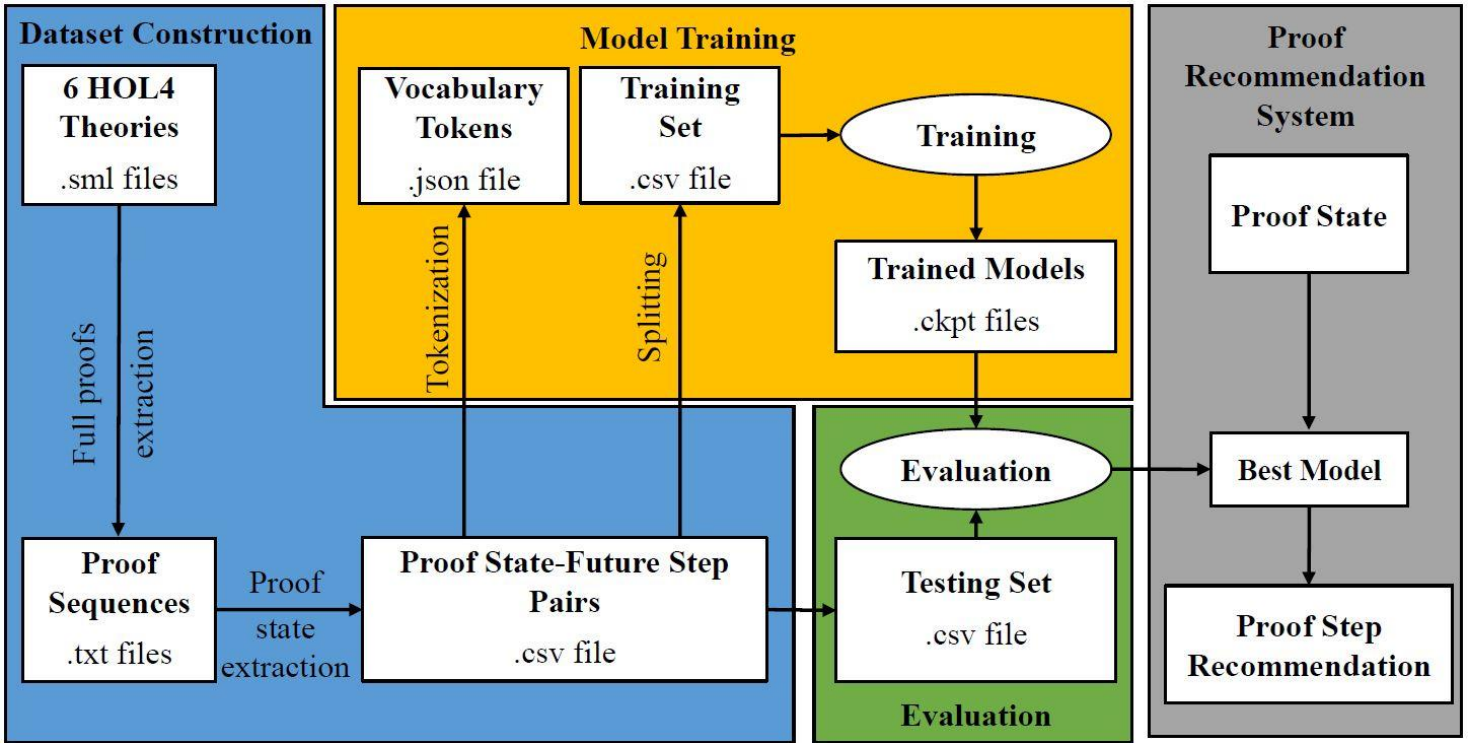
Related Work

Related Work	Approach	Prover	Success Rate
Gauthier et al., 2020	Machine Learning (k-NN)	HOL4	66.4%
Blaauwbroek et al., 2020	Machine Learning	Coq	23.4%
Luan et al., 2021	Deep Learning (LSTM)	Coq	87%
Yeh et al., 2023	Deep Learning (T5)	PVS	70%

HOL4PRS: Proof Recommendation System for HOL4 Theorem Prover



HOL4PRS Tool Structure

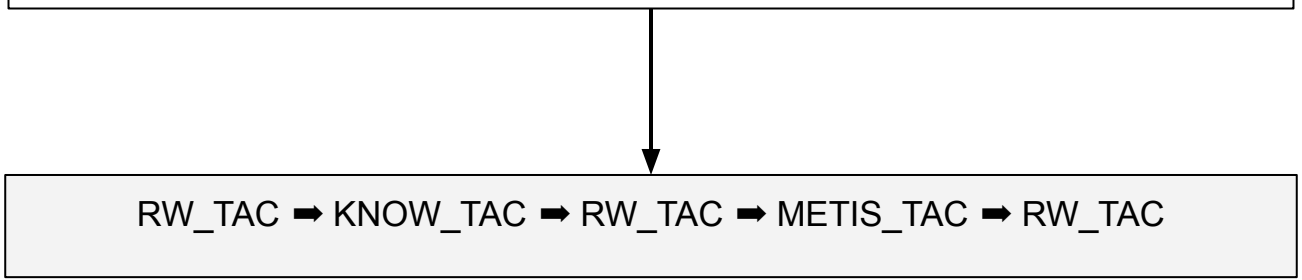


Dataset Construction

.*sml* File

```
val rule_18 = store_thm("rule_18",
  ``!X Y. D_AND X Y = D_OR (P_AND X Y)(P_AND Y X)``,

RW_TAC std_ss[D_AND_def, P_AND_def, D_OR_def]
THEN KNOW_TAC(``  !s.( max (X s) (Y s)) =
    (
         min (if X s <= Y s then Y s else PosInf)
           (if Y s <= X s then X s else PosInf))``)
    THEN1 (RW_TAC std_ss[extreal_max_def, extreal_min_def] THEN
METIS_TAC[extreal_lt_def, extreal_le_def, lt_le, le_lt, le_trans,
lt_trans, lt_infty, le_infty])
THEN RW_TAC std_ss[] ) ;
```



Tactic Sequence

Datasets

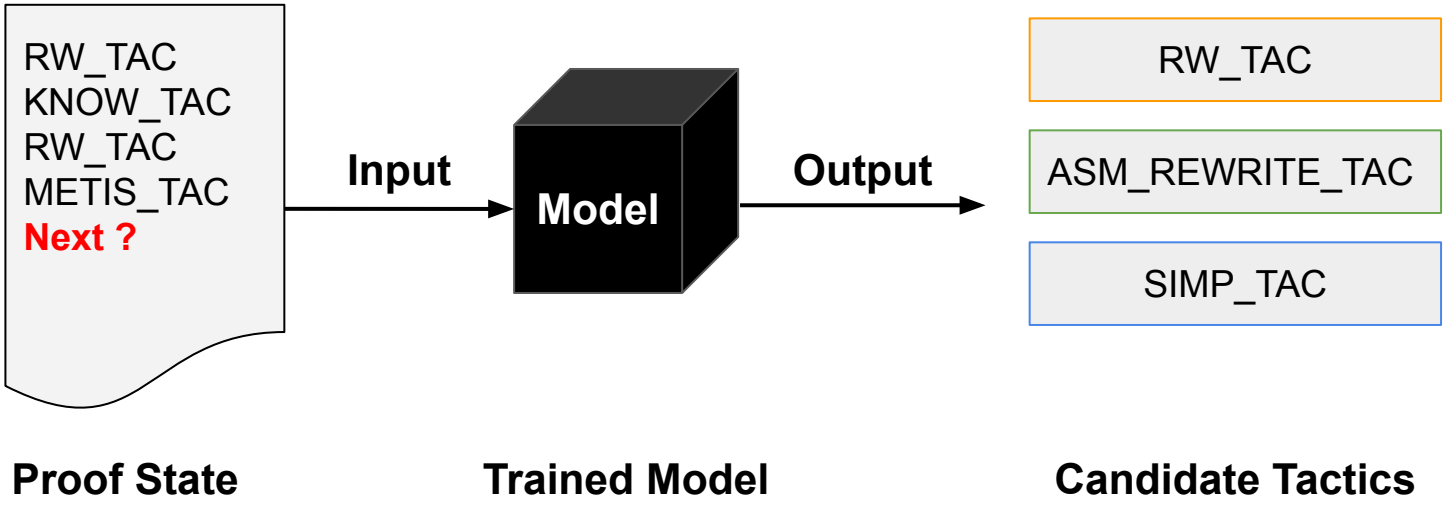
Proof: A sequence of n tactics used to prove a theorem.

Proof States: All possible tactic sequences recorded from a proof.

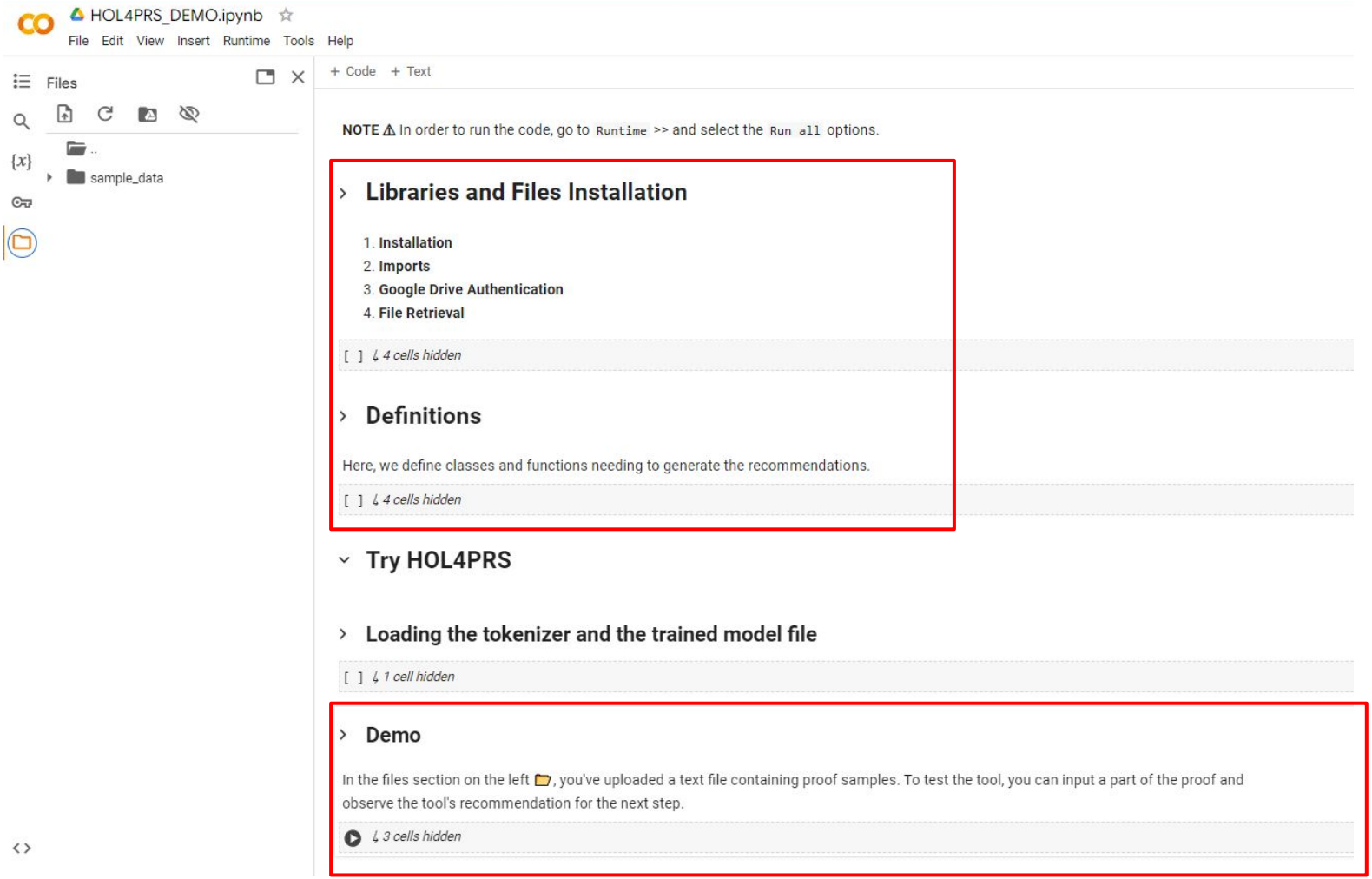
	Dataset 1	Dataset 2	Dataset 3	Dataset 4	Dataset 5	Dataset 6	Dataset 7
Proofs	1,873	2,475	153	295	61	279	5,136
Proof States	43,167	57,602	2,973	7,371	1,784	3,259	116,156

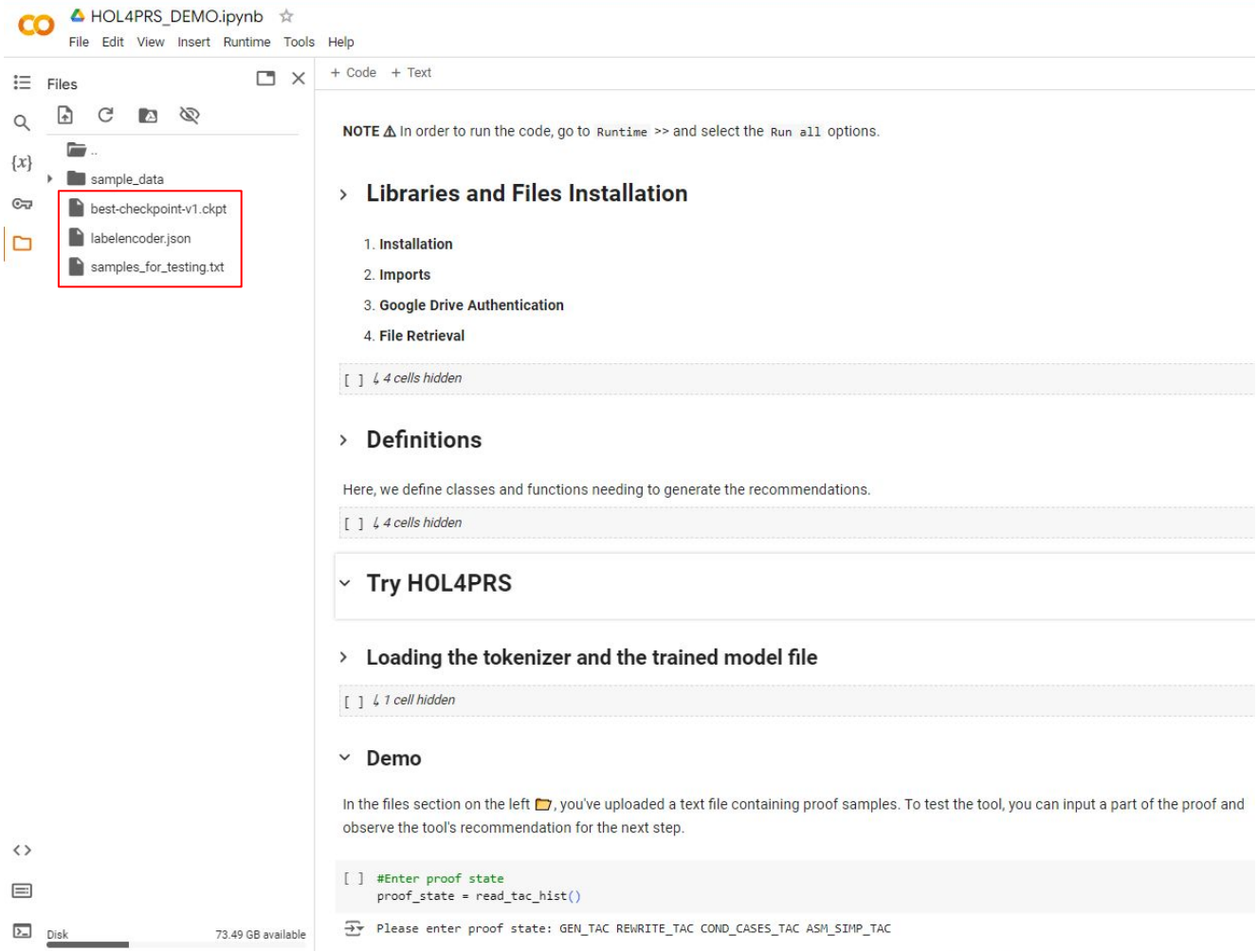
Dataset 7 is a combination of the Datasets 1 to 6.

Train Model


RW_TAC
KNOW_TAC
RW_TAC
METIS_TAC
Next ?
Input
Model
Output
RW_TAC
ASM_REWRITE_TAC
SIMP_TAC
Proof State
Trained Model
Candidate Tactics

Demo


HOL4PRS_DEMO.ipynb
File Edit View Insert Runtime Tools Help
Files
sample_data
+ Code + Text
NOTE ⚠ In order to run the code, go to Runtime >> and select the Run all options.
Libraries and Files Installation
1. Installation
2. Imports
3. Google Drive Authentication
4. File Retrieval
[ ] 4 cells hidden
Definitions
Here, we define classes and functions needing to generate the recommendations.
[ ] 4 cells hidden
Try HOL4PRS
Loading the tokenizer and the trained model file
[ ] 1 cell hidden
Demo
In the files section on the left, you've uploaded a text file containing proof samples. To test the tool, you can input a part of the proof and observe the tool's recommendation for the next step.
3 cells hidden


HOL4PRS_DEMO.ipynb
File Edit View Insert Runtime Tools Help
Files
sample_data
best-checkpoint-v1.ckpt
labelencoder.json
samples_for_testing.txt
Disk 73.49 GB available
+ Code + Text
NOTE ⚠ In order to run the code, go to Runtime >> and select the Run all options.
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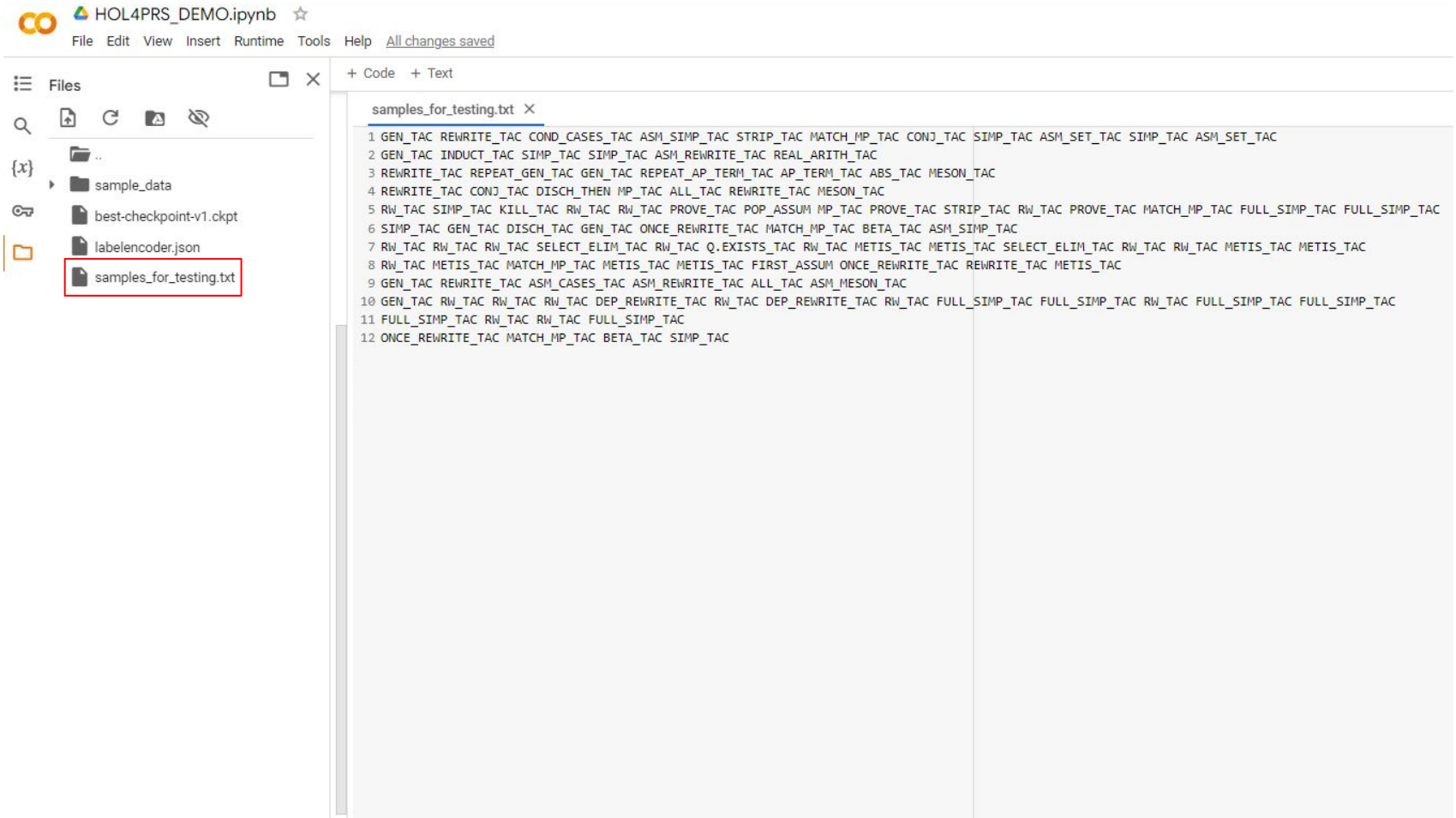
Demo
In the files section on the left, you've uploaded a text file containing proof samples. To test the tool, you can input a part of the proof and observe the tool's recommendation for the next step.
[ ] #Enter proof state
proof_state = read_tac_hist()
Please enter proof state: GEN_TAC REWRITE_TAC COND_CASES_TAC ASM_SIMP_TAC


HOL4PRS_DEMO.ipynb
File Edit View Insert Runtime Tools Help All changes saved
Files
+ Code + Text
sample_data
best-checkpoint-v1.ckpt
labelencoder.json
samples_for_testing.txt
samples_for_testing.txt
1 GEN_TAC REWRITE_TAC COND_CASES_TAC ASM_SIMP_TAC STRIP_TAC MATCH_MP_TAC CONJ_TAC SIMP_TAC ASM_SET_TAC SIMP_TAC ASM_SET_TAC
2 GEN_TAC INDUCT_TAC SIMP_TAC SIMP_TAC ASM_REWRITE_TAC REAL_ARITH_TAC
3 REWRITE_TAC REPEAT_GEN_TAC GEN_TAC REPEAT_AP_TERM_TAC AP_TERM_TAC ABS_TAC MESON_TAC
4 REWRITE_TAC CONJ_TAC DISCH_THEN MP_TAC ALL_TAC REWRITE_TAC MESON_TAC
5 RW_TAC SIMP_TAC KILL_TAC RW_TAC RW_TAC PROVE_TAC POP_ASSUM MP_TAC PROVE_TAC STRIP_TAC RW_TAC PROVE_TAC MATCH_MP_TAC FULL_SIMP_TAC FULL_SIMP_TAC
6 SIMP_TAC GEN_TAC DISCH_TAC GEN_TAC ONCE_REWRITE_TAC MATCH_MP_TAC BETA_TAC ASM_SIMP_TAC
7 RW_TAC RW_TAC RW_TAC SELECT_ELIM_TAC RW_TAC Q.EXISTS_TAC RW_TAC METIS_TAC METIS_TAC SELECT_ELIM_TAC RW_TAC RW_TAC METIS_TAC METIS_TAC
8 RW_TAC METIS_TAC MATCH_MP_TAC METIS_TAC METIS_TAC FIRST_ASSUM ONCE_REWRITE_TAC REWRITE_TAC METIS_TAC
9 GEN_TAC REWRITE_TAC ASM_CASES_TAC ASM_REWRITE_TAC ALL_TAC ASM_MESON_TAC
10 GEN_TAC RW_TAC RW_TAC RW_TAC DEP_REWRITE_TAC RW_TAC DEP_REWRITE_TAC RW_TAC FULL_SIMP_TAC FULL_SIMP_TAC RW_TAC FULL_SIMP_TAC FULL_SIMP_TAC
11 FULL_SIMP_TAC RW_TAC RW_TAC FULL_SIMP_TAC
12 ONCE_REWRITE_TAC MATCH_MP_TAC BETA_TAC SIMP_TAC

Demo

In the files section on the left 📂, you've uploaded a text file containing proof samples. To test the tool, you can input a part of the proof and observe the tool's recommendation for the next step.

```
#Enter proof state
proof_state = read_tac_hist()
```

```
Please enter proof state:
```

```
[ ] #Generate recommendation
    get_recom(proof_state, roberta_model, tokenizer)
```

samples_for_testing.txt

```
1 GEN_TAC REWRITE_TAC COND_CASES_TAC ASM_SIMP_TAC STRIP_TAC MATCH_MP_TAC CONJ_TAC SIMP_TAC ASM_SET_TAC SIMP_TAC ASM_SET_TAC
2 GEN_TAC INDUCT_TAC SIMP_TAC SIMP_TAC ASM_REWRITE_TAC REAL_ARITH_TAC
3 REWRITE_TAC REPEAT_GEN_TAC GEN_TAC REPEAT_AP_TERM_TAC AP_TERM_TAC ABS_TAC MESON_TAC
4 REWRITE_TAC CONJ_TAC DISCH_THEN MP_TAC ALL_TAC REWRITE_TAC MESON_TAC
5 RW_TAC SIMP_TAC KILL_TAC RW_TAC RW_TAC PROVE_TAC POP_ASSUM MP_TAC PROVE_TAC STRIP_TAC RW_TAC PROVE_TAC MATCH_MP_TAC FULL_SIMP_TAC FULL_SIMP_TAC
6 SIMP_TAC GEN_TAC DISCH_TAC GEN_TAC ONCE_REWRITE_TAC MATCH_MP_TAC BETA_TAC ASM_SIMP_TAC
7 RW_TAC RW_TAC RW_TAC SELECT_ELIM_TAC RW_TAC Q.EXISTS_TAC RW_TAC METIS_TAC METIS_TAC SELECT_ELIM_TAC RW_TAC RW_TAC METIS_TAC METIS_TAC
```

Demo

In the files section on the left 📂, you've uploaded a text file containing proof samples. To test the tool, you can input a part of the proof and observe the tool's recommendation for the next step.

```
[10] #Enter proof state
     proof_state = read_tac_hist()

Please enter proof state: REWRITE_TAC CONJ_TAC DISCH_THEN MP_TAC ALL_TAC REWRITE_TAC
```

```
#Generate recommendation
get_recom(proof_state, roberta_model, tokenizer)

HOL4PRS reccommendations are:  ['MAPEVERY', 'MESON_TAC', 'REALARITH_TAC', 'REPEAT GEN_TAC', 'REPEAT STRIP_TAC', 'REWRITE_TAC', 'SET_TAC']
```

samples_for_testing.txt

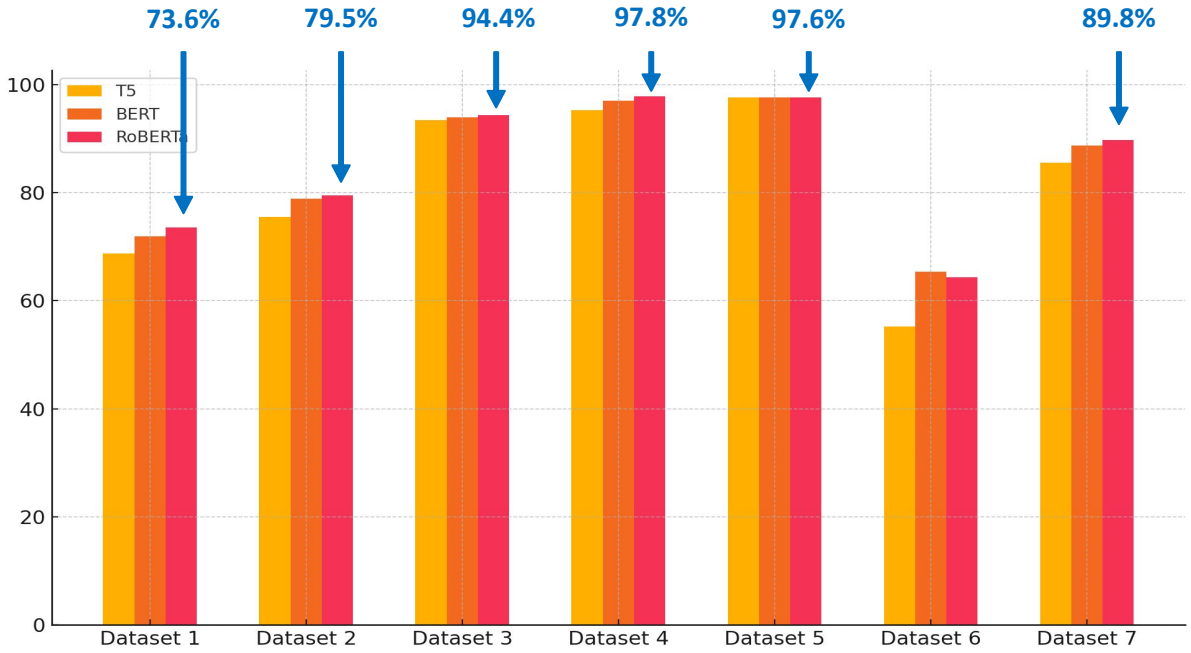
```
1 GEN_TAC REWRITE_TAC COND_CASES_TAC ASM_SIMP_TAC STRIP_TAC MATCH_MP_TAC CONJ_TAC SIMP_TAC ASM_SET_TAC SIMP_TAC ASM_SET_TAC
2 GEN_TAC INDUCT_TAC SIMP_TAC SIMP_TAC ASM_REWRITE_TAC REAL_ARITH_TAC
3 REWRITE_TAC REPEAT_GEN_TAC GEN_TAC REPEAT_AP_TERM_TAC AP_TERM_TAC ABS_TAC MESON_TAC
4 REWRITE_TAC CONJ_TAC DISCH_THEN MP_TAC ALL_TAC REWRITE_TAC MESON_TAC
5 RW_TAC SIMP_TAC KILL_TAC RW_TAC RW_TAC PROVE_TAC POP_ASSUM MP_TAC PROVE_TAC STRIP_TAC RW_TAC PROVE_TAC MATCH_MP_TAC FULL_SIMP_TAC FULL_SIMP_TAC
6 SIMP_TAC GEN_TAC DISCH_TAC GEN_TAC ONCE_REWRITE_TAC MATCH_MP_TAC BETA_TAC ASM_SIMP_TAC
7 RW_TAC RW_TAC RW_TAC SELECT_ELIM_TAC RW_TAC Q.EXISTS_TAC RW_TAC METIS_TAC METIS_TAC SELECT_ELIM_TAC RW_TAC RW_TAC METIS_TAC METIS_TAC
```

Experimental Results

N-Correctness Rate: Probability that the correct tactic is among the top N recommendations.

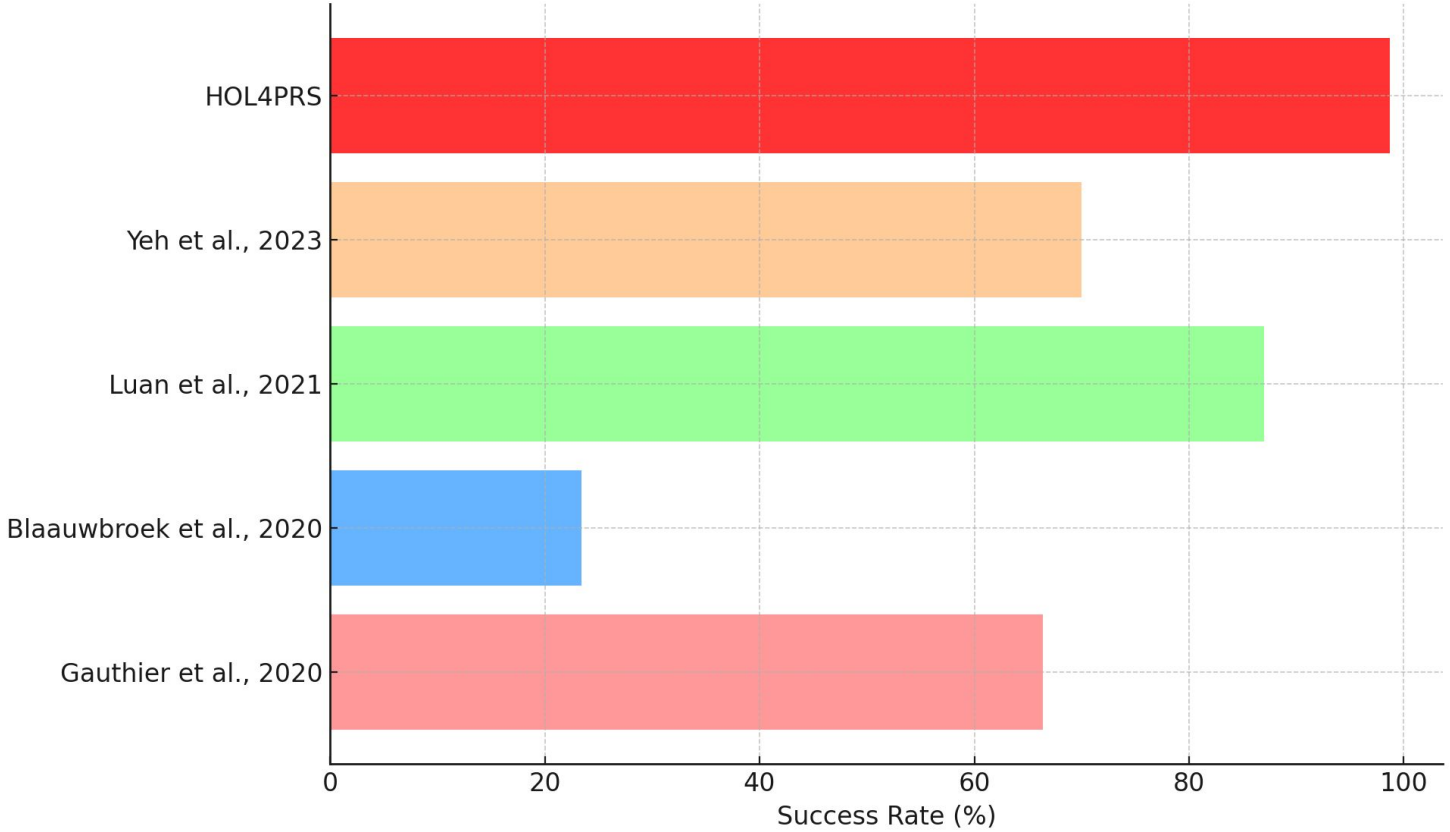
Datasets	T5			BERT			RoBERTa		
	Top 3	Top 7	Top 10	Top 3	Top 7	Top 10	Top 3	Top 7	Top 10
Dateset 1	51.3%	68.7%	76.4%	52.7%	71.9%	79.9%	54.5%	73.6%	93.7%
Dateset 2	60.4%	75.5%	80.5%	60.5%	78.9%	86.3%	59.7%	79.5%	85.8%
Dateset 3	69.8%	93.4%	95.4%	76.1%	93.9%	97%	78.4%	94.4%	97.5%
Dateset 4	77.3%	95.3%	97.2%	87.3%	97.0%	98.5%	89.5%	97.8%	98.8%
Dateset 5	76.6%	97.6%	98.2%	76.6%	97.6%	98.2%	76.6%	97.6%	97.6%
Dateset 6	39.9%	55.2%	61.9%	45.1%	65.4%	72.7%	43.4%	64.3%	73.8%
Dataset 7	**72.9%**	**85.6%**	**87.8%**	**75.4%**	**88.7%**	**92.3%**	**77.3%**	**89.8%**	**93.7%**

Experimental Results



HOL4PRS deploys RoBERTA for Top-7 recommendations

Results Comparaison



Future Work

- Integrate more contextual information with the input to improve the accuracy and relevance of proof step recommendations.
- Expand HOL4PRS to include more HOL4 theories.
- Extend the system to support other interactive theorem provers (ITPs) beyond HOL4.
- Explore premise selection approaches.
- Autonomously generate complete proofs without human intervention.

Proof Recommendation System for HOL4 Theorem Prover

For more information visit

https://hvg.ece.concordia.ca/projects/fvai/pr1/

https://github.com/DkNour/HOL4PRS-Proof-Recommendation-System-for-the-HOL4-Theorem-Prover.git

Thank you!
Q&A

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