Can AI prove creatively?

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Abstract

This paper explores how the perspective of thought experiments can aid in analyzing the challenge of creative artificial intelligence (AI). Via observation of perceptual properties and surpassing prescribed rules, thought experiments help reorganize information and reveal elegant solutions.

1 Creative art or creative task?

Shortly after Gowers, Green, Manners, Tao (2023) published the Polynomial Freiman-Ruzsa Conjecture. It was verified with AI proof assistant Lean. It is impressive that AI can help proving, but what level of complexity does such a proof entail? Human mathematicians tend to decrease complexity through creative solutions. Is AI capable of finding such solutions?

Extensive work in Computational Creativity within AI primarily focuses on generating creative artifacts, such as paintings and poems. However, there is a notable lack of emphasis on creativity within task-oriented domains, particularly in mathematics. This paper posits that both the artistic and task-oriented aspects must be considered for the analysis and automation of mathematical creativity.

A recent survey of Creative Problem Solving (CPS), a subfield of AI dedicated to non-standard solutions, utilizes the notion of conceptual space: "This space is associated with states or actions, where manipulation of either the action space or the state space leads to the discovery of new actions or states. CPS occurs when the agent's initial conceptual space is insufficient to complete the task, prompting the agent to expand its conceptual space to achieve the task's goal." (Gizzi et al. 2022)

2 Creative problem solving in mathematics

In mathematics, a similar definition of a Creative solution was introduced by John McCarthy (a father of AI): "a solution to a problem is creative if it involves concepts not present in the statement of the problem and the general knowledge surrounding it." (MacCarthy 1999). His example is the famous *mutilated chessboard problem* (MCP).

The general question is whether it is possible to cover the n x n region with two diagonally opposite squares removed with 1 x 2 rectangles. Exhaustive search for n=8 is already beyond human capacity and “a tough nut for proof procedures” (MacCarthy 1998). A creative solution can be seen as a thought experiment: imagine that it is a chessboard without opposite corners. Can it be covered by dominos? The answer is “No”, because both of the absent corners are white, and since each domino covers black and white adjacent squares, we are left with two black squares uncovered. The colors are unavailable to the formulation of the problem, but it is a key invariant for the argument.

How can we teach AI to search for such solutions? Specifically, how can we instruct AI to establish connections between mathematics and chess, or geometric combinatorics and matching theory? Color, although not inherently a mathematical concept, becomes essential when added to the task. However, incorporating such a property into the task necessitates expanding the domain and surpassing conventional practices. This poses a challenge to automated proving. Thus far, domain flexibility, the synergies between the traditionally separate research areas remain significant limitations for AI.

The so-called 'conceptual blending,' introduced by Fauconnier & Turner (2008), is a well-studied approach in computational creativity. However, its potential for enhancing creative problem-solving in AI remains largely unexplored. Lieto et al. (2019) have employed conceptual blending, a mode of combining knowledge, to discover novel concepts within a semantic network. Similarly, Yu & Song (2019) have demonstrated the feasibility of learning conceptual blending. Nevertheless, the application of such techniques to the challenge of task learning and planning in AI remains an open question.

The traditional 'planning-solving' approach fails to capture creativity as it relies solely on rules and algorithms. Creativity, on the other hand, involves manipulating the environment, context, or representation. In Straikova and Van Bendegem's work (2022), we demonstrated that all automated solutions for the Mutilated Chessboard Problem (MCP) somehow rely on creative approaches. Furthermore, such creative approaches to solve MCP, once discovered, can be formalised and coded, but are not presently expected to be discovered automatically by proof assistant software such as Lean. Our attempt to have ChatGPT solve MCP resulted in grammatically sound and well-informed yet incorrect answers, claiming that such cover is possible.

> Mathful: “Hmm, that's an interesting question! Let's think about it together. Would you like me to explain what a 1 x 2 rectangle is?”
>
> AIMath: “Yes, it is possible to cover the n x n region with two diagonally opposite squares removed using 1 x 2 rectangles.”

3 The framework of thought experiments

One can distinguish four principal components of Creative Problem Solving (CPS): problem formulation (concepts), representation (symbolic and non-symbolic), information manipulation (over-representation), and evaluation of outcomes. Understanding the creativity of problem-solving necessitates considering these components.

Thought experiments encompass all these aspects. They allow for the addition of elements to tasks, as seen in the case of MCP. They facilitate visual representation and imaginary manipulations in the conceptual space. Moreover, they enable the acceptance of what may seem impossible in practical scenarios as plausible in hypothetical situations. Examples include Euler's bold proof of the Basel problem, Lobachevsky's discovery of hyperbolic geometry, and Gromov's discovery of large kale geometry of generated groups, among others (Starikova 2024).

In conclusion, the exploration of AI creativity in mathematical problem-solving, presents a multifaceted challenge that necessitates integrating diverse perspectives and methodologies. While traditional approaches may fall short in capturing the nuances of creative problem-solving, the incorporation of concepts such as thought experiments offers promising avenues for advancement. By embracing the interplay between domain-specific knowledge and imaginative exploration, we can unlock new frontiers in AI-driven creativity.

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