Learning to Identify Useful Lemmas from Failure

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Learning to Identify Useful Lemmas from Failure

Lemmas to Aid Automated Proof Search

Explore the benefit of identifying/using lemmas to aid proof search

- Lemmas can make the proof shorter
- Lemmas can make selecting the next inference harder
- Ideally, we would like to identify just a few relevant lemmas
- Similar to premise selection, but we assume no given premise set

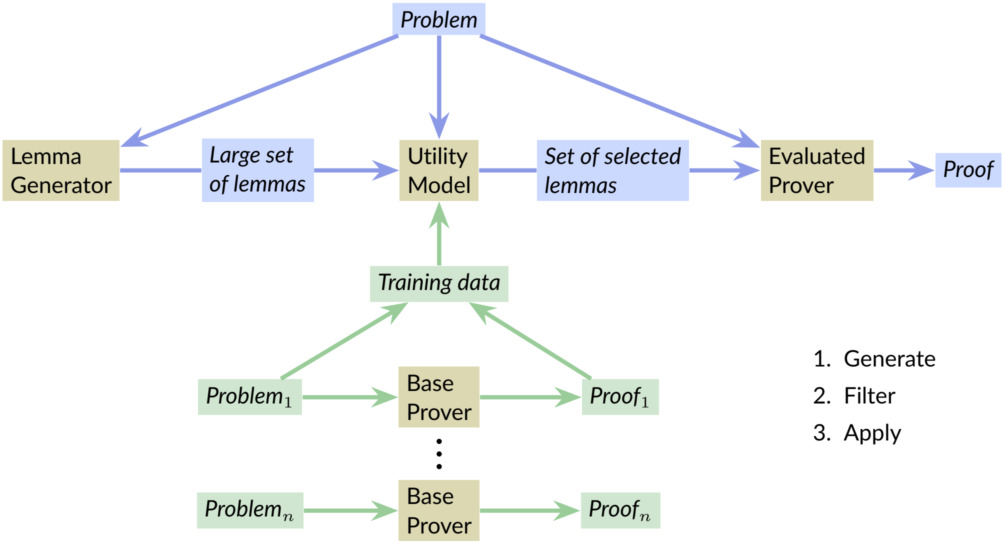
Rawson, Wernhard, Zombori, Bibel. Lemmas: Generation, Selection, Application. To appear at TABLEAUX2023

Dataset

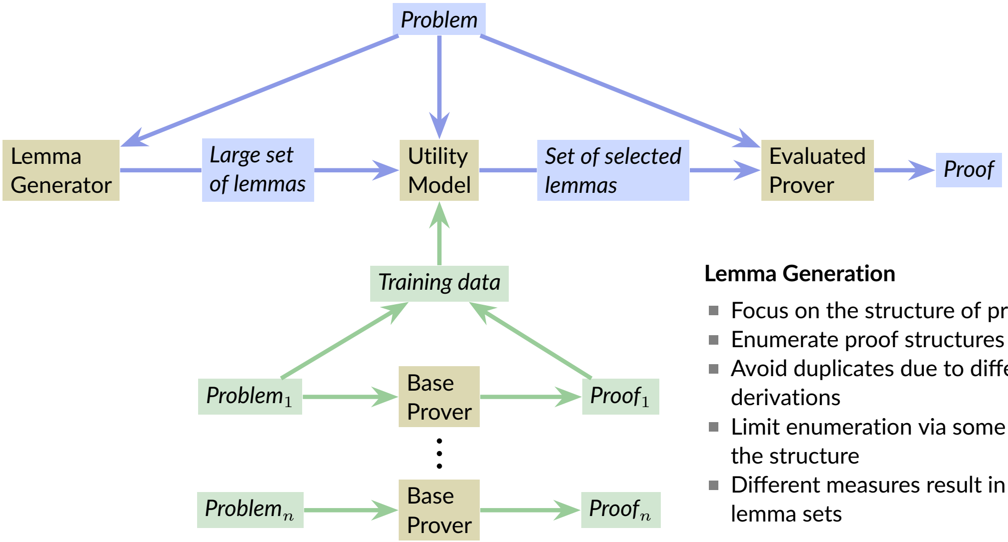
Restrict attention to Condensed Detachment (CD) problems

Detachment axiom	$\mathsf{P}(\mathsf{i}(x, y)) \land \mathsf{P}(x) \to \mathsf{P}(y)$	
Proper axioms	units	e.g. $\mathsf{P}(\mathsf{i}(\mathsf{i}(\mathsf{i}(x, y), z), \mathsf{i}(\mathsf{i}(z, x), \mathsf{i}(u, x))))$
Goal	negative ground unit	e.g. $\neg\mathsf{P}(\mathsf{i}(\mathsf{a}, \mathsf{i}(\mathsf{b}, \mathsf{a})))$

- Horn, first-order variables, binary function symbol, cyclic predicate dependency
- Generalization to arbitrary Horn problems is possible
- Proofs have a simple regular tree structure (D-terms)
- D-terms are convenient for feature extraction and for structure enumeration

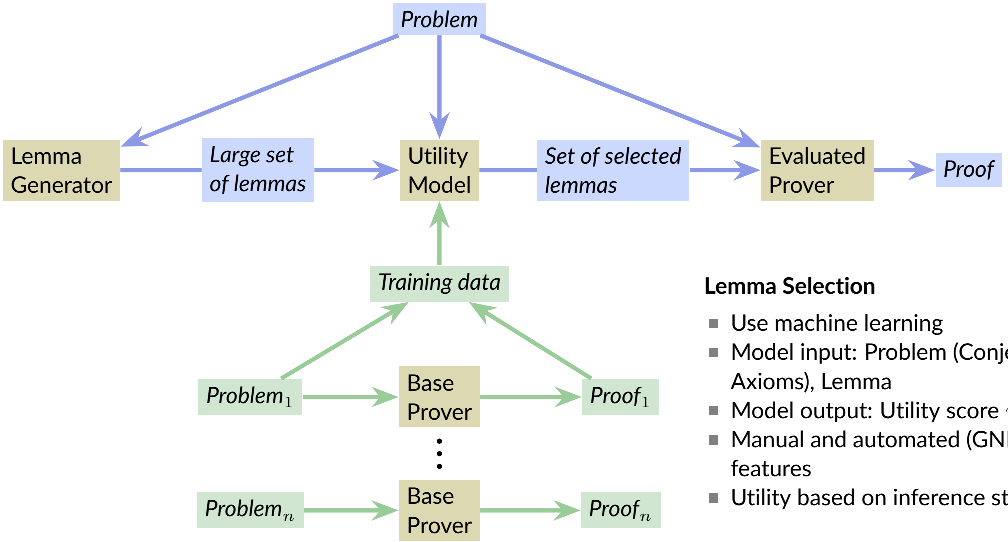


1. Generate
2. Filter
3. Apply



Lemma Generation

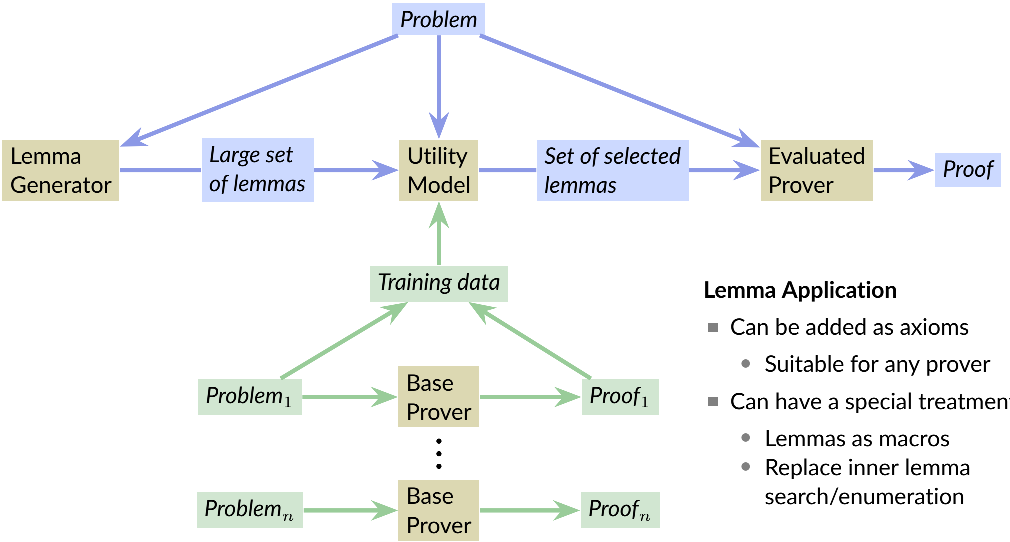
- Focus on the structure of proofs
- Enumerate proof structures
- Avoid duplicates due to different derivations
- Limit enumeration via some measure on the structure
- Different measures result in very different lemma sets



Lemma Selection

- Use machine learning
- Model input: Problem (Conjecture + Axioms), Lemma
- Model output: Utility score $u \in [0, 1]$
- Manual and automated (GNN) input features
- Utility based on inference step reduction

Method overview



Lemma Application

- Can be added as axioms
  - Suitable for any prover
- Can have a special treatment
  - Lemmas as macros
  - Replace inner lemma search/enumeration

Iterative Improvement

- Start from a set of problems
- Search from proofs
- Learn from proof attempts
- Fit a model
- Start search again, using the learned model

Learning Requirements, Considered Provers

- Lemma generation requires proof structure enumeration (SGCD)
- We require provers that emit proofs as D-terms (SGCD, Prover9, CMProver, CCS)
- Any prover can be used for evaluation

	SGCD	Prover9	CMProver	leanCoP	CCS-Vanilla	Vampire	E
Goal-driven	●/–	–	●	●	●	○	○
CM-CT	○	–	●	●	–	–	–
Proof Structure Enumeration	●	–	●	○	●	–	–
Resolution / Superposition	–	●	–	–	–	●	●
Output proof as D-term	●	●	●	–	●	–	–
Input lemmas that replace search	●	–	–	–	●	–	–

Learning from Successful Proof Attempts

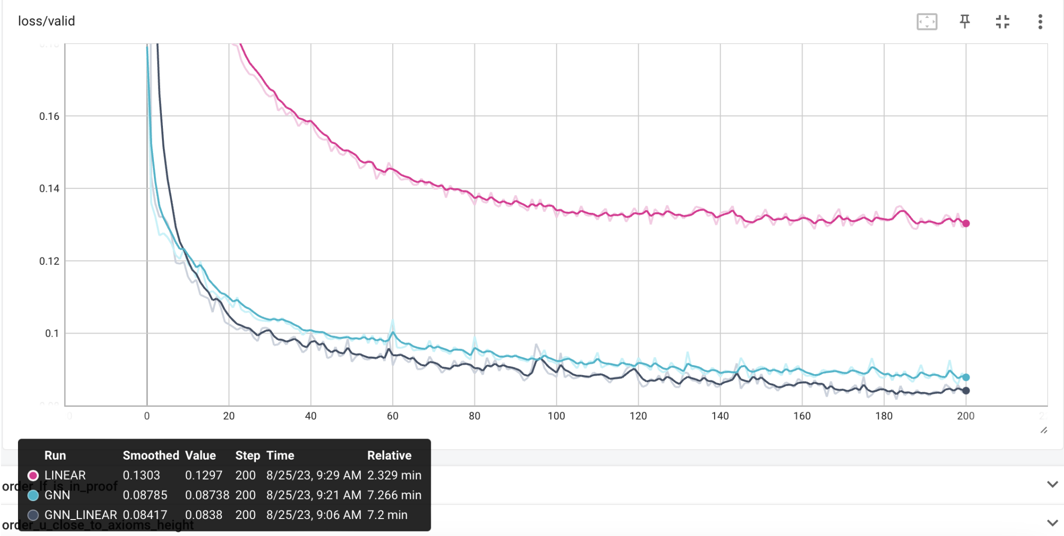
- Utility measure calculation requires a prover that can produce a proof tree structure
- Given a proof, any substructure can be considered as a lemma that we can learn from
- Lots of training signal from a single proof, if the proof is long
- Different proofs of the same problem can be used

Learning from Failed Proof Attempts

- Any proof attempt constructs a sequence of incomplete proof structures
- Most of these have complete substructures
- These are proof terms of formulas proven as a byproduct of proof search
- We can use any such substructures as a proof to learn from
- Similar to Hindsight Experience Replay [Andrychowicz et al., 2017]
  - Pretend that we wanted to prove what we accidentally proved
- Provides huge amounts of training data from failed proofs
  - ˜100K samples with 10 sec timeout

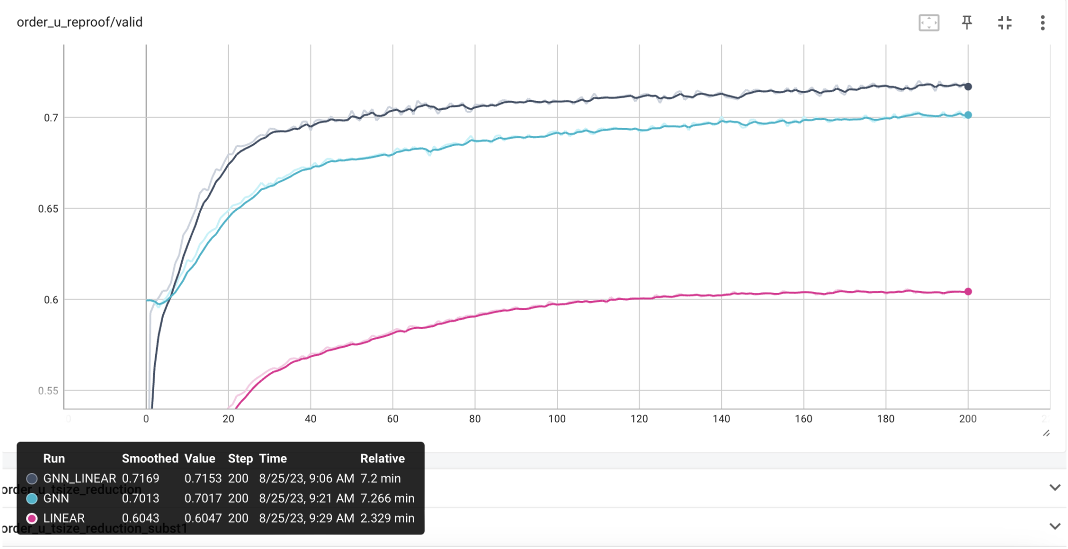
Model Fitting: Linear Model vs Graph Neural Network

Validation Loss



Model Fitting: Linear Model vs Graph Neural Network

Ability to predict correct order



Problemwise learning from failed attempts

- Prover: SGCD (provecd_sgcd_s1.pl)
- Time limit: 10 sec
- Total problems: 312

Train a separate model for each problem

- Prover: SGCD (provecd_sgcd_s1.pl)
- Time limit: 10 sec
- Total problems: 411

Train a single model for all problems.

Learn from	Iteration					Total
	0	1	2	3	4	
success	199	203	206	216	205	222 (+23)
failure	199	211	219	209	205	229 (+30)
both	199	212	207	**223**	200	**230 (+31)**

Learning both from failed and successful proof attempts

- Prover: portfolio of diverse SGCD strategies (f_sgcd_tsize)
- Time limit: 10 sec
- Total problems: 411

Learn from	Iteration					Total
	0	1	2	3	4	
both	236	257	246	249	244	263 (+27)

Learning to Identify Useful Lemmas from Failure

1. Learning to Identify Useful Lemmas
2. Learning from Successful as well as Failed Proof Attempts
3. Experiments
4. **Learning Subtree/Unit Lemmas**
5. Conclusion

Conclusion

- Lemmas are helpful to find a proof
- Generate, filter, apply lemmas
- A lot of signal can be extracted from failed proof attempts that is useful for learning
- Lemma generation brings a bit of resolution into non-resolution based provers
- Blurs the distinction between forward and backward reasoning

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