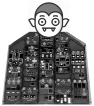
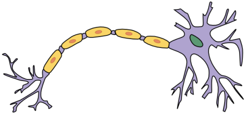
One Year With Deepire: Lessons Learned and Where to Go Next?

Martin Suda[1]

Czech Technical University in Prague, Czech Republic

AITP, September 2021

[1]Supported by Czech Science Foundation project 20-06390Y.

Deepire =  + 

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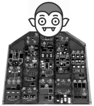
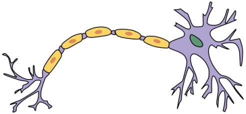
Deepire =  + 

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- state-of-the-art saturation-based prover

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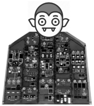
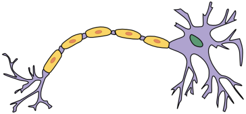
Deepire =  + 

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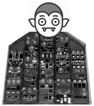
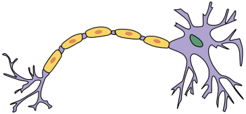
Deepire =  + 

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➥ The special bit: uses a recursive neural network (RvNN) based solely on clause derivation history

Deepire Is One Year Old!

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The story so far:

- [AITP20] - introduced the first prototype
- [CADE21] - improved Vampire's theory reasoning on SMTLIB
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In this talk:

- overview of the main results, insights, and future outlooks

Outline

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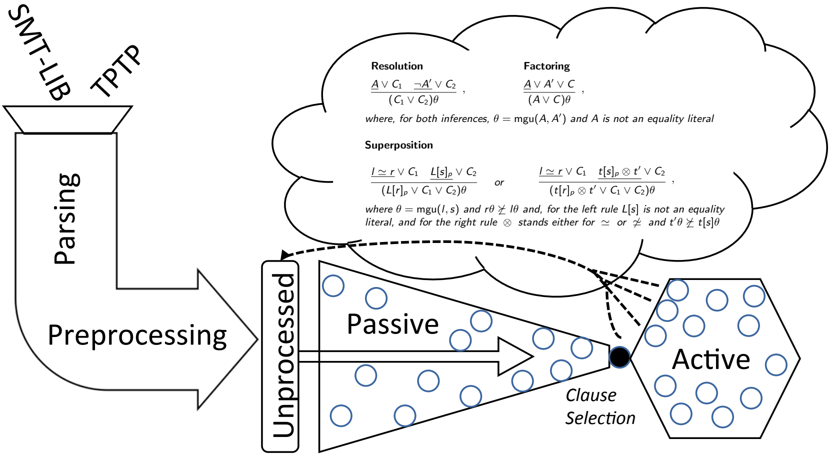
1. Saturation, Clause Selection, and Machine Learning
2. Recursive Neural Networks over Clause Derivations
3. Notes on Implementation and Training
4. Experiments on Mizar
5. Conclusion

Saturation-based Theorem Proving



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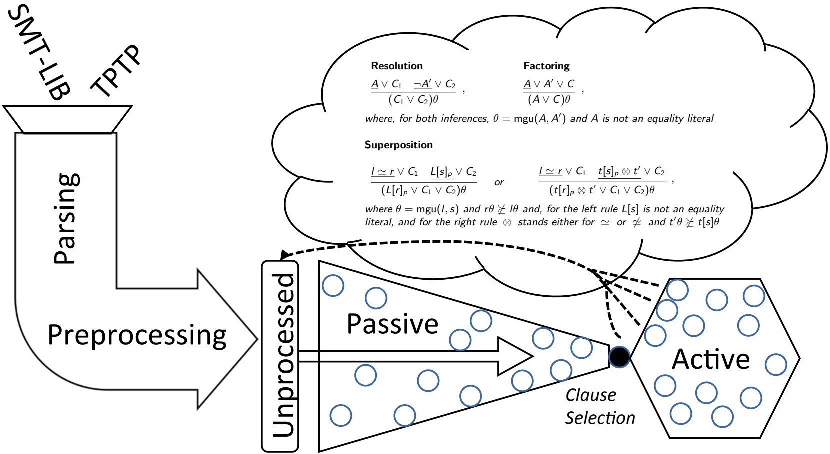

At a typical successful end: $|Passive| \gg |Active| \gg |Proof|$

How is clause selection traditionally done?

Take simple clause evaluation criteria:

- <u>age</u>: prefer clauses that were generated long time ago
- <u>weight</u>: prefer clauses with fewer symbols

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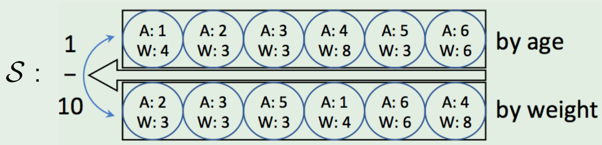
Example (Organizing *Passive* via two priority queues)



Machine-learned Clause Selection Guidance

The core idea

Learn to recognize and prefer for selection clauses that look like those that contributed to a proof in past successful runs.

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- integrating the learned advice back to the saturation loop

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Priority:

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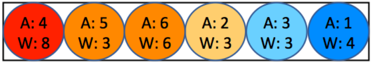
- 

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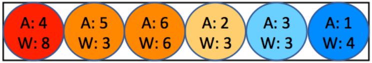
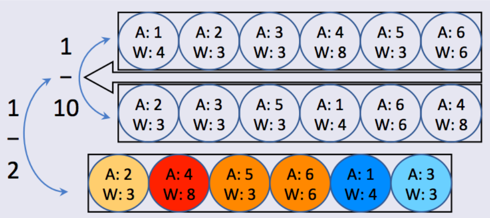
- 

Combine with the original strategy

$\mathcal{S} \oplus \mathcal{M}^{1,0}$:



What Worked the Best?

Layered Clause Selection [Tammet19,G&S20]:

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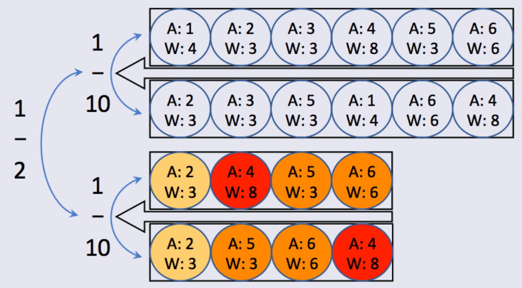

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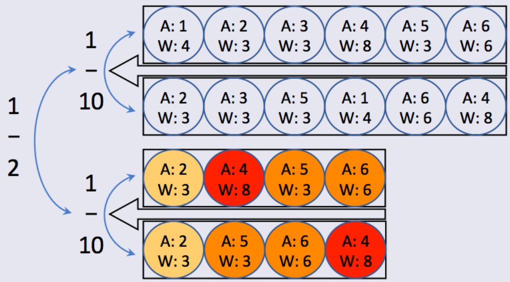


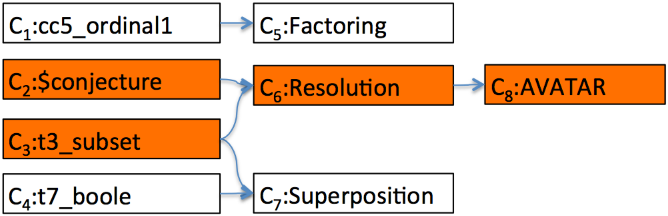
Advantages of LCS:

- keep using the well-tuned $\mathcal{S}$ also for the positively classified
- allows for the lazy evaluation trick [AITP20,CADE21]
- a smooth transition from the original to the ML-boosted

Outline

Represent Clauses by Their Derivation History

"Don't look at what the clause says, only where it's coming from."



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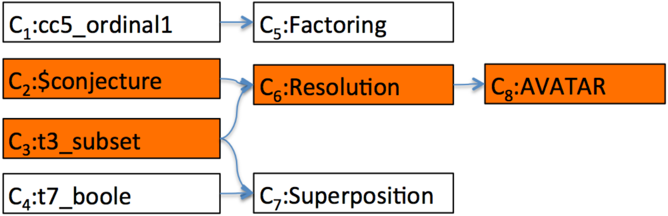

Focusing on the MIZAR dataset here:

- a large set of axioms $\mathcal{A}$ referenced by all the problems
- each problem P consists of a conjecture C_P and a $\mathcal{A}_P \subseteq \mathcal{A}$
- a small set of inference rules labeling the internal nodes

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Example (Evaluating the derivation from the previous slide)

$$v_{C_2} := I_{\$conjecture}$$
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$$v_{C_6} := D_{Resolution}(v_{C_2}, v_{C_3})$$
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➥ NB: Constant work per clause!

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3) Conjecture relatedness

- in each problem, we mark conjecture clauses by $l_{\$conjecture}$
- the network learns to incorporate the right level of goal directedness

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Dynamic computational graphs:

- elegant and flexible, but
- training needs to keep building them over and over!

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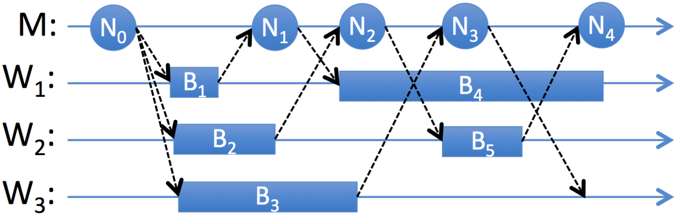

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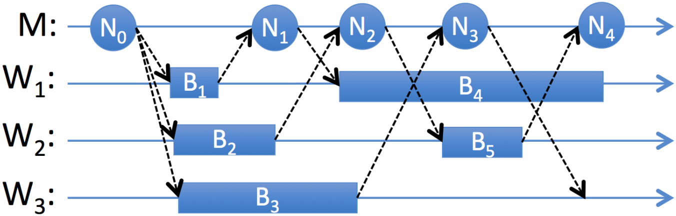
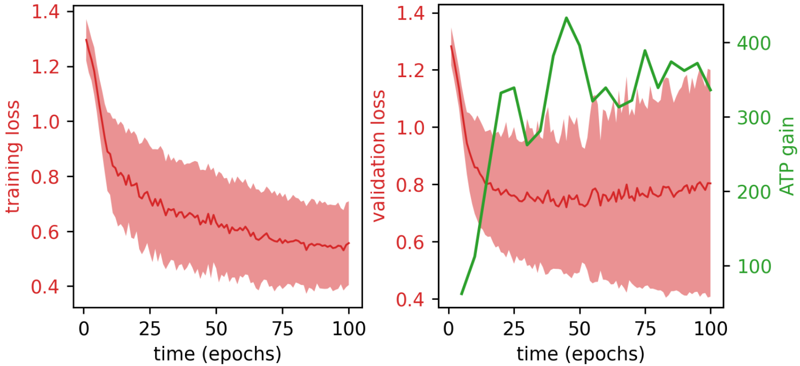


➥ A funny "drift" effect that actually regularizes!

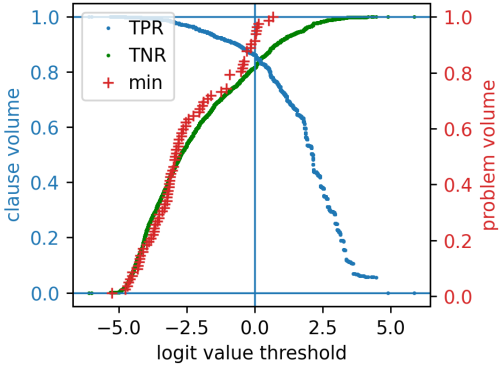
Validate and Compare to the ATP Performance



(from [CADE21]: Deepire for theory reasoning on SMTLIB)

Tweak Your Positive Bias

How often is $\mathcal{M}$ 100% correct?



[CADE21]: leaning a bit positively improves ATP performance

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Mizar40 benchmark [Urban&Kaliszyk15]

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Fixed for the whole experiment:

- a base strategy $\mathcal{V}$:
  - previously shown to work well on Mizar40
- 10 s time limit

Training the First Models

Data preparation:

- $\mathcal{V}$ was able to solve 20 197 problems
- 800 MB of successful derivations (when zipped)
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revealed axioms m	0.5K	1K	1K	1K	2K
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➥ Large capacity generalizes best!

Evaluation with the Prover

Num.probs solved by $\mathcal{V}$ and its RvNN boosted variants

strategy	$\mathcal{V}$	$\mathcal{H}^{n128}$	$\mathcal{M}^{n64}$	$\mathcal{M}^{n128}$	$\mathcal{M}^{n256}$	$\mathcal{D}^{n128}$
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$\mathcal{V}+$	+0	+5022	+5879	+6129	+5707	+6277
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- Could be even more greedy about the revealed axioms (m)
- Going over the embedding size ($n = 128$) makes it too slow
- It's fine to spend 40% of time *just thinking what the next clause should be* if it results in a good enough advice!

Looping and a Comparison to ENIGMA

Looping [Jakubův&Urban19]

- iterate the learning and solving phases
- keep learning also from the newly discovered proofs

➥ There: boosted tree learner over hand-crafted features

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Performance comparison

	ENIGMA [J&U19]		Deepire		
loop	solved	$+\mathcal{S}$ %	solved	$+\mathcal{V}$ %	note
0	14 933	0.0	20 197	0.0	
1	20 366	35.8	26 014	28.8	$m = 2000$
2	22 839	52.3	27 348	35.4	$m = 3000$
3	23 467	56.5	28 947	43.3	$m = 5000$
4	23 753	58.4			
4’	25 397	70.0			

Points to note:

- both show the effect of diminishing returns

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Performance comparison

	ENIGMA [J&U19]		Deepire		
loop	solved	$+\mathcal{S}$ %	solved	$+\mathcal{V}$ %	note
0	14 933	0.0	20 197	0.0	
1	20 366	35.8	26 014	28.8	$m = 2000$
2	22 839	52.3	27 348	35.4	$m = 3000$
3	23 467	56.5	28 947	43.3	$m = 5000$
4	23 753	58.4			
4'	25 397	70.0			

Points to note:

- both show the effect of diminishing returns
- ENIGMA climbs higher (relatively) from lower numbers

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Thank you for attention!