# Deep Learning for Temporal Logics

*Frederik Schmitt*, Christopher Hahn, Jens U. Kreber,
Markus N. Rabe, Bernd Finkbeiner

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INFORMATION SECURITY

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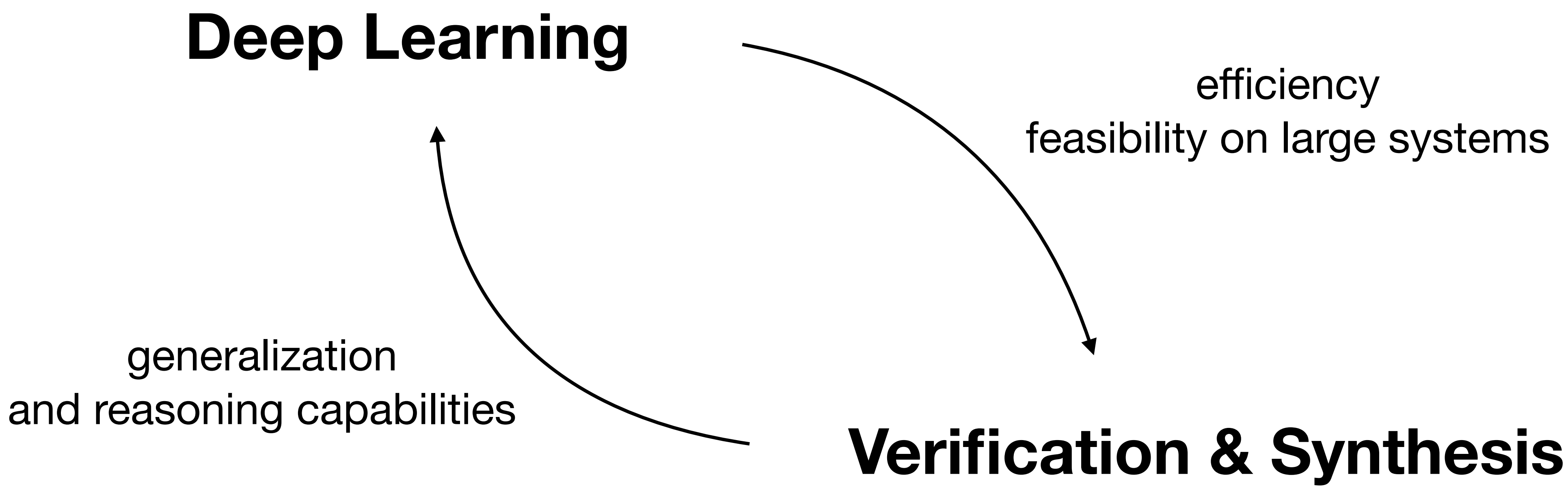
DES
SAARLANDES

Google Research

# Deep Learning for Formal Methods



# Examples of Related Work

## NeuroSAT

Selsam, D., Lamm, M., Bünz, B., Liang, P., de Moura, L., Dill, D.L.: Learning a SAT Solver from Single-Bit Supervision. ICLR 2019

## FastSMT

Balunović, M., Bielik, P., Vechev, M.: Learning to Solve SMT Formulas. NeurIPS 2018

## DeepHOL

Bansal, K., Loos, S.M., Rabe, M.N., Szegedy, C., Wilcox, S.: HOList: An Environment for Machine Learning of Higher-Order Theorem Proving. ICML 2019

## DeepMath

Alemi, A. A., Chollet, F., Een, N., Irving, G., Szegedy, C., Urban, J.: DeepMath: Deep Sequence Models for Premise Selection. NeurIPS 2016

**...**

# Deep Learning for Temporal Logics

**Deep Learning**

efficiency
feasibility on large systems

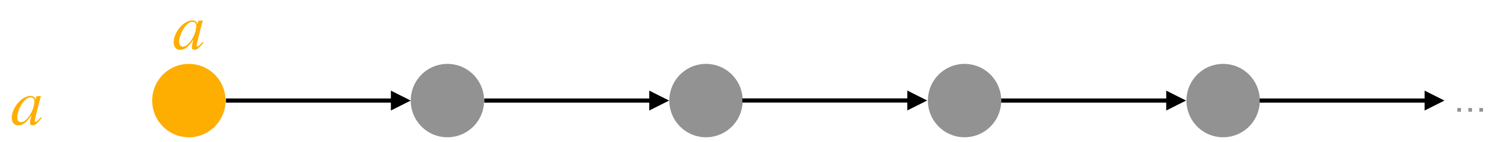
generalization
and reasoning capabilities

***Temporal Logics***

- *Trace generation Problem (PSPACE)*
- *Circuit synthesis Problem (2EXPTIME)*

# Linear-time Temporal Logic (LTL)[1]

$\varphi, \psi ::= a \mid \mathsf{true} \mid \neg\varphi \mid \varphi \wedge \psi \mid \bigcirc \varphi \mid \varphi \,\mathsf{U}\, \psi$ where $a \in AP$




[1] Pnueli, A.: The Temporal Logic of Programs. 18th Annual Symposium on Foundations of Computer Science, Providence, Rhode Island, USA, 1977

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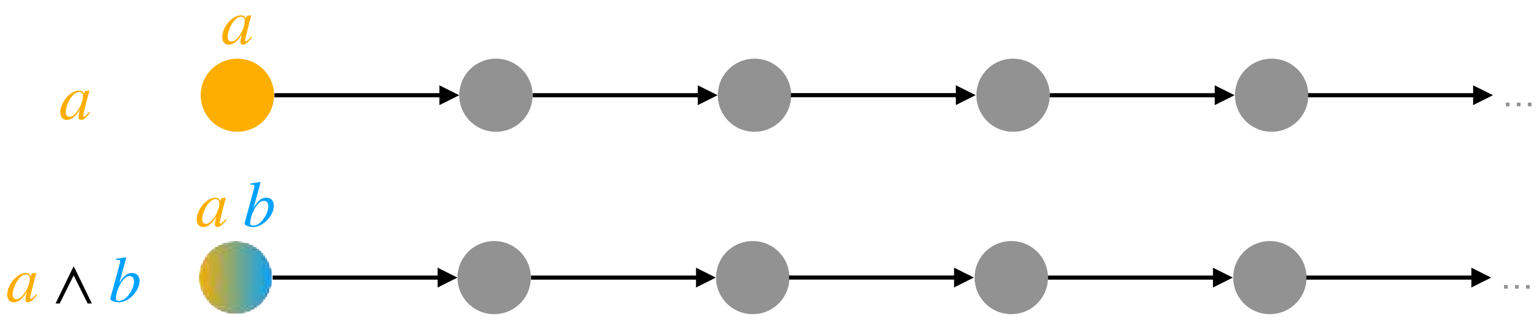


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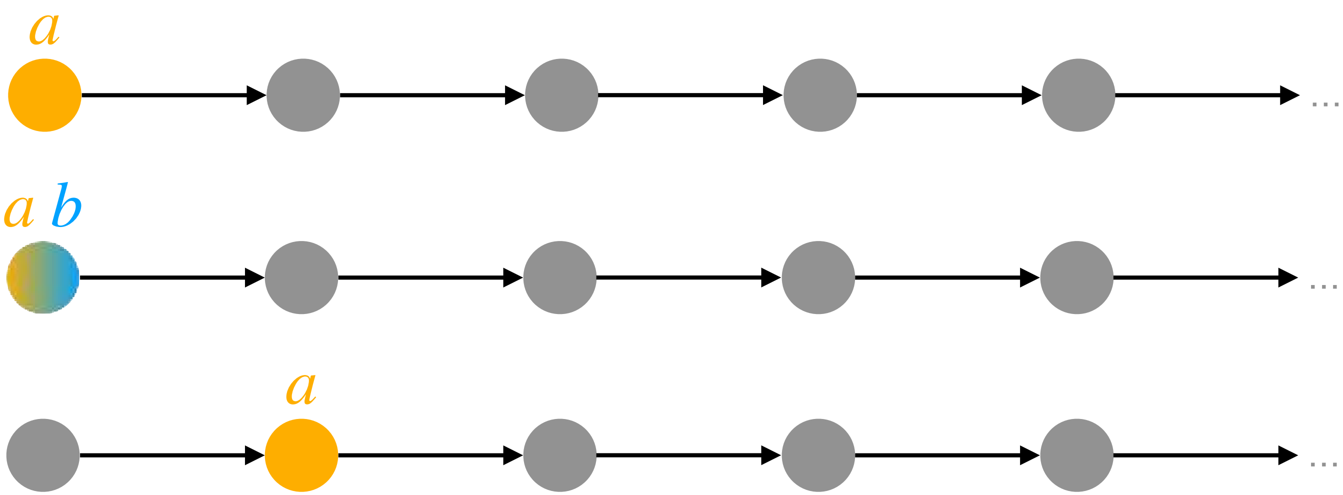
$a$

$a \wedge b$

Next $\bigcirc a$

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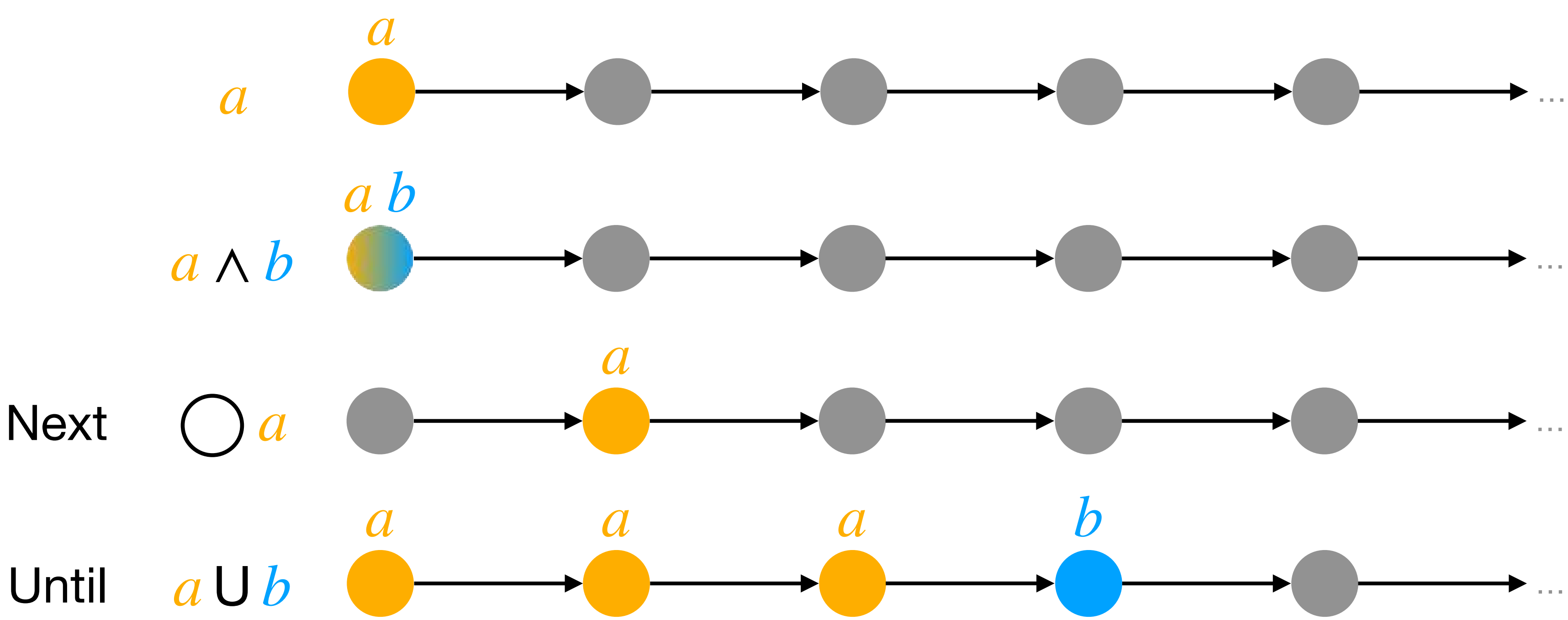


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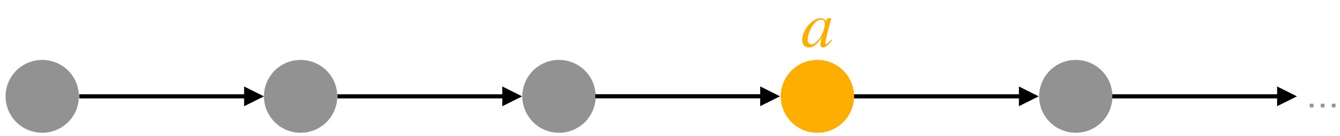
# Linear-time Temporal Logic (LTL)[1]

Eventually

$\Diamond a := \text{true} \, \mathsf{U} \, a$ 


Globally

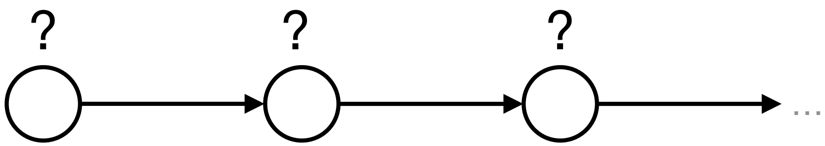
$\Box a := \neg \Diamond \neg a$ 


[1] Pnueli, A.: The Temporal Logic of Programs. 18th Annual Symposium on Foundations of Computer Science, Providence, Rhode Island, USA, 1977

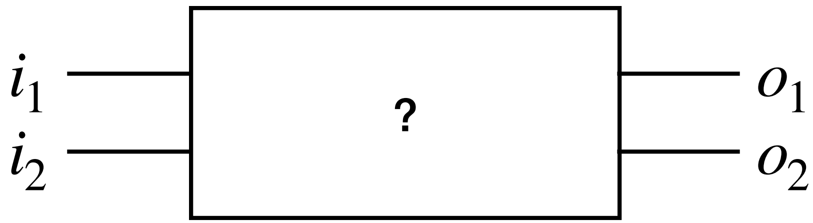
# Part 1:
# Trace Generation

# Part 2:
# Circuit Synthesis

Trace $\pi \models$ LTL Formula $\varphi$




Circuit $C \models$ LTL Specification $\varphi$

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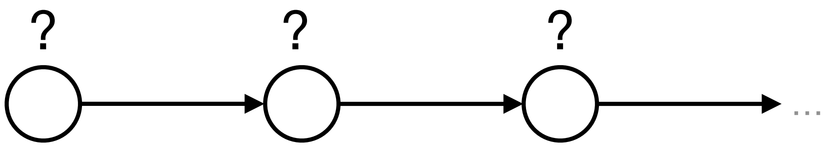
# Part 1: Trace Generation

# Part 2: Circuit Synthesis

Trace $\pi \models$ LTL Formula $\varphi$

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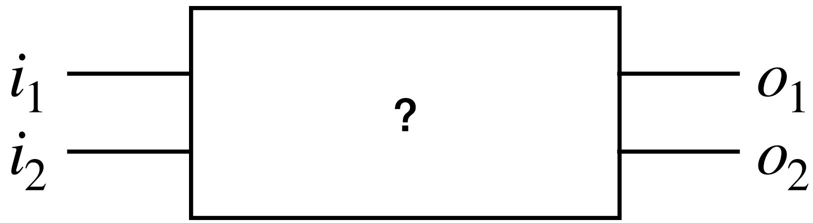



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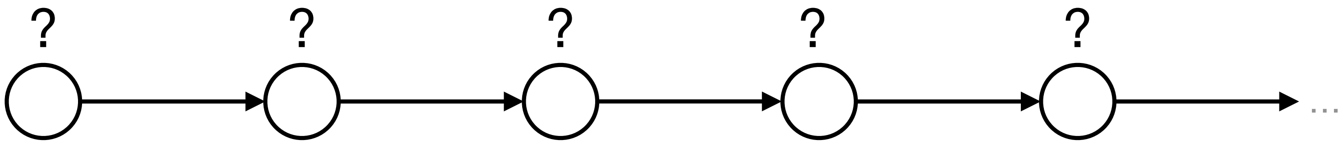
# Trace Generation Problem

$$(a \mathsf{U} (b \wedge c)) \wedge (a \mathsf{U} (\neg b \wedge c)) \wedge (a \mathsf{U} (\neg b \wedge \neg c))$$



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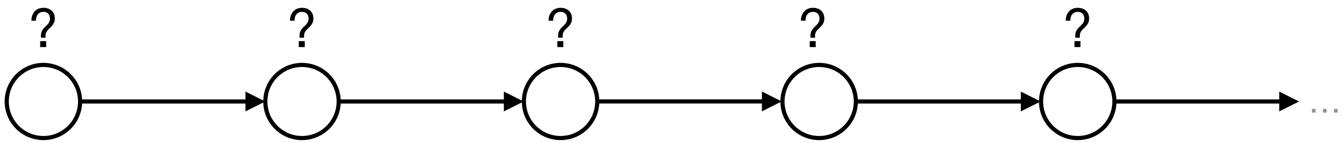
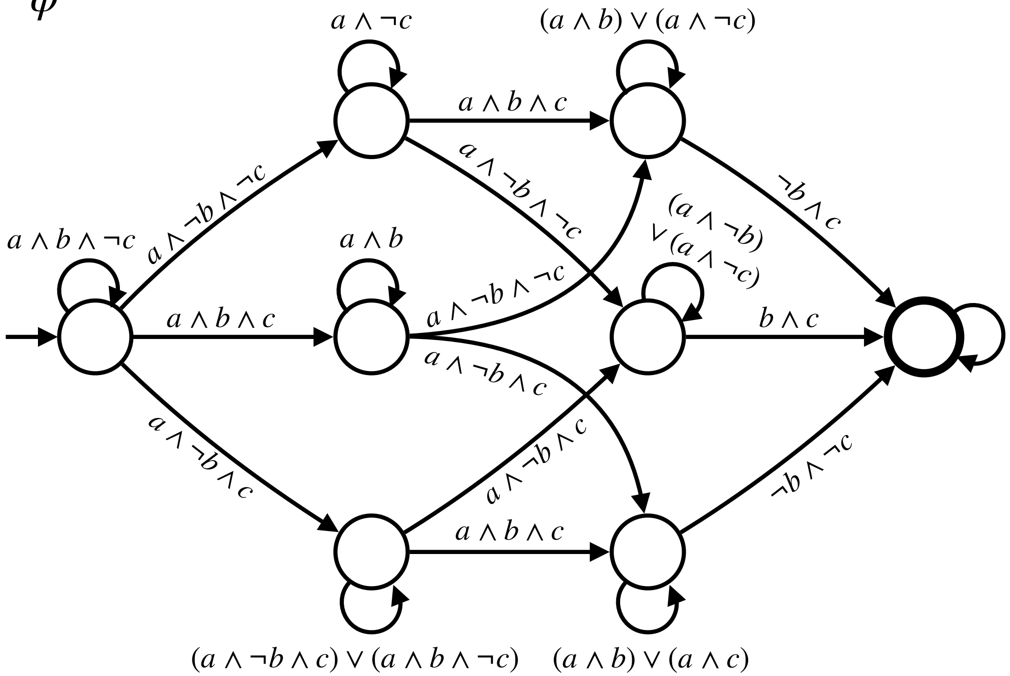



The LTL Trace Generation Problem is PSPACE-complete.

# Classic Trace Generation

$$\varphi = (a \,\mathsf{U}\, (b \land c)) \land (a \,\mathsf{U}\, (\neg b \land c)) \land (a \,\mathsf{U}\, (\neg b \land \neg c))$$

Büchi Automaton $A_\varphi$



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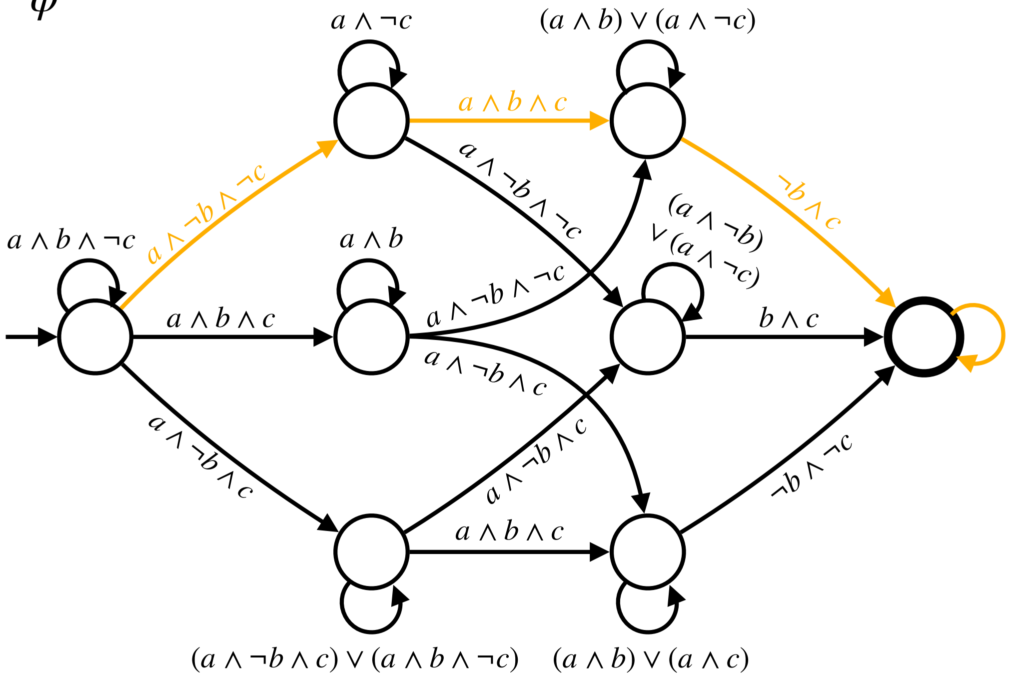

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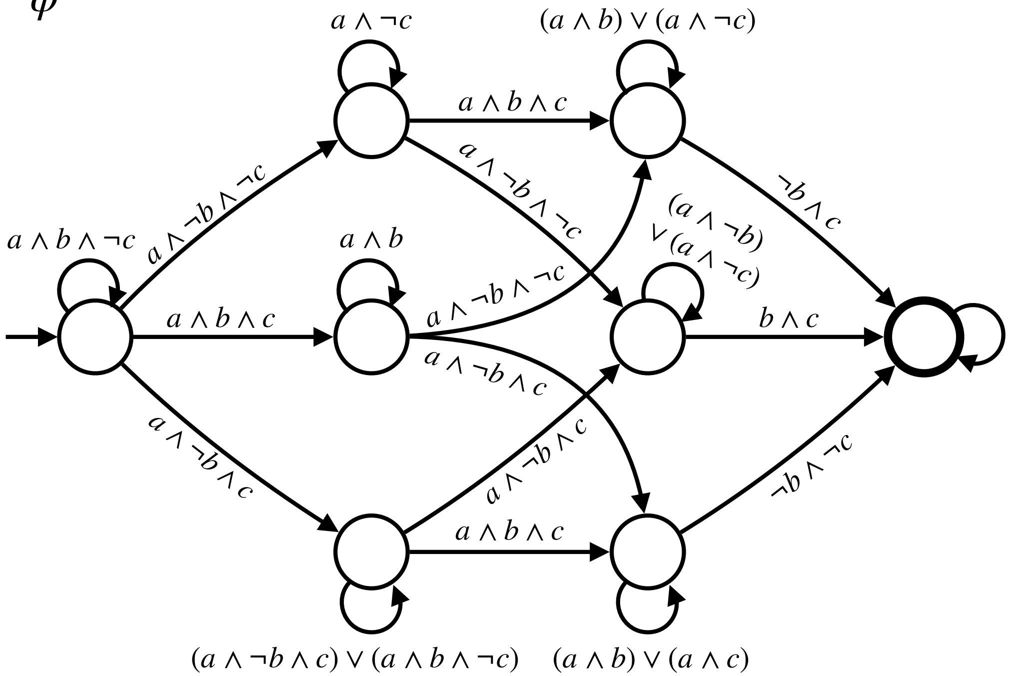


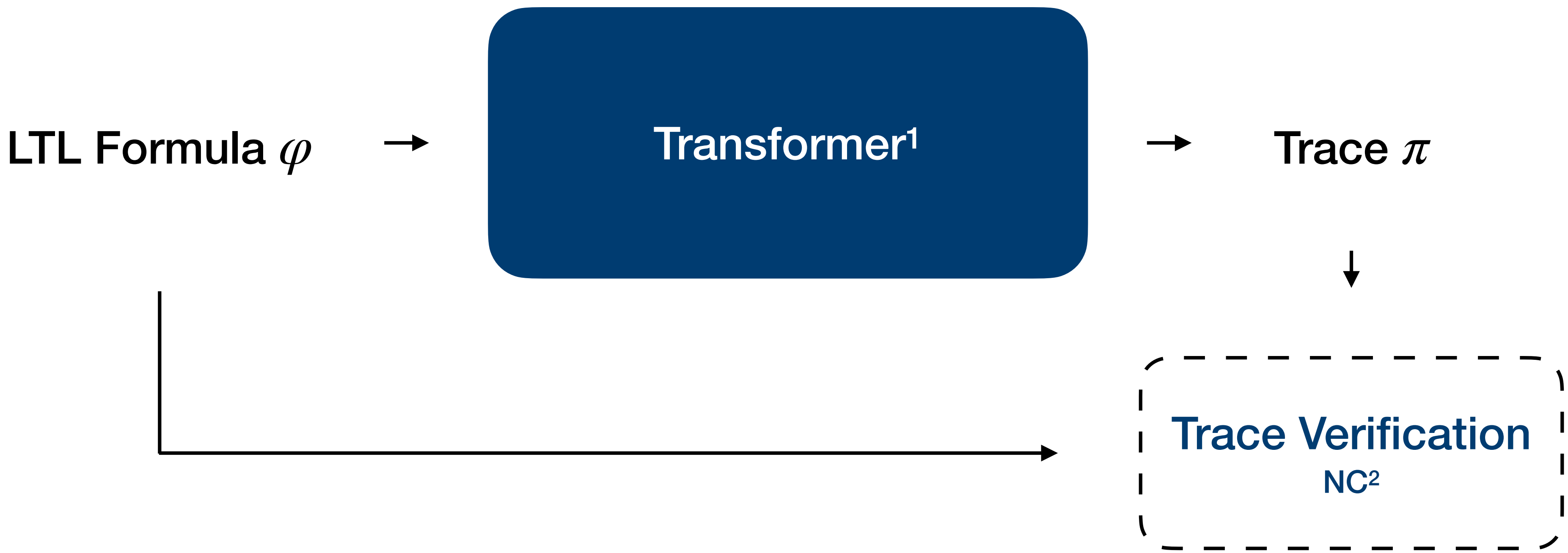

The number of states can be exponential in $|\varphi|$.

# *Neural* Trace Generation

LTL Formula $\varphi$ → Transformer[1] → Trace $\pi$

[1] Vaswani, A., Shazeer, N., Parmar, N., Uszkoreit, J., Jones, L., Gomez, A.N., Kaiser, L., Polosukhin, I.: Attention is All you Need. NeurIPS 2017

# Neural Trace Generation




[1] Vaswani, A., Shazeer, N., Parmar, N., Uszkoreit, J., Jones, L., Gomez, A.N., Kaiser, L., Polosukhin, I.: Attention is All you Need. NeurIPS 2017

[2] Kuhtz, L., Finkbeiner, B.: LTL Path Checking is Efficiently Parallelizable. ICALP 2009

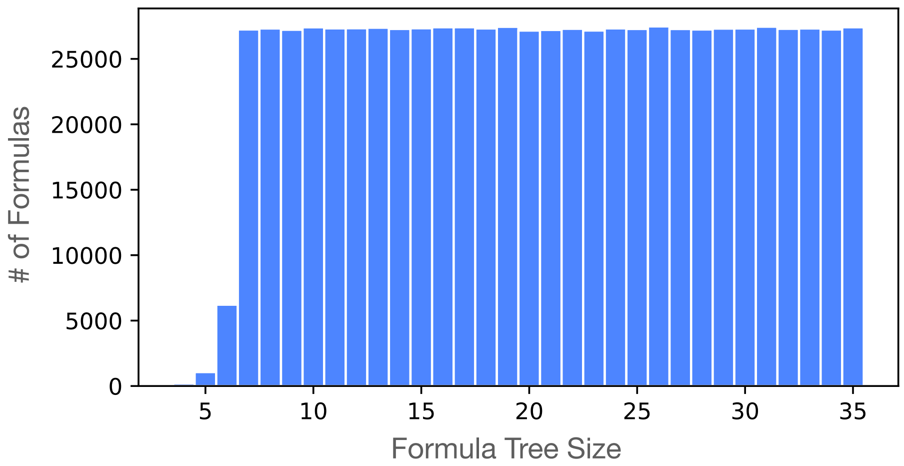
# Neural Trace Generation
## Datasets

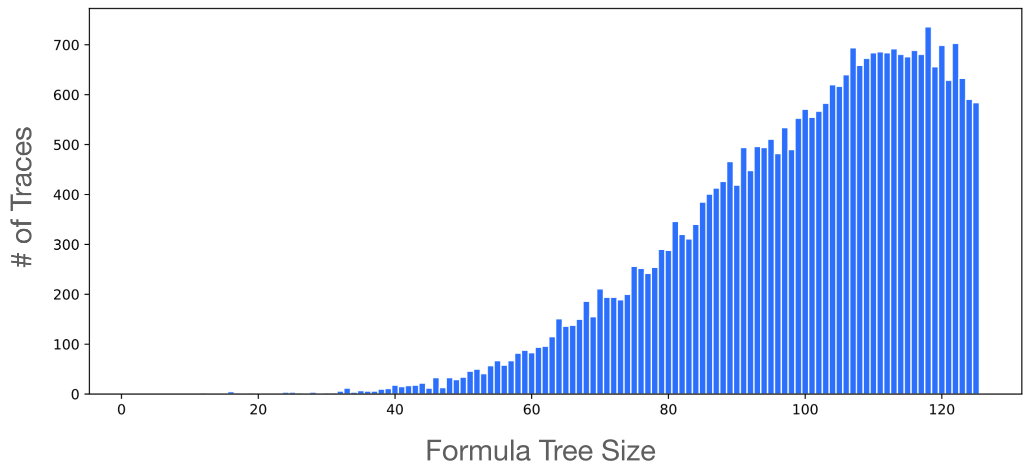
### LTLRandom35

- 1,000,000 formula-trace pairs
- Generated randomly
- Uniformly distributed in formula size

### LTLPattern126

- 1,664,487 formula-trace pairs
- Constructed from formula patterns[1]
- Conjunctions of patterns are hard to solve

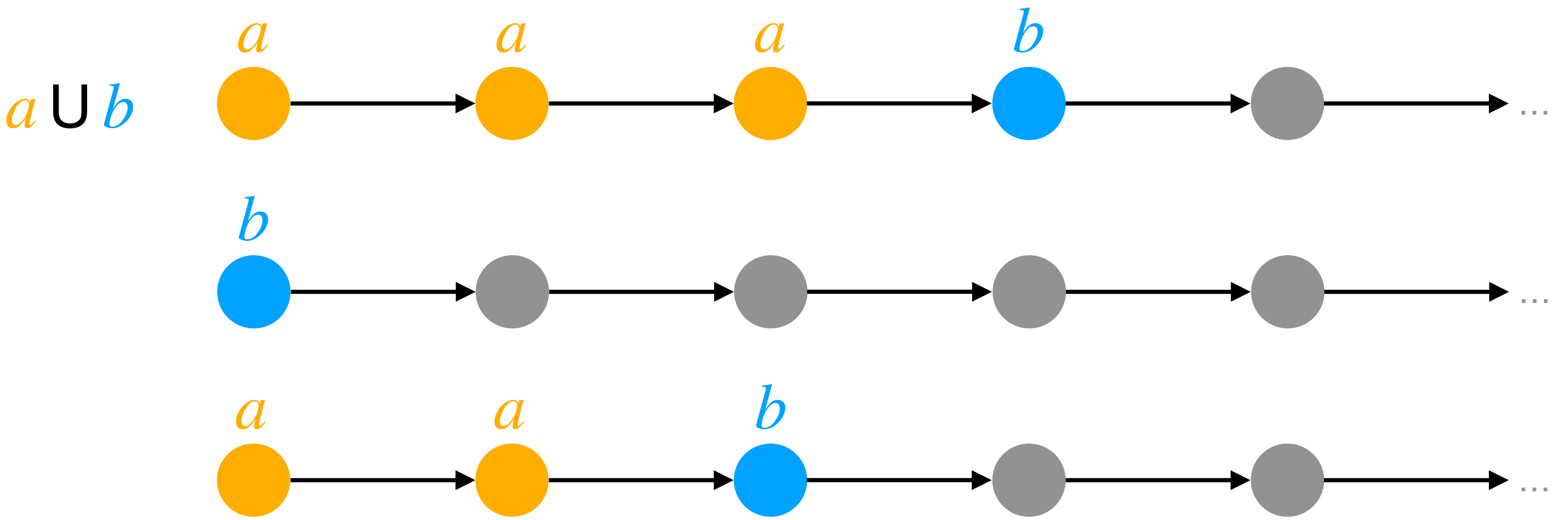



[1] Dwyer, M. B., Avrunin, G.S., Corbett, J.C.: Property Specification Patterns for Finite-State Verification. 2017

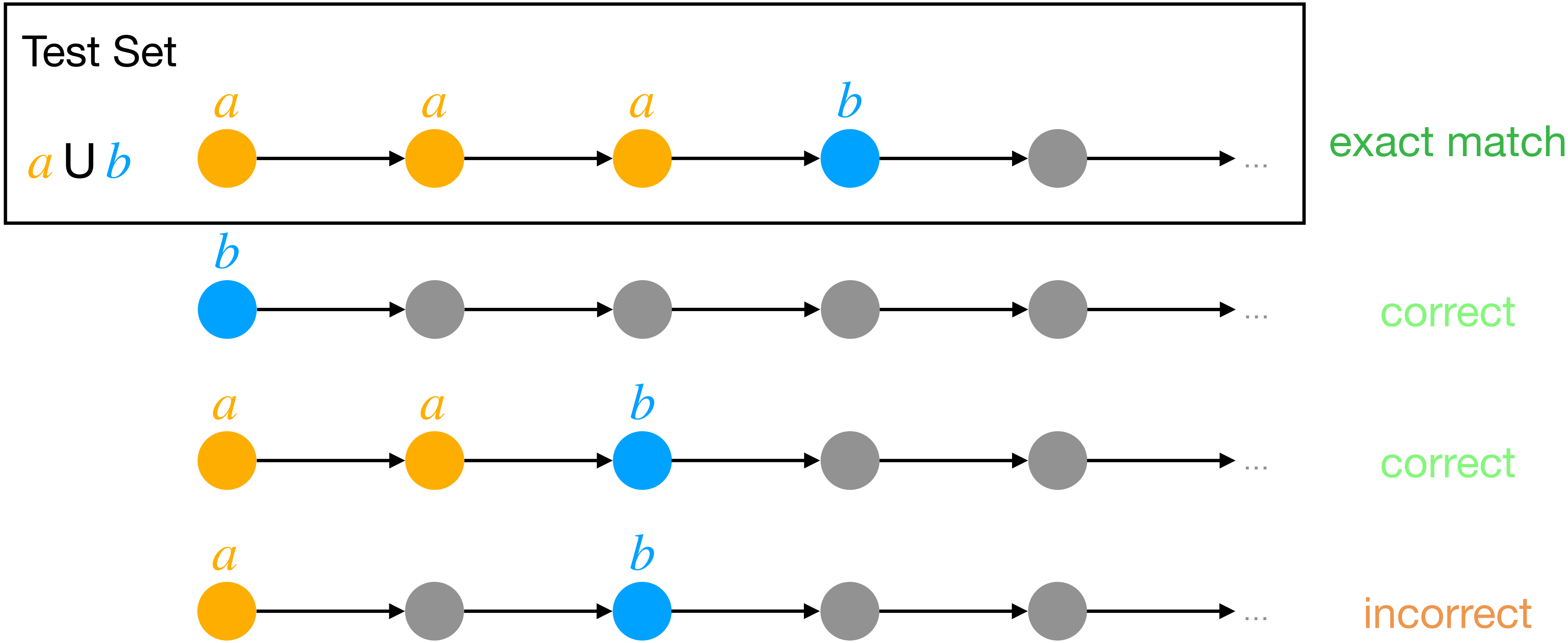
# Neural Trace Generation

## Performance Measures



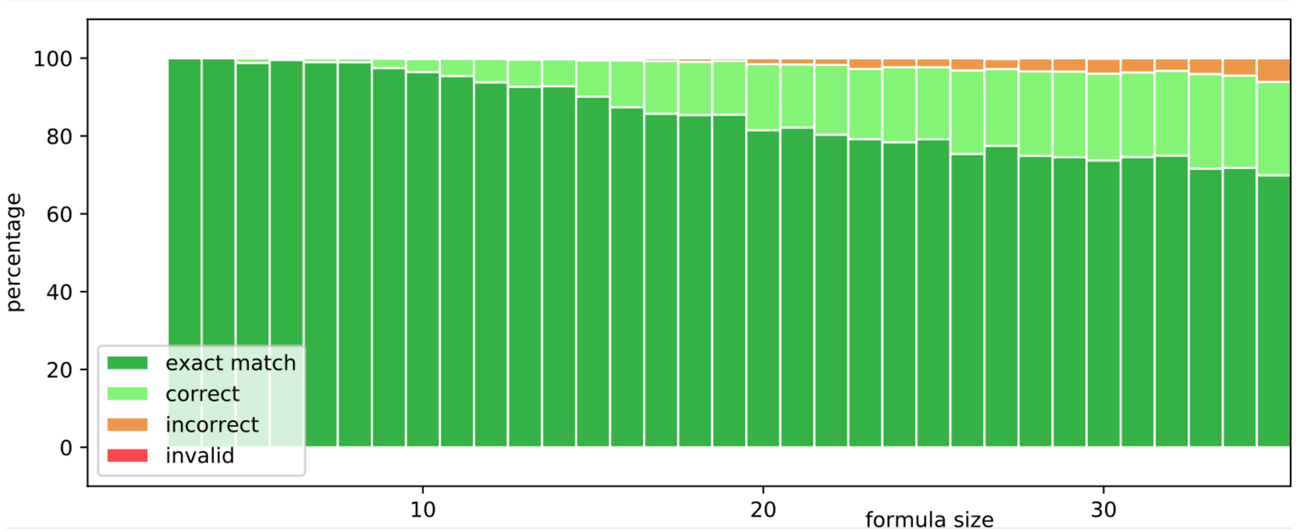
# Neural Trace Generation

## Performance Measures



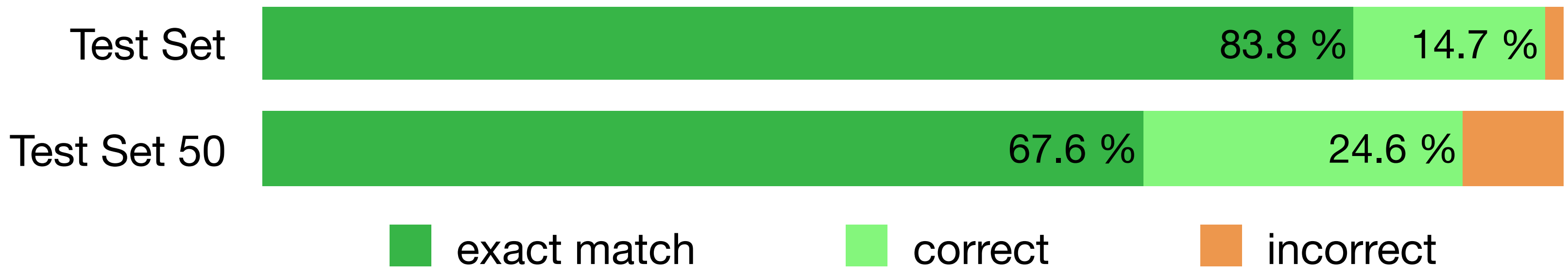
# Neural Trace Generation

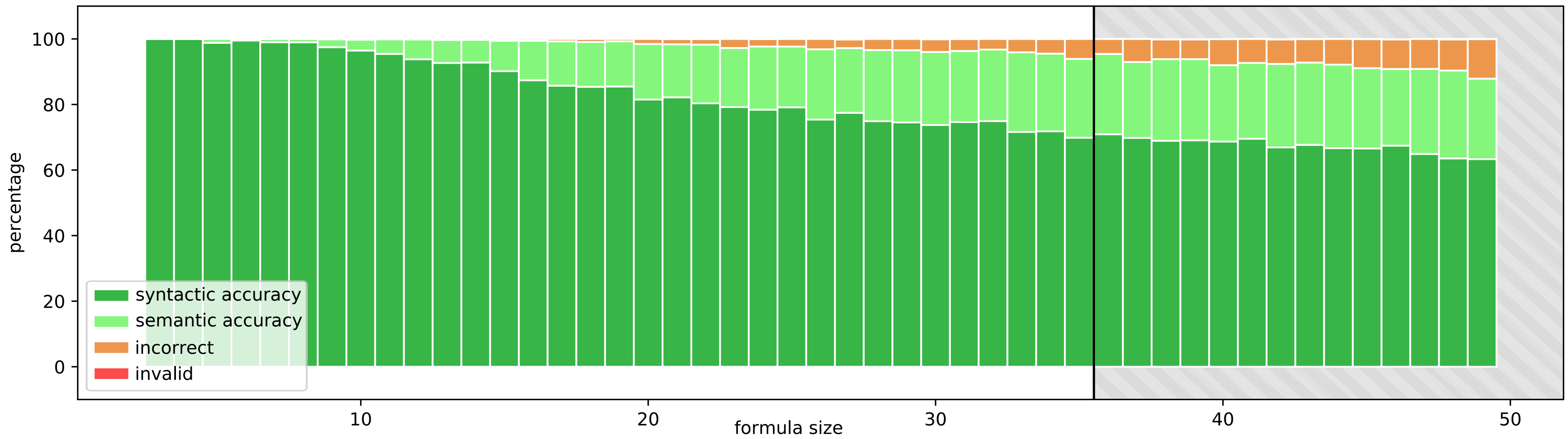
## Results - LTLRandom35

# Neural Trace Generation

## Results - LTLRandom35

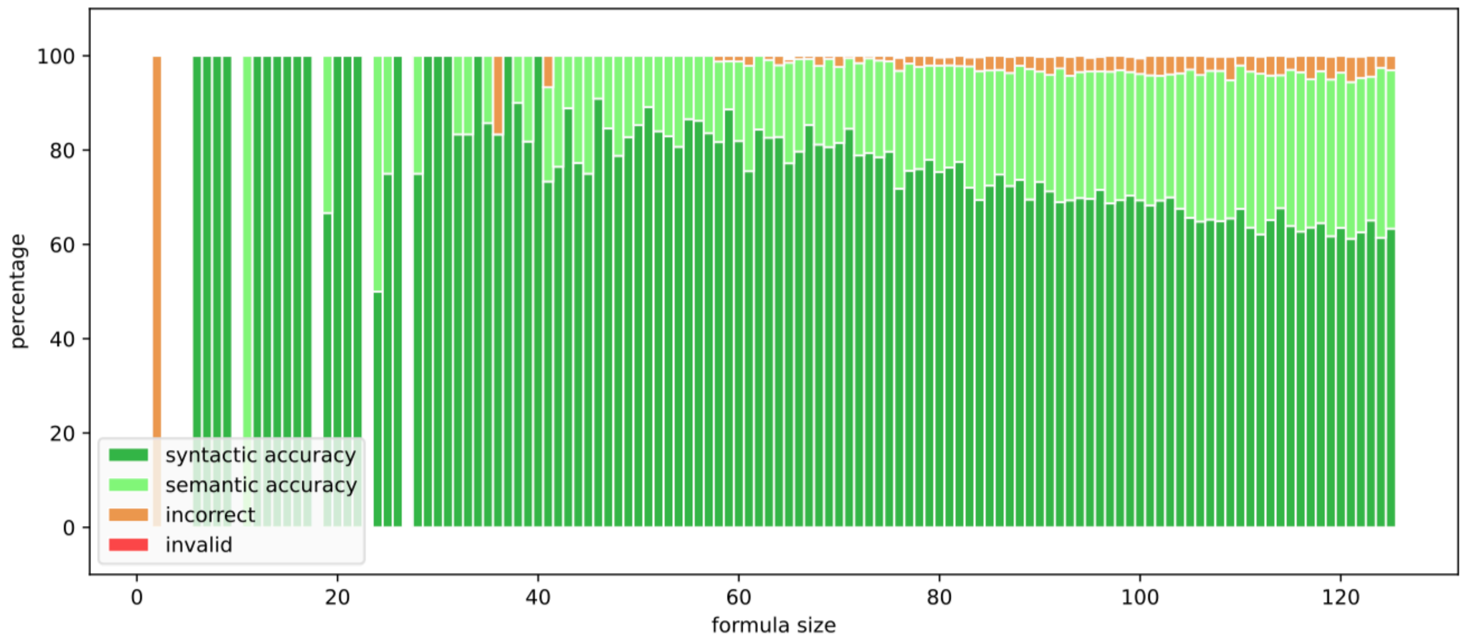



Tree positional encoding: Shiv, V. L. and Quirk, C.: Novel positional encodings to enable tree-based transformers. NeurIPS 2019

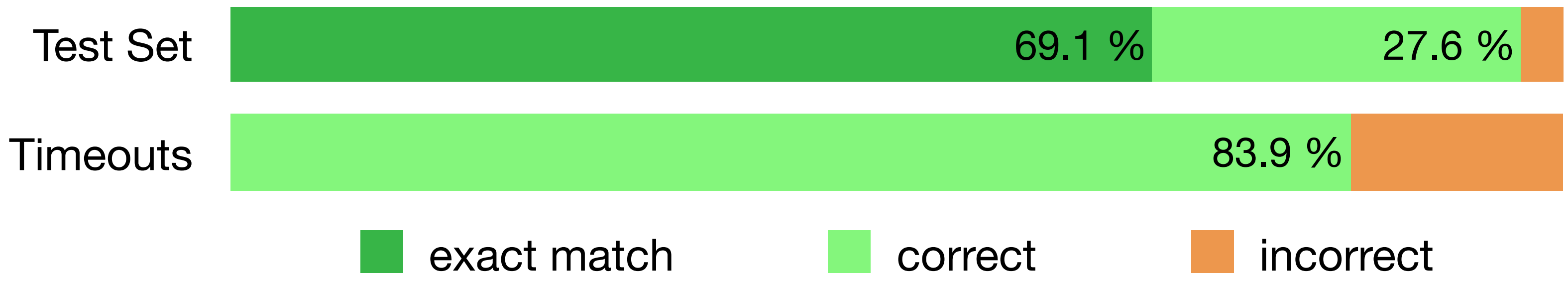
# Neural Trace Generation

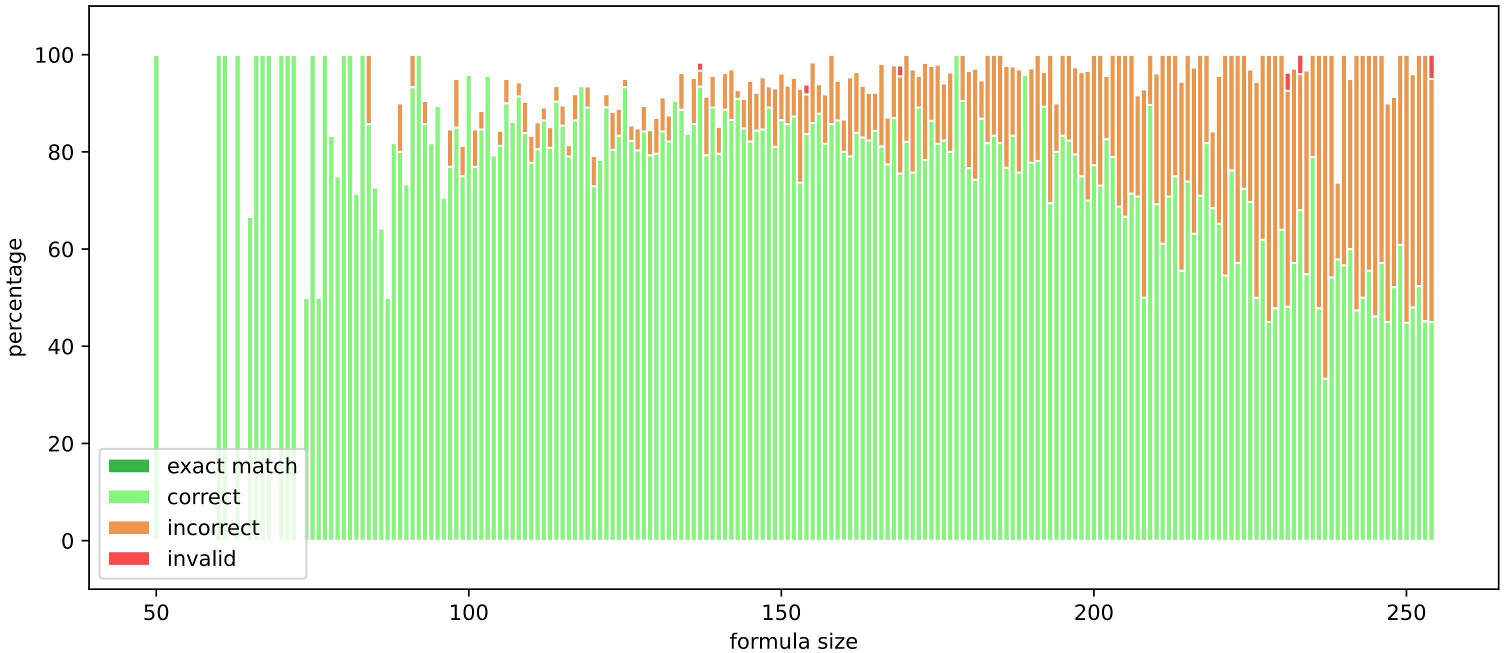
## Results - LTLPattern126

# Neural Trace Generation

## Results - LTLPattern126



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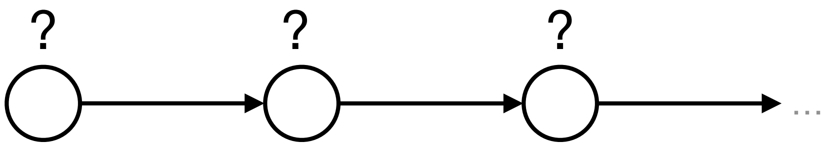
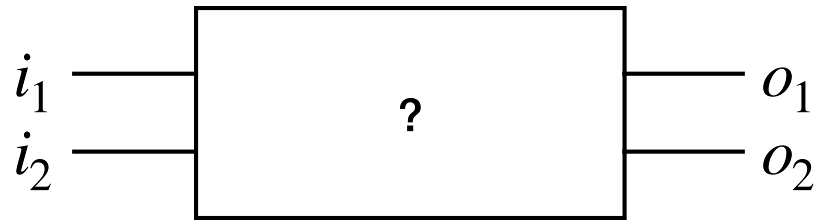
Trace $\pi \models$ LTL Formula $\varphi$




- Semantic generalization
- Generalization to larger formulas with tree positional encoding

## Part 2: Circuit Synthesis

Circuit $C \models$ LTL Specification $\varphi$

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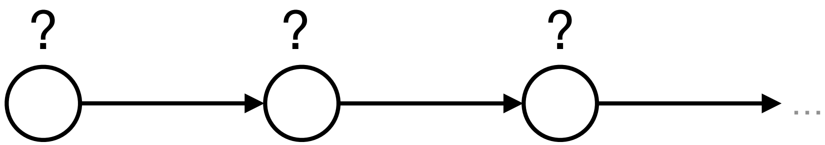


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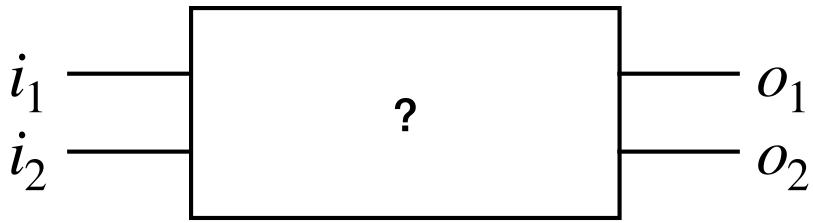


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# Church's Problem

"Given a requirement which a circuit is to satisfy (…). The synthesis problem is then to find recursion equivalences representing a circuit that satisfies the given requirement (or alternatively, to determine that there is no such circuit)."

**Alonzo Church, 1957**

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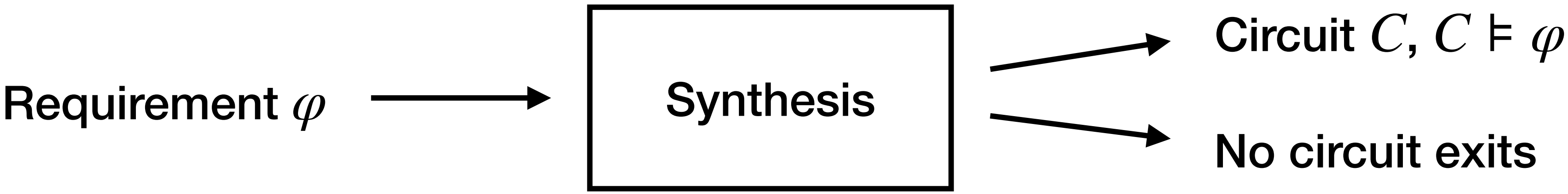

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**Alonzo Church, 1957**

Requirement $\varphi$ → Synthesis → Circuit $C$, $C \models \varphi$

Synthesis → No circuit exits

# Classic LTL Synthesis

**Bounded Synthesis**[1]

**Game-based Synthesis**[2]

Specification

increase bound

Constraint System

Deterministic Automaton

Game on State Graph

Circuit / Unrealizable

The LTL Synthesis Problem is 2EXPTIME-complete[3].

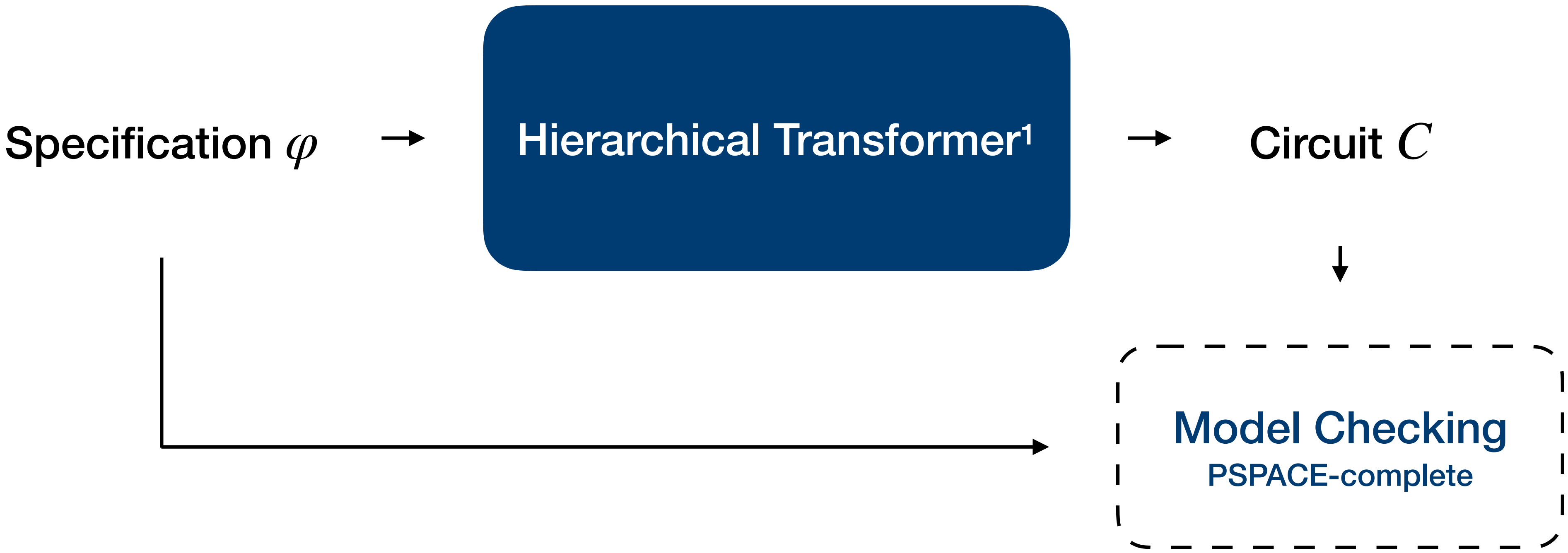
[1] Schewe, S., Finkbeiner, B.: Bounded Synthesis. ATVA 2007
[2] Büchi, J.R., Landweber, L.H.: Solving Sequential Conditions by Finite-State Strategies. Transactions of the American Mathematical Society Vol. 183 1969
[3] Pnueli, A., Rosner, R.: On the Synthesis of a Reactive Module. POPL 1989

# *Neural* LTL Synthesis

Specification $\varphi$ → Hierarchical Transformer[1] → Circuit $C$

[1] Li, W., Yu, L., Wu, Y., Paulson, L.C.: IsarStep: a Benchmark for High-level Mathematical Reasoning. ICLR 2021

# Neural LTL Synthesis

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# Neural LTL Synthesis

## Data Generation from Specification Patterns

$$\begin{aligned} & \square(r_1 \Rightarrow \diamondsuit g_1) \\ \wedge\ & \square(r_2 \Rightarrow \diamondsuit g_2) \\ \wedge\ & \square \neg(g_1 \wedge g_2) \end{aligned}$$

Response

Response

Mutual Exclusion

$$\begin{aligned} AP &= I \cup O \\ I &= \{r_1, r_2\} \\ O &= \{g_1, g_2\} \end{aligned}$$

# Neural LTL Synthesis

## Data Generation from Specification Patterns

$$\square(r_1 \Rightarrow \Diamond g_1)$$ Response

$$\boxed{\wedge}\ \square(r_2 \Rightarrow \Diamond g_2)$$ Response

$$\boxed{\wedge}\ \square\neg(g_1 \wedge g_2)$$ Mutual Exclusion

$$AP = I \cup O$$
$$I = \{r_1, r_2\}$$
$$O = \{g_1, g_2\}$$

Conjunctions of smaller guarantees

# Neural LTL Synthesis

## Data Generation from Specification Patterns

$$\square(r_1 \Rightarrow \lozenge g_1)$$
$$\boxed{\wedge}\ \square(r_2 \Rightarrow \lozenge g_2)$$
$$\boxed{\wedge}\ \square\neg(g_1 \wedge g_2)$$

Response

Response

Mutual Exclusion

$$AP = I \cup O$$
$$I = \{r_1, r_2\}$$
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Conjunctions of smaller guarantees

Frequent patterns

# Neural LTL Synthesis

## Data Generation from Specification Patterns

$$\square(r_1 \Rightarrow \bigcirc(\neg g_2 \mathsf{U} g_1))$$ Prioritized Response

$$\wedge\ \square(r_2 \Rightarrow \Diamond g_2)$$ Response

$$\wedge\ \square\neg(g_1 \wedge g_2)$$ Mutual Exclusion

$$AP = I \cup O$$
$$I = \{r_1, r_2\}$$
$$O = \{g_1, g_2\}$$

Conjunctions of smaller guarantees

Frequent patterns

# Neural LTL Synthesis

## Data Generation from Specification Patterns

$$\boxed{\Rightarrow}\quad \Box\Diamond\neg r_1$$

$$\Box(r_1 \Rightarrow \bigcirc(\neg g_2 \mathsf{U} g_1))$$

$$\boxed{\wedge}\ \Box(r_2 \Rightarrow \Diamond g_2)$$

$$\boxed{\wedge}\ \Box\neg(g_1 \wedge g_2)$$

Infinitely Often $\neg r_1$

Prioritized Response

Response

Mutual Exclusion

$$AP = I \cup O$$
$$I = \{r_1, r_2\}$$
$$O = \{g_1, g_2\}$$

Conjunctions of smaller guarantees

Frequent patterns

Assumptions

# Neural LTL Synthesis

## Data Generation from Specification Patterns

$\square \Diamond \neg r_1$ — Infinitely Often $\neg r_1$

$\boxed{\Rightarrow}$

$\square (r_1 \Rightarrow \bigcirc(\neg g_2 \mathsf{U} g_1))$ — Prioritized Response

$\boxed{\wedge}\ \square (r_2 \Rightarrow \Diamond g_2)$ — Response

$\boxed{\wedge}\ \square \neg(g_1 \wedge g_2)$ — Mutual Exclusion

$$AP = I \cup O$$
$$I = \{r_1, r_2\}$$
$$O = \{g_1, g_2\}$$

Conjunctions of smaller guarantees

Frequent patterns

Assumptions

$$\text{assumption}_1 \wedge \ldots \wedge \text{assumption}_m \Rightarrow \text{guarantee}_1 \wedge \ldots \wedge \text{guarantee}_n$$

# Neural LTL Synthesis

## Data Generation from Specification Patterns

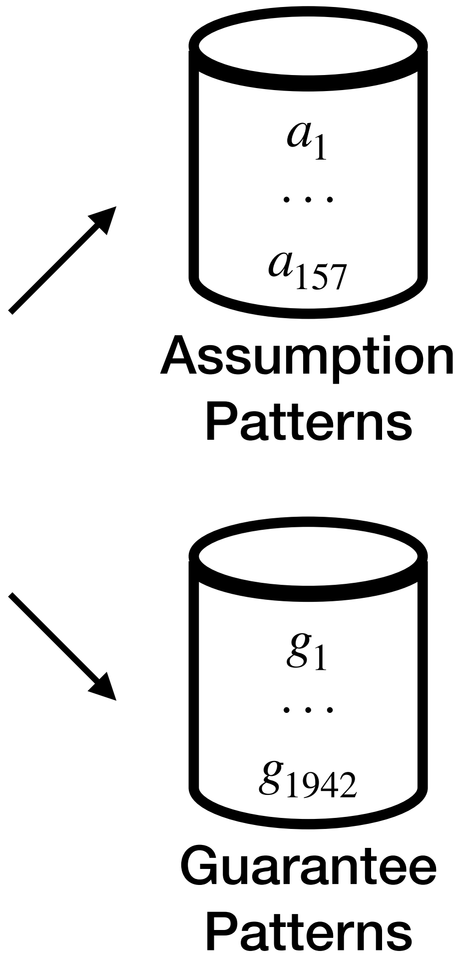
**346 SYNTCOMP[1] Specifications**

[1] http://www.syntcomp.org

# Neural LTL Synthesis

## Data Generation from Specification Patterns

346 SYNTCOMP[1]
Specifications

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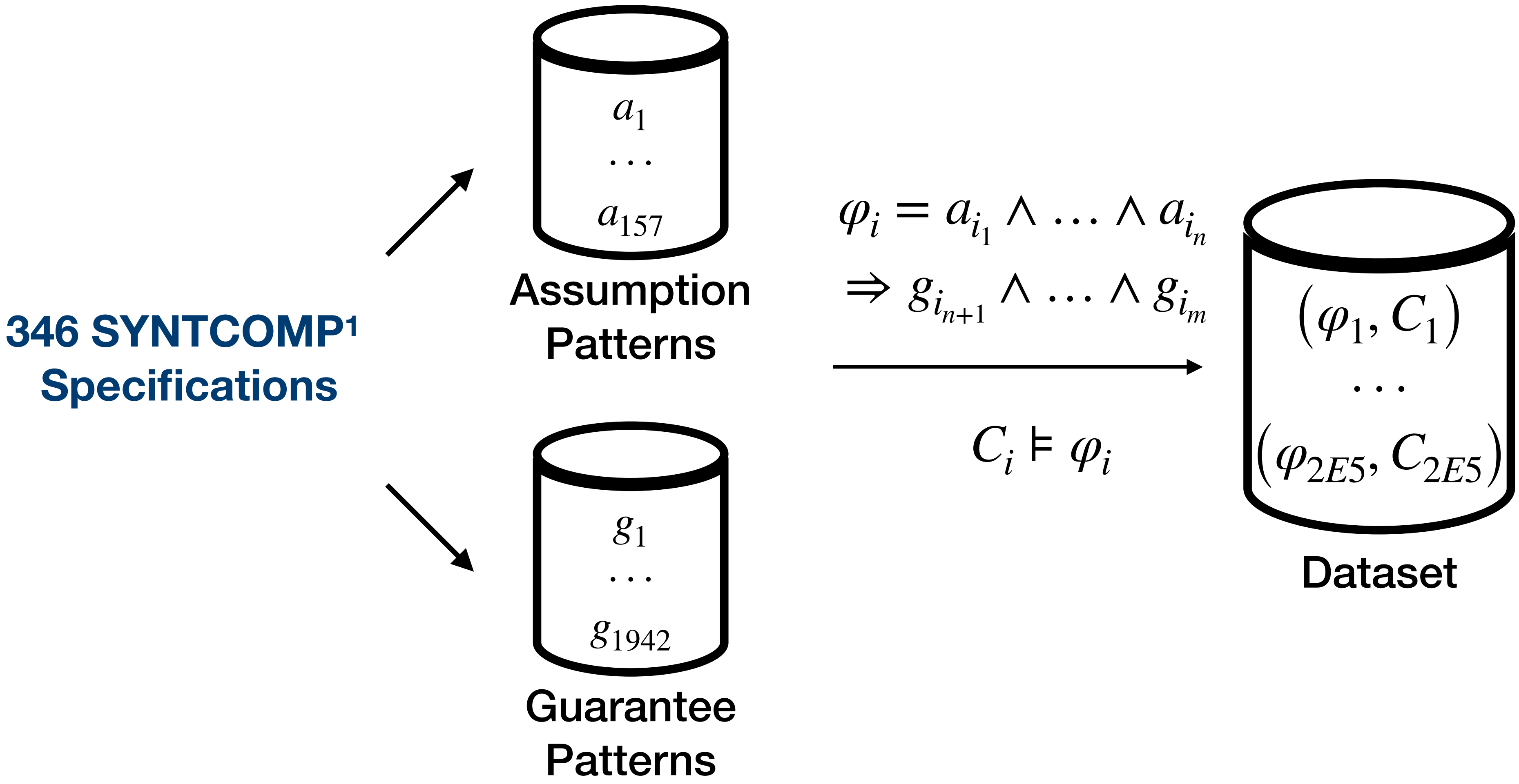



[1] http://www.syntcomp.org

# Neural LTL Synthesis

## Hierarchical Transformer[1]

AIGER Circuit $C$

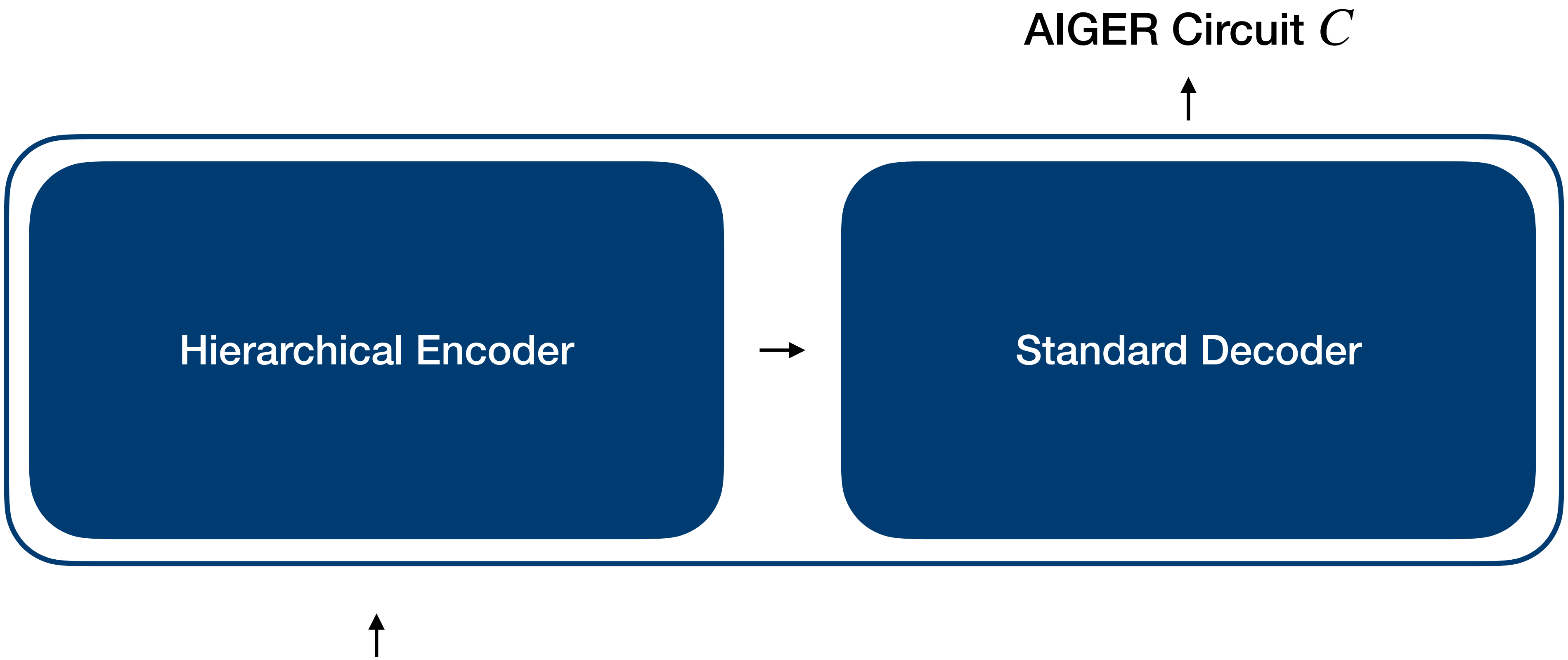
Hierarchical Transformer

$\square(r_1 \Rightarrow \Diamond g_1)$ $\square(r_2 \Rightarrow \Diamond g_2)$ $\square\neg(g_1 \wedge g_2)$

[1] Li, W., Yu, L., Wu, Y., Paulson, L.C.: IsarStep: a Benchmark for High-level Mathematical Reasoning. ICLR 2021

# Neural LTL Synthesis

## Hierarchical Transformer[1]

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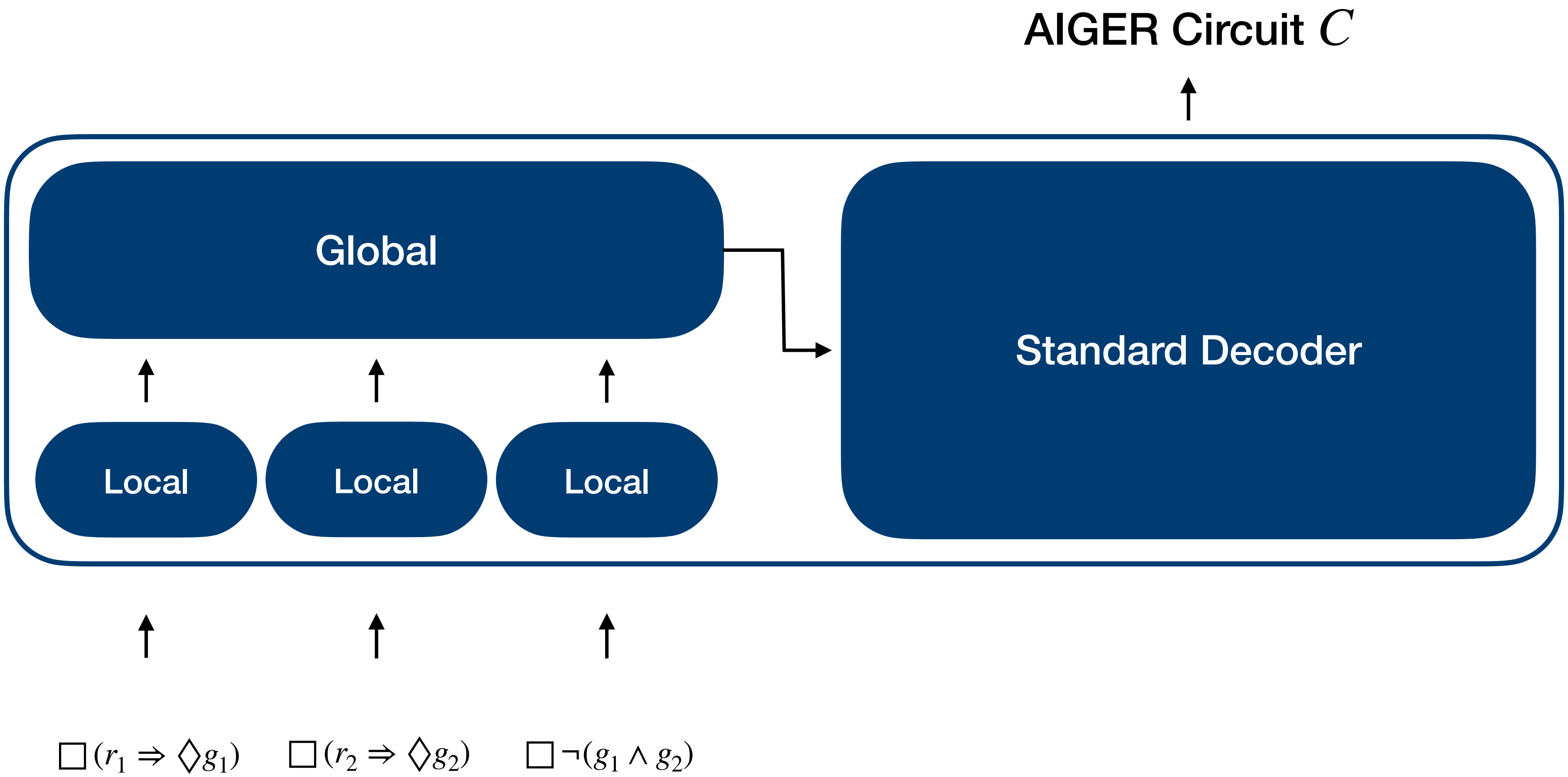


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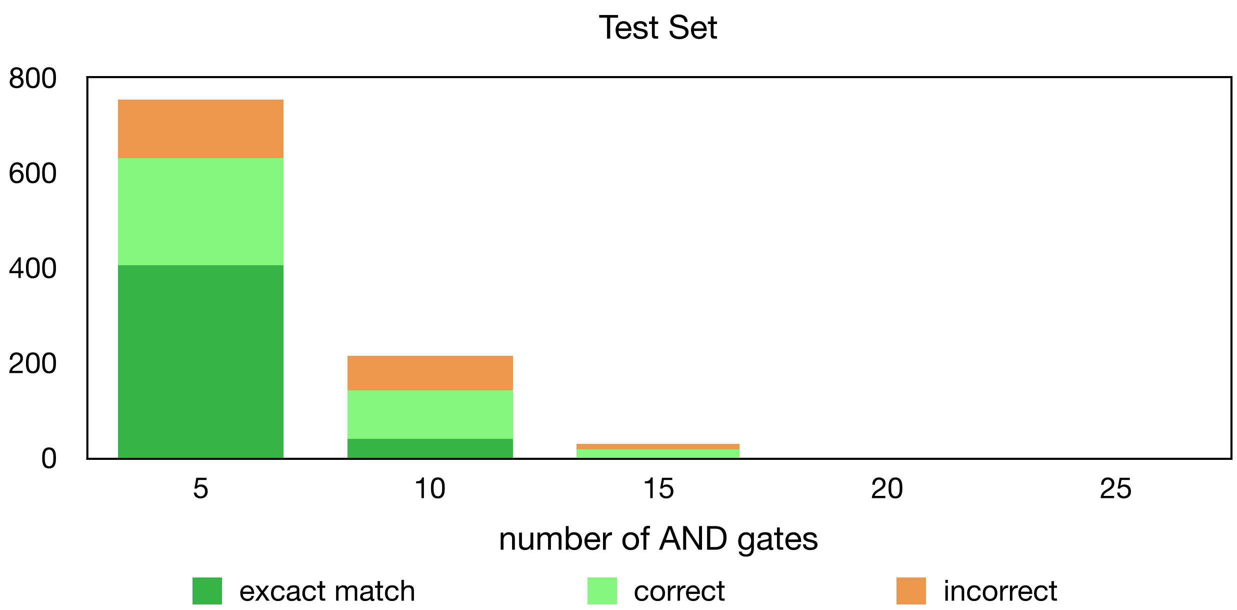
# Neural LTL Synthesis Results

## Test Set

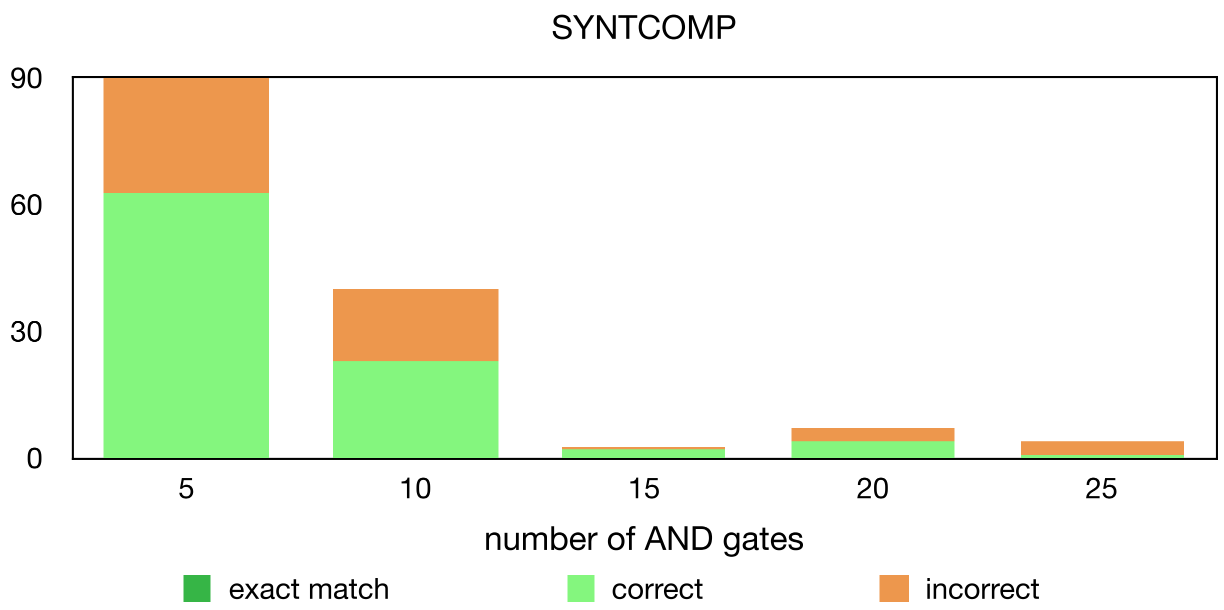
| | Beam Size 1 | Beam Size 4 | Beam Size 8 | Beam Size 16 |
|---|---|---|---|---|
| Test Set | 53.6 | 70.4 | 75.8 | 79.9 |



# Neural LTL Synthesis Results

## SYNTCOMP[1]

| | Beam Size 1 | Beam Size 4 | Beam Size 8 | Beam Size 16 |
|---|---|---|---|---|
| Test Set | 53.6 | 70.4 | 75.8 | 79.9 |
| SYNTCOMP | 51.9 | 60.0 | 63.6 | 66.8 |

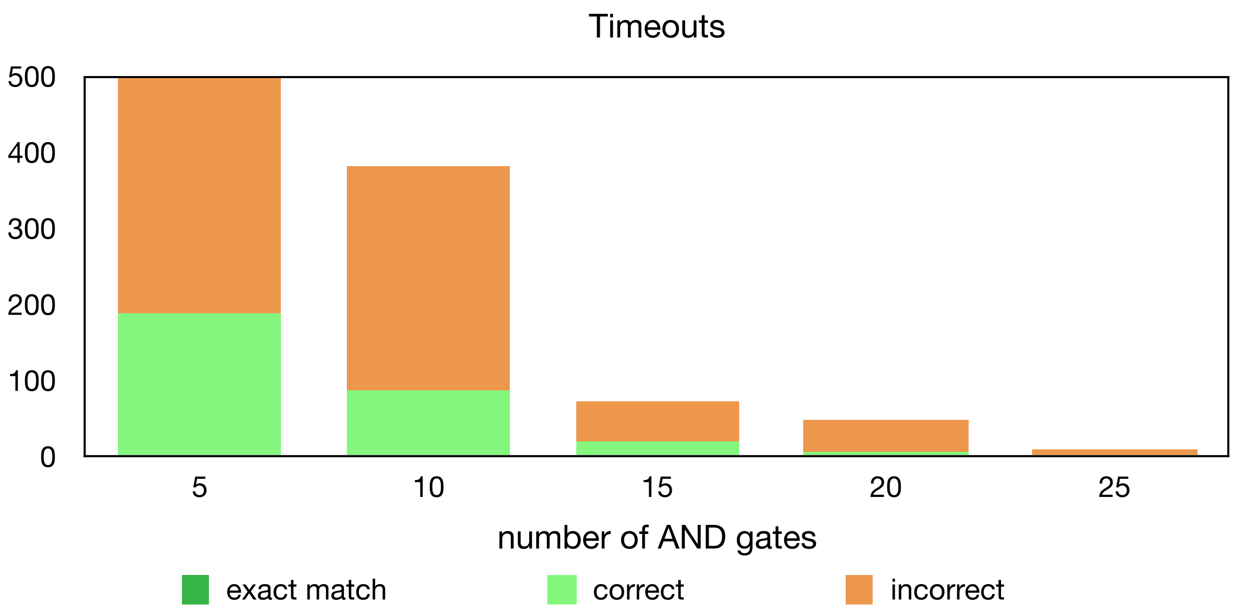



[1] http://www.syntcomp.org

# Neural LTL Synthesis Results

## Timeouts

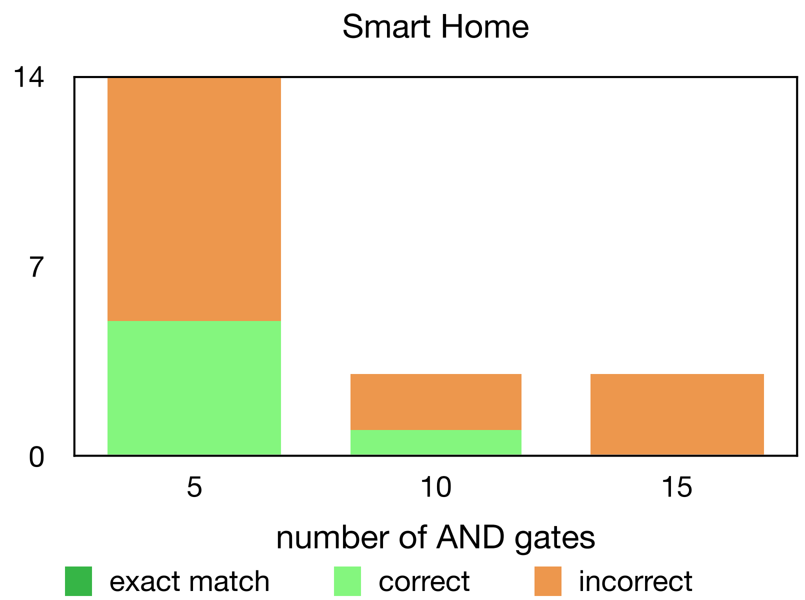
| | Beam Size 1 | Beam Size 4 | Beam Size 8 | Beam Size 16 |
|---|---|---|---|---|
| Test Set | 53.6 | 70.4 | 75.8 | 79.9 |
| SYNTCOMP | 51.9 | 60.0 | 63.6 | 66.8 |
| Timeouts | 11.7 | 21.1 | 25.9 | 30.1 |



# Neural LTL Synthesis Results

## Smart Home[1]

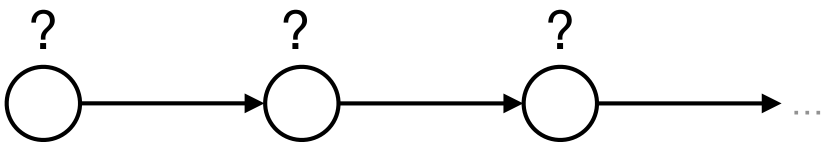
| | Beam Size 1 | Beam Size 4 | Beam Size 8 | Beam Size 16 |
|---|---|---|---|---|
| Test Set | 53.6 | 70.4 | 75.8 | 79.9 |
| SYNTCOMP | 51.9 | 60.0 | 63.6 | 66.8 |
| Timeouts | 11.7 | 21.1 | 25.9 | 30.1 |
| Smart Home | 22.9 | 31.4 | 44.8 | 40.0 |




[1] J.A.R.V.I.S. TSL/TLSF Benchmark Suite, 2021.

# Part 1: Trace Generation

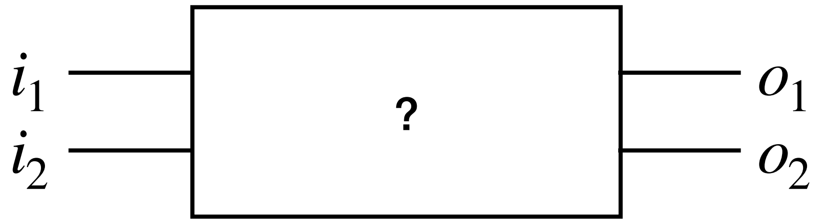
**Trace $\pi \models$ LTL Formula $\varphi$**




- Semantic generalization
- Generalization to larger formulas with tree positional encoding

# Part 2: Circuit Synthesis

**Circuit $C \models$ LTL Specification $\varphi$**




- Circuit synthesis end-to-end
- Generalizes to SYNTCOMP benchmarks

Hahn, C., S., F., Kreber, J.U., Rabe, M.N., Finkbeiner, B.: Teaching Temporal Logics to Neural Networks. ICLR 2021
S., F., Hahn, C., Rabe, M.N., Finkbeiner, B.: Neural Circuit Synthesis From Specification Patterns. arXiv Preprint 2021

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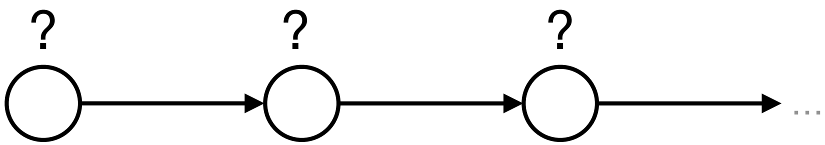
**Trace $\pi \models$ LTL Formula $\varphi$**




- Semantic generalization
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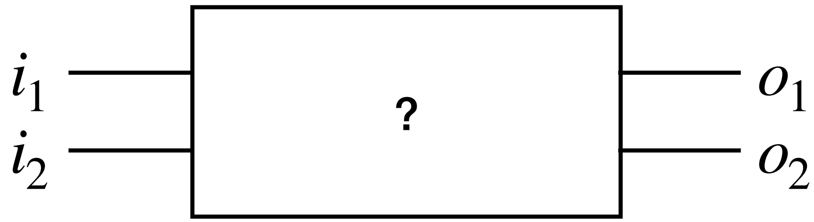
# Part 2: Circuit Synthesis

**Circuit $C \models$ LTL Specification $\varphi$**




- Circuit synthesis end-to-end
- Generalizes to SYNTCOMP benchmarks

With deep learning new types of fast algorithms for verification and synthesis can be developed.

Hahn, C., S., F., Kreber, J.U., Rabe, M.N., Finkbeiner, B.: Teaching Temporal Logics to Neural Networks. ICLR 2021
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