

mædmax at School

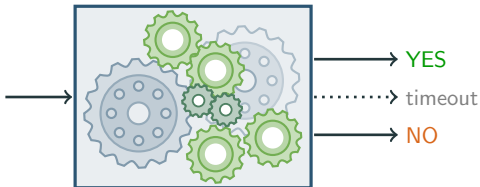
Learning Selection in Equational Reasoning

Sarah Winkler
University of Innsbruck

4th Conference on Artificial Intelligence and Theorem Proving
10 April 2019

Equational Theorem Proving

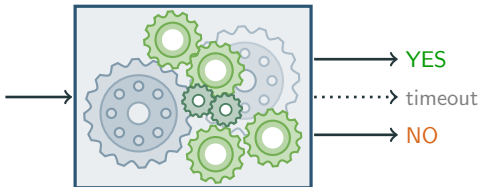
$$\begin{aligned}0 + x &\approx x \\ (-x) + x &\approx 0 \\ x + (y + z) &\approx (x + y) + z \\ x + y &\approx y + x \\ x \cdot (y \cdot z) &\approx (x \cdot y) \cdot z \\ (x + y) \cdot z &\approx (x \cdot z) + (y \cdot z) \\ a \cdot 0 &\approx 0\end{aligned}$$



- ▶ input: set of equations \mathcal{E}_0 and goal $s \approx t$
- ▶ output: YES if $\mathcal{E}_0 \models s \approx t$ or NO otherwise

Equational Theorem Proving

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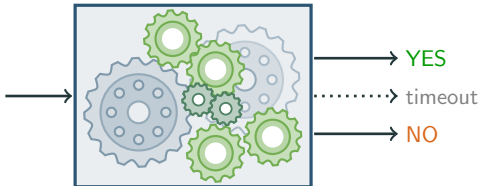
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maedmax

equational theorem proving tool based on maximal ordered completion

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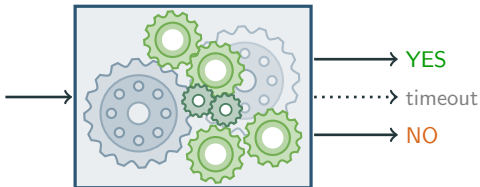
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(**maedmax**: equational **d**eduction **max**imized)

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S. Winkler and G. Moser.

Maedmax: A Maximal Ordered Completion Tool.

In *Proc. 9th IJCAR*, LNCS 10900, pp. 472-480, 2018.

Maximal Ordered Completion

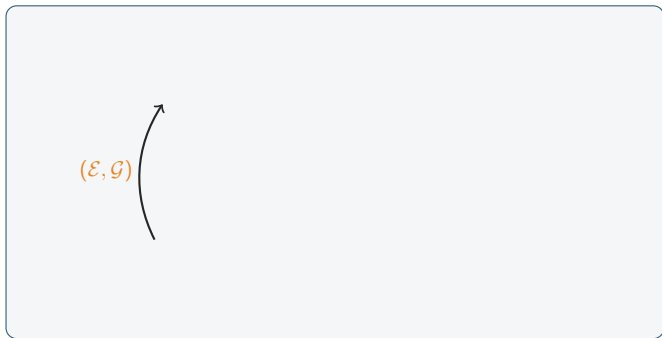
Learning Experiments

Equation Selection

Proof Progress

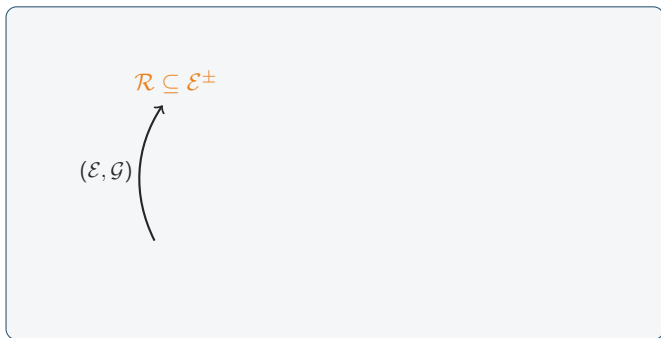
Conclusion

Maximal Ordered Completion



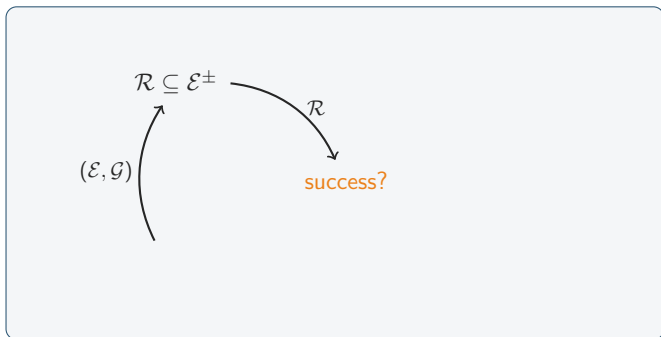
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Maximal Ordered Completion



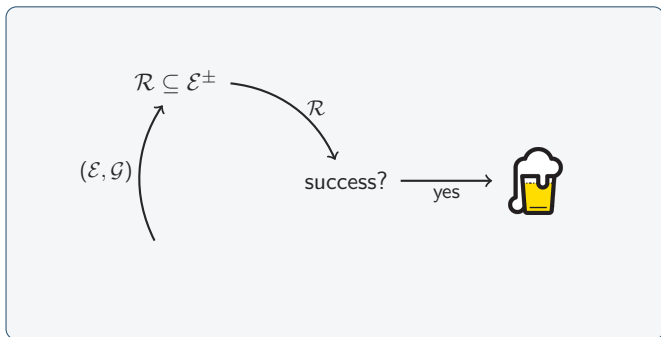
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- 1 get **terminating** rewrite system \mathcal{R} from \mathcal{E}

Maximal Ordered Completion



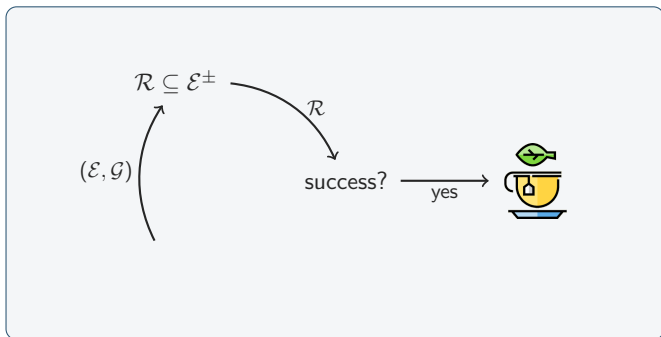
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Maximal Ordered Completion



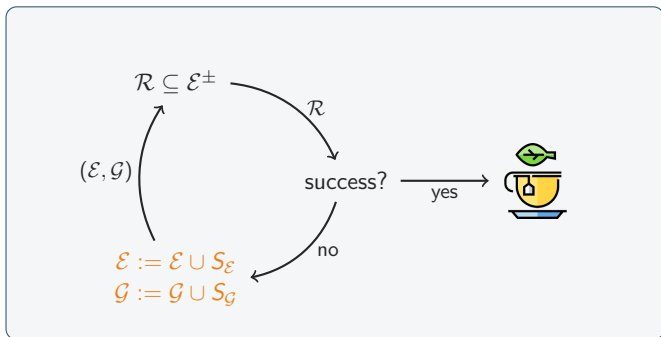
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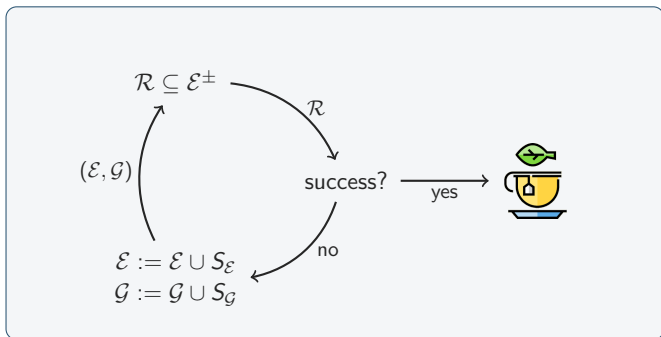
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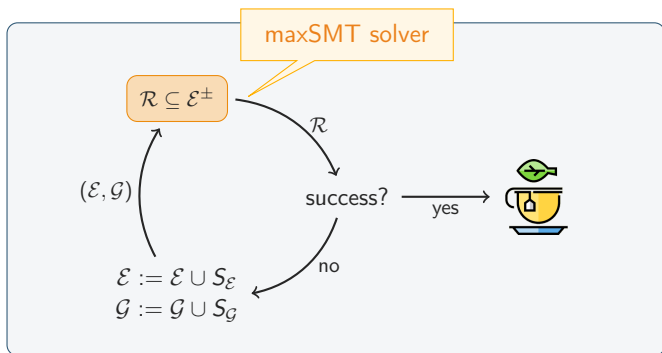
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- 1 get terminating rewrite system \mathcal{R} from \mathcal{E}
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- 3 add some **critical pairs** $S_{\mathcal{E}} \subseteq \text{CP}_>(\mathcal{R} \cup \mathcal{E})$ and $S_{\mathcal{G}} \subseteq \text{CP}_>(\mathcal{R} \cup \mathcal{E}, \mathcal{G})$

Maximal Ordered Completion



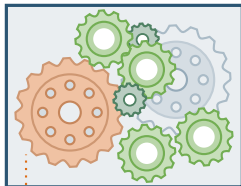
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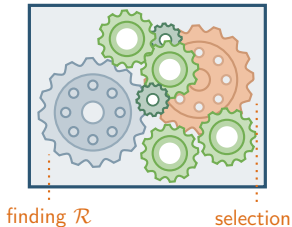
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Critical Choice Points



finding \mathcal{R}

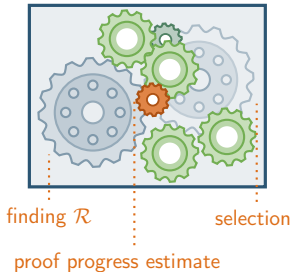
Critical Choice Points



Selecting things

- ▶ selection of $S_{\mathcal{E}}$ and $S_{\mathcal{G}}$ is highly critical
- ▶ on average only 15% of selected equations and goals used for proof
- ▶ **learn** better choice criteria

Critical Choice Points



Estimating proof progress

- ▶ heuristic proof progress estimate: if stuck
 - ▶ add additional (old) equations
 - ▶ or ultimately restart
- ▶ **learn** better estimate

it's all about the next (small) thing

Maximal Ordered Completion

Learning Experiments

Equation Selection

Proof Progress

Conclusion

Learning Equation Selection

Features

- ▶ **state**: number of iterations, equations, and goals
- ▶ **equation**:
 - ▶ **hand-crafted**: polarity, size, size difference, age, orientability, linearity, duplicatingness, # of matches and critical pairs on \mathcal{E}

Learning Equation Selection

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- ▶ state: number of iterations, equations, and goals
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 - ▶ term structure: ~ 200 *pq-gram* counts



N. Augsten, M. Böhlen, and J. Gamper.

The *pq-gram* distance between ordered labeled trees.

ACM Transactions on Database Systems, 35(1):1–36, 2010.

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Example (pq -Grams)

$$f(i(f(i(x), y)), i(x)) \approx i(y)$$

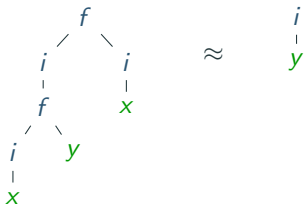
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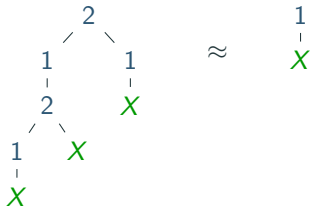
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- ▶ abstract symbol names to arities, variables to X



Learning Equation Selection

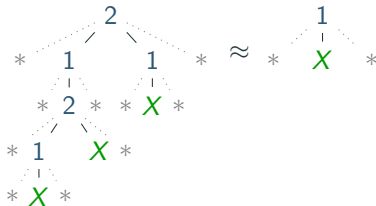
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- ▶ abstract symbol names to arities, variables to X
- ▶ add **dummy** nodes



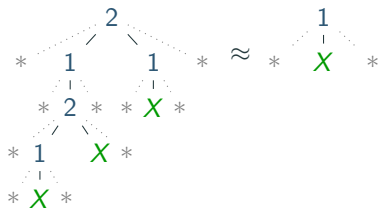
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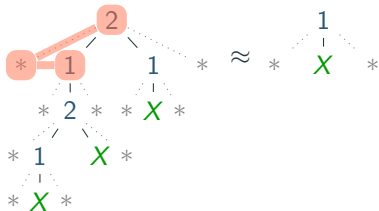
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 $2.*.1$

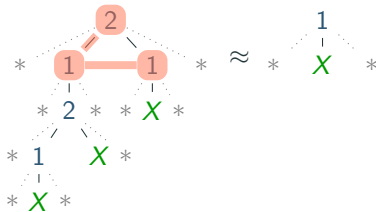
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2.*.1, 2.1.1

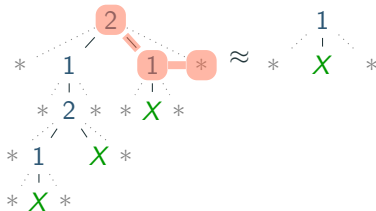
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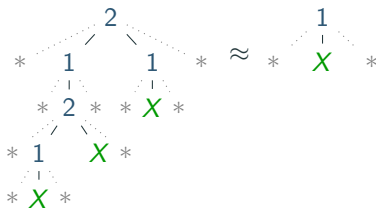
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2.*.1, 2.1.X, 2.X.*, 1.*.X, 1.X.*,
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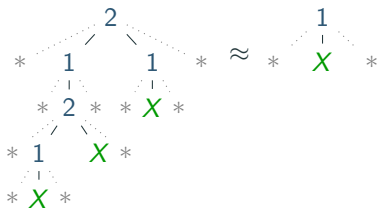
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 $2.*.1, 2.1.X, 2.X.*, 1.*.X, 1.X.*,$
 $1.*.X, 1.X.*$ and $1.*.X, 1.X.*$
- ▶ count up to arity 3

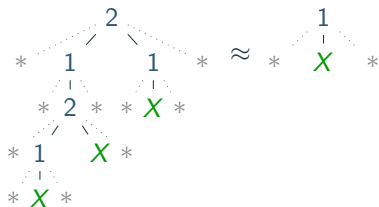
Learning Equation Selection

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Example (pq -Grams)

$$f(i(f(i(x), y)), i(x)) \approx i(y)$$



- ▶ abstract syntactic variables to
- ▶ add dummy
- ▶ pq -grams.
- ▶ for $p = 2$,
2.*.1, 2.1.1
2.*.1, 2.1.X
1.*.X, 1.X.
- ▶ count up to

1	polarity	0
2	size	10
3	size diff	6
4	age	42
	⋮	
16	2.2.1	0
17	2.*.1	2
18	2.1.1	1
19	2.1.0	0
20	1.*.X	2
21	1.X.*	2
	⋮	
121	2.2.1	0
122	2.*.1	0
123	2.1.1	0
124	2.1.0	0
125	1.*.X	1
126	1.X.*	1
	⋮	

Binary classification

classify equation as **positive** (useful) if occurs in proof, **negative** otherwise

Binary classification

classify equation as positive (useful) if occurs in proof, negative otherwise

Data collection

- ▶ run `mædmax` in different strategies, **recording selections**
- ▶ check classification of all selected literals upon successful proof
- ▶ only 15% positives: **duplicate positives** for balancing

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`random forest` of 100 trees and maximal depth 14

- ▶ fast evaluation, in tests slightly better recall than SVC, extra trees

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Setup

- ▶ evaluate data with Python Scikit
- ▶ export random forest to json, load into `mædmax` (OCaml)

Reinforcement loop experiment

- ▶ start with **random selection** \mathcal{M}_0

Results

	solved	useful
\mathcal{M}_0	206	8%

- ▶ run on 897 UEQ problems in TPTP, 60s timeout

Reinforcement loop experiment

- ▶ start with random selection \mathcal{M}_0
- ▶ collect selections \mathcal{S}_0 running \mathcal{M}_0 with 3 different strategies

Results

	solved	useful	$ \mathcal{S}_i $
\mathcal{M}_0	206	8%	114K

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Reinforcement loop experiment

- ▶ start with random selection \mathcal{M}_0
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- ▶ build **classifier**

Results

	solved	useful	$ \mathcal{S}_i $	precision	recall	f1
\mathcal{M}_0	206	8%	114K	0.86	0.94	0.9

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- ▶ build classifier: \mathcal{M}_1 selects literals e with $P_{\text{positive}}(e) > 0.4$

Results

	solved	useful	$ \mathcal{S}_i $	precision	recall	f1
\mathcal{M}_0	206	8%	114K	0.86	0.94	0.9
\mathcal{M}_1	409	14%				

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- ▶ **selections \mathcal{S}_1** : run \mathcal{M}_1 with 3 different strategies, add to \mathcal{S}_0

Results

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\mathcal{M}_0	206	8%	114K	0.86	0.94	0.9
\mathcal{M}_1	409	14%	358K	0.77	0.83	0.8
\mathcal{M}_2	423	15%	528K	0.76	0.83	0.79
\mathcal{M}_3	433	14%	704K	0.78	0.8	0.79

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Results

time spent on selection rises from 0% (random) to 8-10%

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- ▶ run on 897 UEQ problems in TPTP, 60s timeout
- ▶ other baselines: size solves 575, fifo 366, **best 609**

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- ▶ selections \mathcal{S}_1 : run \mathcal{M}_1 with 3 different strategies, add to \mathcal{S}_0
- ▶ ...

combination with old strategy solves 625

Results

	solved	useful	$ \mathcal{S}_i $	precision	recall	f1
\mathcal{M}_0	206	8%	114K	0.86	0.94	0.9
\mathcal{M}_1	409	14%	358K	0.77	0.83	0.8
\mathcal{M}_2	423	15%	528K	0.76	0.83	0.79
\mathcal{M}_3	433	14%	704K	0.78	0.8	0.79

- ▶ run on 897 UEQ problems in TPTP, 60s timeout
- ▶ other baselines: size solves 575, fifo 366, best 609

Reinforcement loop experiment

- ▶ start with random selection \mathcal{M}_0
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- ▶ run on 897 UEQ problems in TPTP, 60s timeout
- ▶ other baselines: size solves 575, fifo 366, best 609
- ▶ **feature importance**: 60% *pq*-grams, 40% hand crafted
most useful: literal size, # active literals, # matches and CPs

State features

- ▶ size of \mathcal{E}
- ▶ # SMT checks
- ▶ cost of last maxSMT check
- ▶ memory used
- ▶ # CPs with \mathcal{R}
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Learning Proof Progress

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- ▶ store difference of proof state vectors between two iterations, along with progress classification
- ▶ get data of about 20K iterations

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Overall gain

+4.5% solved problems combining new selection classifier with old selection, and adding new progress estimate

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- ▶ a bit more



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What did not work

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What did not work

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What's next

- ▶ more data
- ▶ more/other features
 - ▶ more state features?
 - ▶ longer pq -grams, or vertical ENIGMA features?
- ▶ more experiments
 - ▶ how to combine with previous selection strategy?
 - ▶ use proof progress estimate also for restarts?
- ▶ ...