Tapping Sources of Mathematical (Big) Data

Michael Kohlhase

Professur für Wissensrepräsentation und -verarbeitung
Informatik, FAU Erlangen-Nürnberg
http://kwarc.info

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- Could use DLAI help (but not in ATP improvements)
- I am looking for good GOFAI Ph.D. students (maybe even DLFAI)

1 Background: Towards a Math Digital Library

Towards a World Digital Library of Mathematics

- Mathematics plays a fundamental role in Science, Technology, and Engineering
  (learn from Math, apply for STEM)
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  - 50 million science articles in 2010 [Jin10] with a doubling time of 8-15 years [Lvl10]

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- We need to preserve this heritage and make it accessible to working mathematicians!
- The EUDML Project digitized large amounts of European Journals
- The (US) National Research Council issued a Plan/Report for a "World Digital Heritage Library of Mathematics" [DLC+14].
  - Form a non-profit organization IMKT (Sloan grant for founding)
  - digitize, standardize, and semanticize math content ($\rightsquigarrow$ added value services)
  - Collaborate with Publishers/Organizations (to obtain rights)
- The International Mathematical Union (IMU) chartered a WG to bring this about.

Background: Mathematical Documents

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- Mathematical knowledge is rich in content, sophisticated in structure, and technical in presentation,
- its conservation, dissemination, and utilization constitutes a challenge for the community and an attractive line of inquiry.
- Challenge: How can/should we do mathematics in the 21st century?
- Mathematical knowledge and objects are transported by documents
- Three levels of electronic documents:
  0. printed (for archival purposes) (∼90%)
  1. digitized (usually from print) (∼50%)
  2. presentational: encoded text interspersed with presentation markup (∼20%)
  3. semantic: encoded text with functional markup for the meaning (≤0.1%)

  transforming down is simple, transforming up needs humans or AI.
- Observation: Computer support for access, aggregation, and application is (largely) restricted to the semantic level.
- This talk: How do we do maths and math documents at the semantic level?

But there is is more Math Knowledge than Documents

- There are large mathematical data bases
  - Zentralblatt Math: the first resource in Maths (http://zbmath.org)
  - MathSciNet: Mathematical Reviews (http://www.ams.org/mathscinet/)
  - LMFDB: *L*-functions & Modular Forms (http://lmfdb.org)
  - OEIS: Open Encyclopedia of Integer Sequences (http://oeis.org)
  - FindStat: Combinatoria Statistics Finder (http://findstat.org)
  - MGP: Math Genealogy Project (http://www.genealogy.math.ndsu.nodak.edu)

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- Idea: Some of this information is already in a semantic/machine-actionable form.
- Problems: licenses, representations, versioning, GUIs, system APIs, ...
- Idea: To arrive at a core DML start at Math DBs and
  - specify open licenses $\leadsto$ data commons
  - standardize representations $\leadsto$ knowledge commons
  - even in maths, data changes $\leadsto$ support versioning
  - system APIs $\leadsto$ collaborate on content, compete on services

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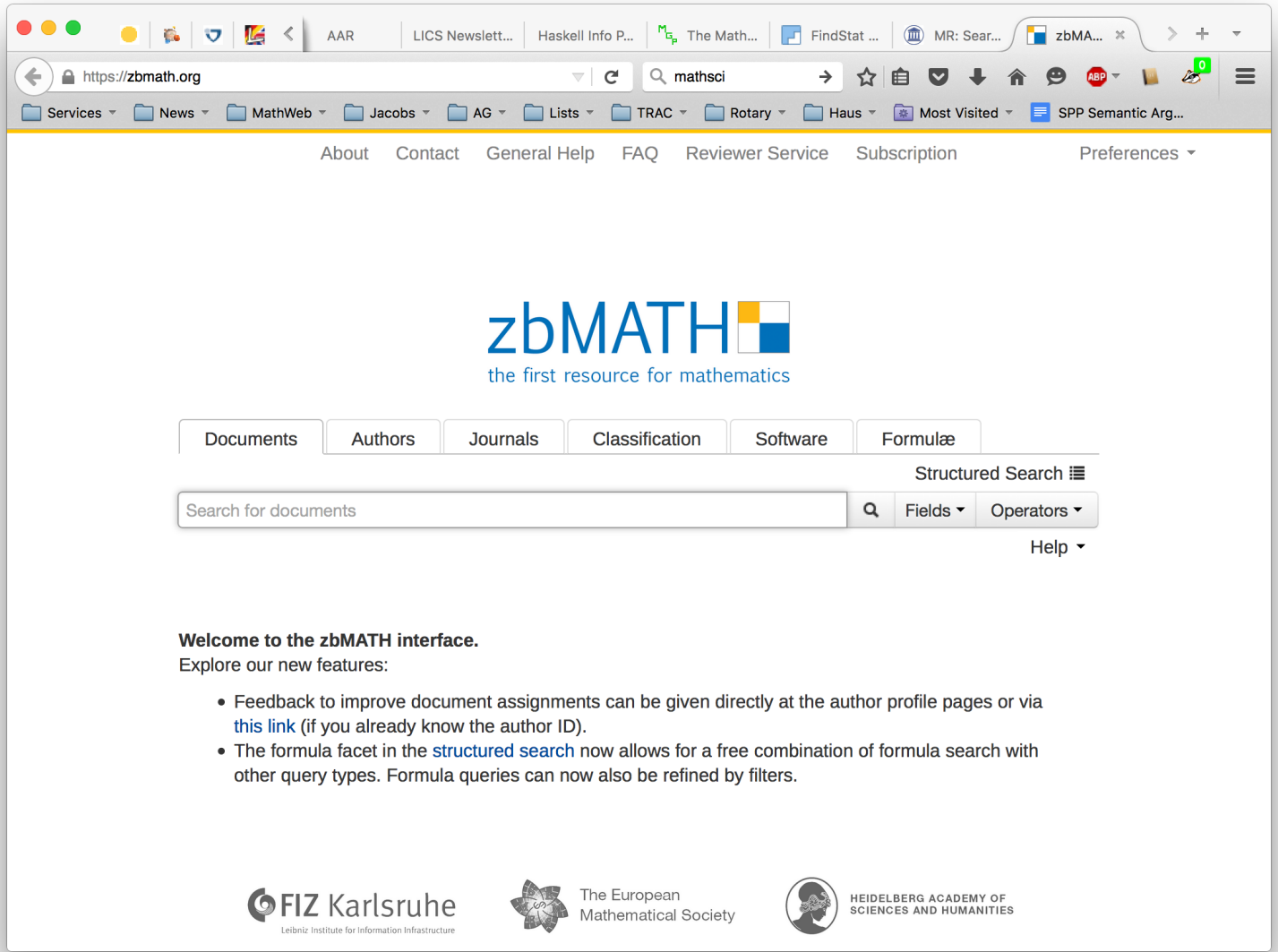
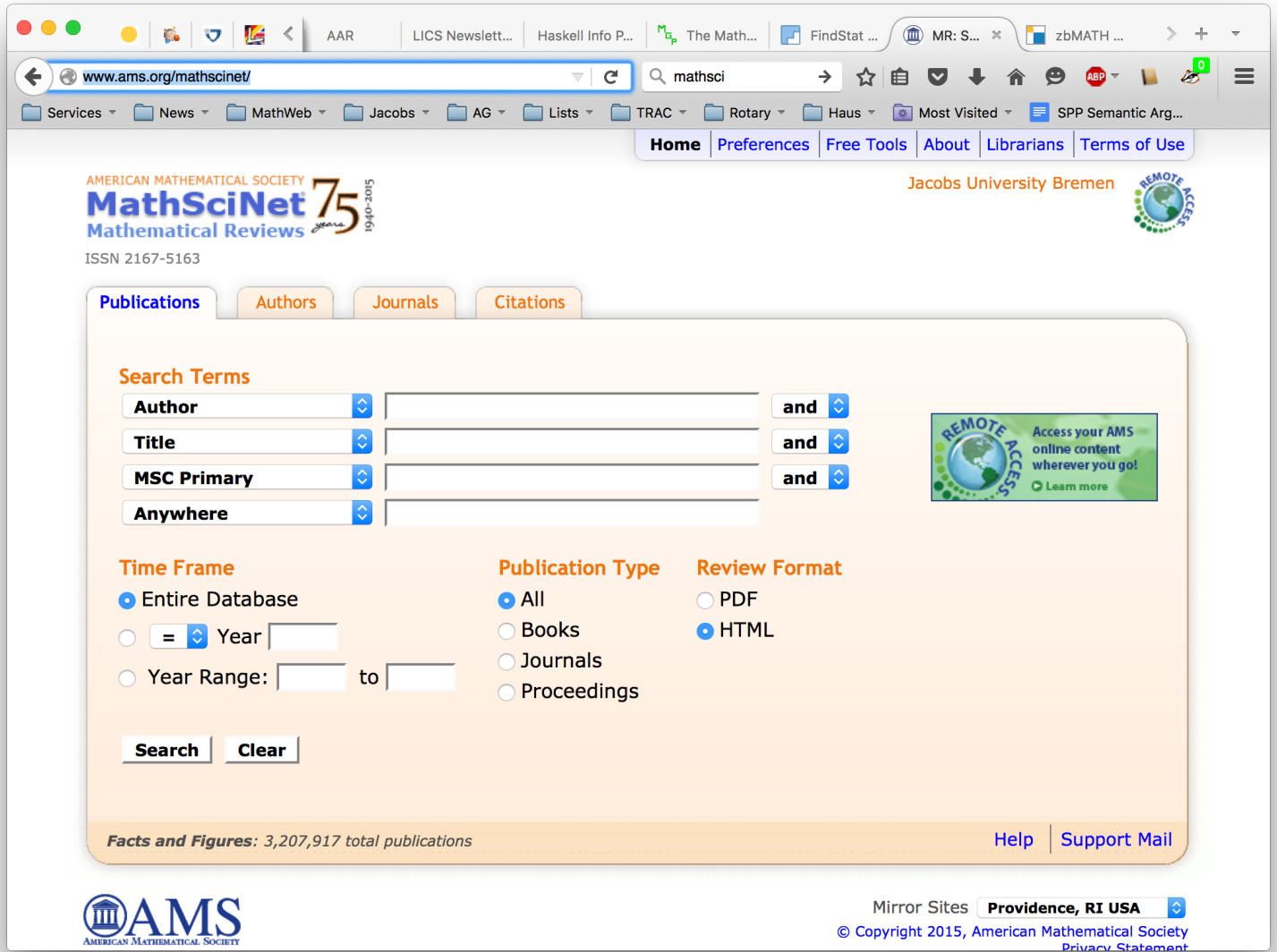
- OpenDreamKit: EU Project 2015-2019 $\leadsto$ Math Virtual Research Environment
  Computer Algebra, HPC, MathUI, KWARC (http://opendreamkit.org)

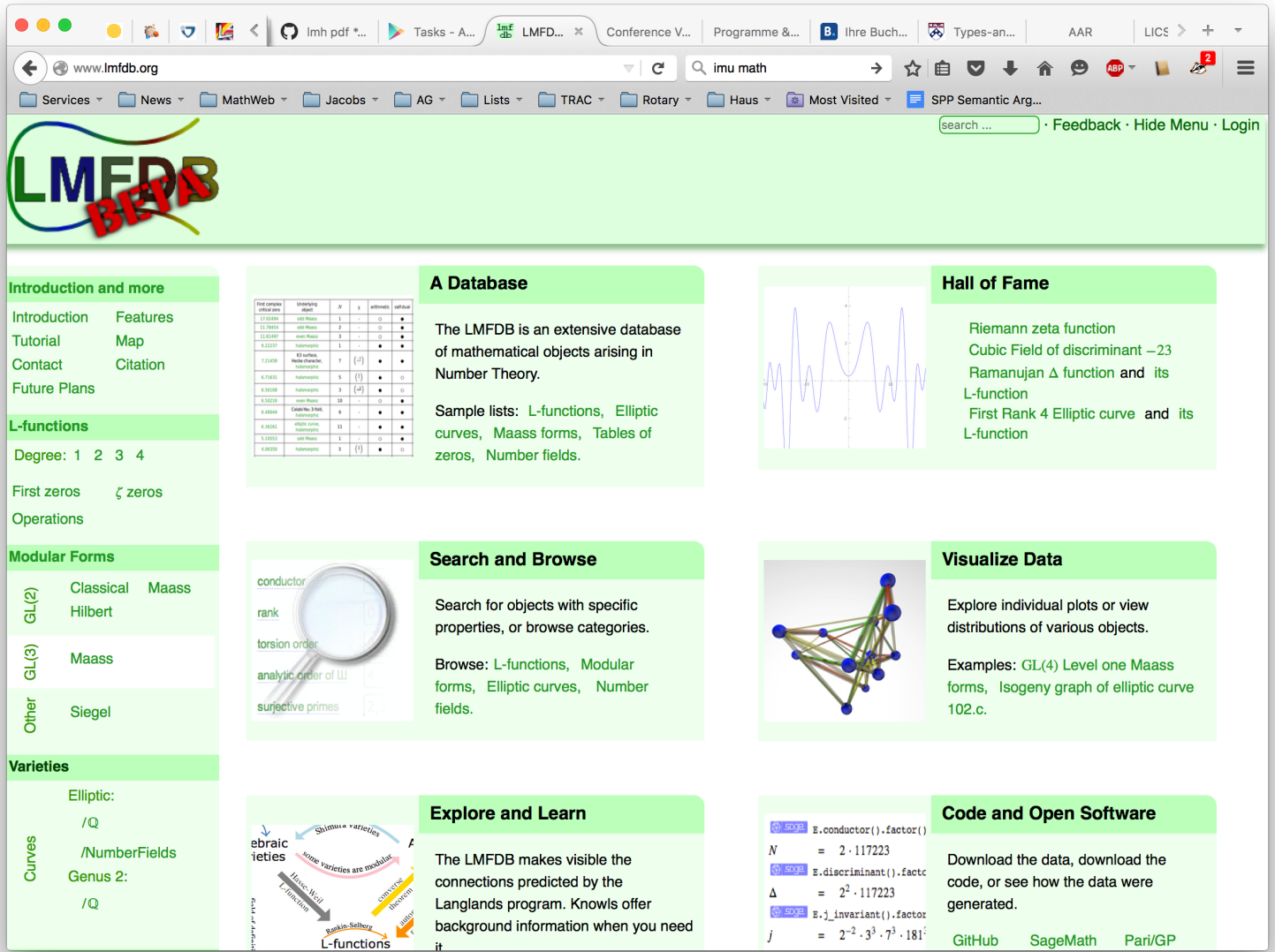
Zentralblatt Math: the first resource in Maths



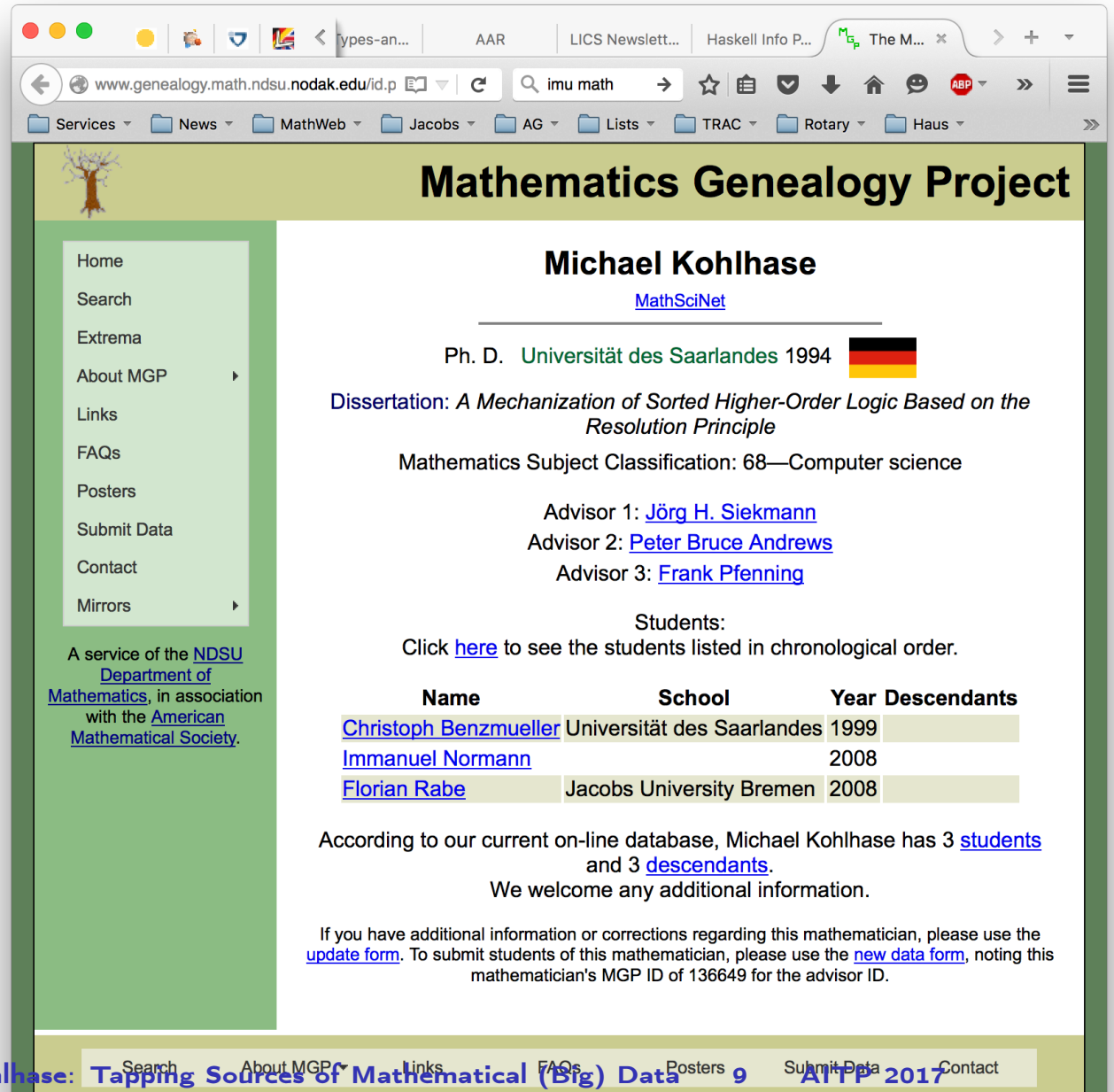
MathSciNet: Mathematical Reviews



LMFDB: *L*-functions & Modular Forms



MGP: Math Genealogy Project



www.genealogy.math.ndsu.nodak.edu/id.p

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A service of the NDSU Department of Mathematics, in association with the American Mathematical Society.

Michael Kohlhase

MathSciNet

Ph. D. Universität des Saarlandes 1994

Dissertation: *A Mechanization of Sorted Higher-Order Logic Based on the Resolution Principle*

Mathematics Subject Classification: 68—Computer science

Advisor 1: Jörg H. Siekmann
Advisor 2: Peter Bruce Andrews
Advisor 3: Frank Pfenning

Students:
Click here to see the students listed in chronological order.

Name	School	Year	Descendants
Christoph Benzmueller	Universität des Saarlandes	1999	
Immanuel Normann		2008	
Florian Rabe	Jacobs University Bremen	2008	

According to our current on-line database, Michael Kohlhase has 3 students and 3 descendants.
We welcome any additional information.

If you have additional information or corrections regarding this mathematician, please use the update form. To submit students of this mathematician, please use the new data form, noting this mathematician's MGP ID of 136649 for the advisor ID.

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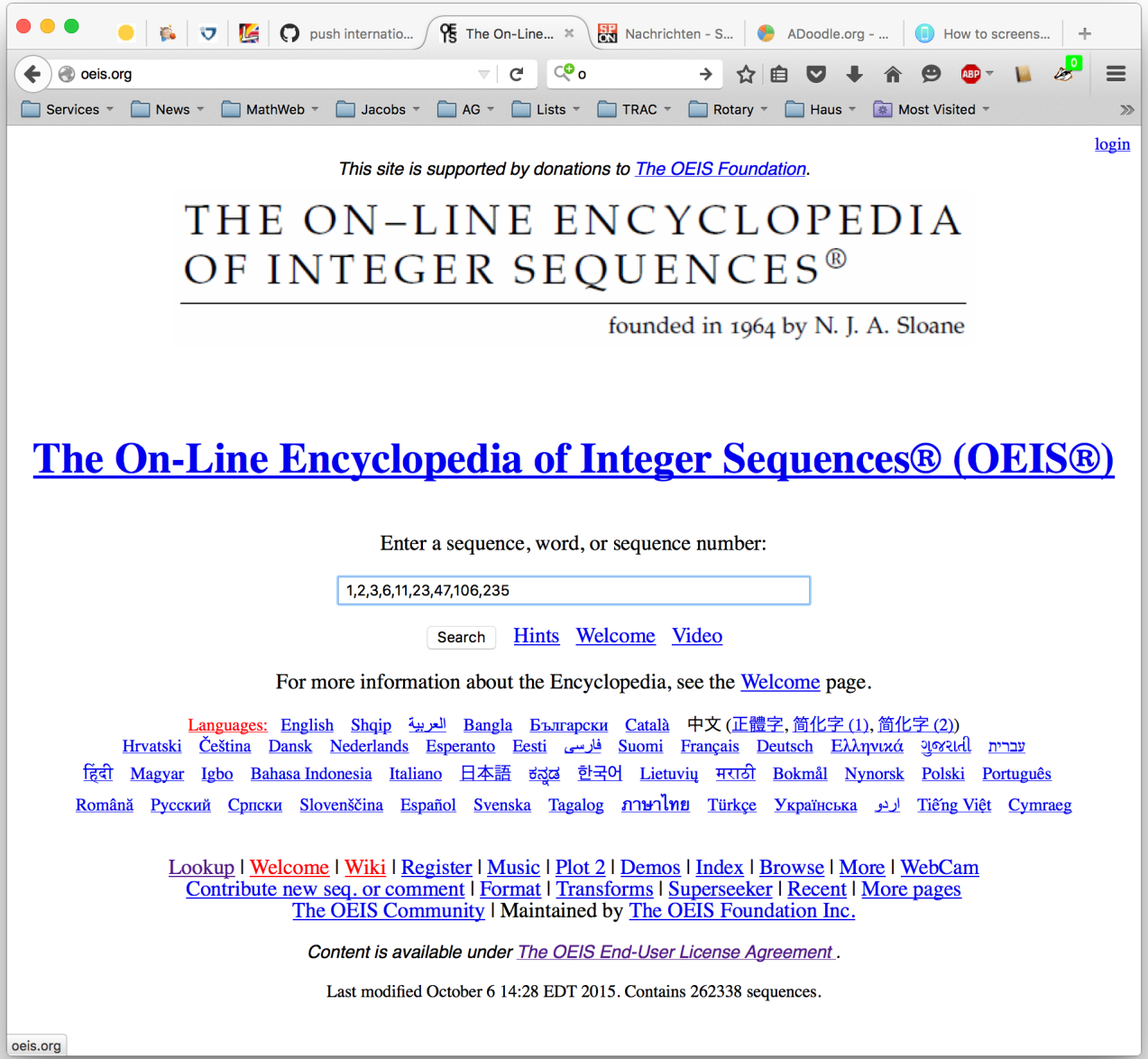
OEIS: Open Encyclopedia of Integer Sequences



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- There is a lot of useful data in maths out there
- But it needs integration, aggregation, and versioning
- Licensing is a major stumbling block

2 Converting the arXiv

The arXMLiv Project: arXiv to semantic XML

- Idea: Develop a large corpus of knowledge in *OMDoc*/PhysML
  - to get around the chicken-and-egg problem of MKM
  - corpus-linguistic methods for semantics recovery (linguists interested)
- **Definition 2.1 (The Cornell Preprint arXiv)** (http://www.arxiv.org)
  Open access to ca. 850K e-prints in Physics, Mathematics, Computer Science and Quantitative Biology.
- **Definition 2.2 (The arXMLiv Project)** (http://arxmliv.kwarc.info)
  - use Bruce Miller's LaTeXML to transform to XHTML+MathML
  - extend to LaTeXML daemon (RESTful web service) (http://latexml.mathweb.org)
  - we have an automated, distributed build system (ca. *Q*2CPU-years)
  - create ca. 12K LaTeXML binding files (8 Jacobs students help)
  - use MathWebSearch to index XML version (realistic search corpus)
- More semantic information will enable more added-value services, e.g.
  - filter hits by model assumptions (expanding, stationary, or contracting universe)
  - use linguistic techniques to add the necessary semantics

Why reimplement the TEX parser? I

- Problem: The TEX parser can change the tokenizer while at runtime (\catcode)
- **Example 2.3 (Obfuscated TEX)** David Carlisle posted the following, when someone claimed that word counting is simple in TEX/LATEX

```
\let~\catcode~‘76~‘A13~‘F1~‘j00~‘P2jdefA71F~‘7113jdefPALLF
PA’’FwPA;;FPAZZFLaLPA//71F71iPAHHFLPAzzFenPASSFthP;A$$FevP
A@@FfPARR717273F737271P;ADDFRgniPAWW71FPATTFvePA**FstRsamP
AGGFRruoPAqq71.72.F717271PAYY7172F727171PA??Fi*LmPA&&71jfi
Fjfi71PAVVFjbigskipRPWGAUU71727374 75,76Fjpar71727375Djifx
:76jelse&U76jfiPLAKK7172F71l7271PAXX71FVLnOSeL71SLRyadR@oL
RrhC?yLRurtKFeLPFovPgaTLtReRomL;PABB71 72,73:Fjif.73.jelse
B73:jfiXF71PU71 72,73:PWs;AMM71F71diPAJJFRdriPAQQFRsreLPAI
I71Fo71dPA!!FRgiePBt’el@ lTLqdrYmu.Q.,Ke;vz vzLqpip.Q.,tz;
;Lql.IrsZ.eap,qn.i. i.eLlMaesLdRcna,;!;h htLqm.MRasZ.ilk,%
s$;z zLqs’.ansZ.Ymi,/sx ;LYegseZRyal,@i;@ TLRlogdLrDsW,@;G
LcYlaDLbJsW,SWXJW ree @rzchLhzsW,;WERcesInW qt.’oL.Rtrul;e
doTsW,Wk;Rri@stW aHAHHFndZPpqar.tridgeLinZpe.LtYer.W,:jbye
```

When formatted by TeX, this leads to the full lyrics of “The twelve days of christmas”. When formattet by LATEXML, it gives

Why reimplement the TEX parser? II

```
<song>
  <verse>
    <line>On the first day of Christmas my true love gave to me</line>
    <line>a partridge in a pear tree.</line>
  </verse>
  <verse>
    <line>On the second day of Christmas my true love gave to me</line>
    <line>two turtle doves</line>
    <line>and a partridge in a pear tree.</line>
  </verse>
  <verse>
    <line>On the third day of Christmas my true love gave to me</line>
    <line>three french hens</line>
    <line>two turtle doves</line>
    <line>and a partridge in a pear tree.</line>
  </verse>
  <verse>
    <line>On the fourth day of Christmas my true love gave to me</line>
    <line>four calling birds</line>
    <line>three french hens</line>
    <line>two turtle doves</line>
    <line>and a partridge in a pear tree.</line>
  </verse>
  ...
```

Why reimplement the TEX parser? III

- But the real reason is: that we can take advantage of the semantics in the LATEX.
- LATEXML does not need to expand macros, we can tell it about XML equivalents.
- **Example 2.4 (Recovering the Semantics of Proofs)**
  Add the following magic incantation to `amsthm.sty.ltxml` (LATEXML binding)

```
DefEnvironment('{proof}',"<xhtml:div class='proof'>#body</xhtml:div>");
```

  The `arXMLiv` approach: Try to cover most packages and classes in the arXiv
  (Jacobs undergrads' intro to research)

Future Plans for arXMLiv

- State: LaTeX-to-XHTML+MathML Format Conversion works (65% success)
- Over the summer: Bump up success rate to 75%, daily downloads, web site, instrumentation,...
- Soon: Integrate user-level quality control (integrate JS feedback into html)
- starting Fall: Extend post-processing by linguistic methods for semantic analysis
  - build semantics blackboard/database for linguistic information (rdf triples)
  - extend build system for arbitrary XML2BB processes
  - invite the linguists over (they leave semantics results in BB)
  - harvest the semantics BB to get OMDoc representations

Current and Possible Applications

- the arxmliv build system `http://arxmliv.kwarc.info`
- the transformation web service `http://tex2xml.kwarc.info`
- LaTeXML daemon to avoid `perl` and LaTeX startup times (Deyan Ginev)
  - keep LaTeXML alive as a daemon that can process multiple files/fragments (patch memory leaks)
  - a LaTeXML client just passes files/fragments along (10/s to 100/s)
- embedding/editing LaTeX in web pages `http://tex2xml.kwarc.info/test`
- a MathML version of the arXiv allows vision-impared readers to understand the texts
- generalization search (need to know sentence structure for detecting universal variables)
- semantic search by academic discipline or theory assumption (need discourse structure)
- development of scientific vocabularies (over the past 18 years; drink from the source)

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- We have problems re-distributing them (Licensing) (working on this)
- Lots of potential Applications
  - Formula Search (`arxivsearch.mathweb.info`, `https://zbmath.org/formulae/`)
  - screenreaders for quantity expressions (semantics extraction, annotation)
  - applicable theorem search (need to identify the universal/existential/constant identifiers)
  - machine translation (need a handle on the math terminology (large, dynamic))

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  All of these are a mixture of DLAI and GOFAI methods
- I am sceptical of DLAI autoformalization (surprise me!)

3 OAF: Assembling a Global Resource of Formalizations

OAF: Open Archive of Formalizations: Motivation

- Idea1 (OAF): Assemble all theorem prover libraries in a common synergy space
- Observation: Formal/symbolic systems and their libraries are non-interoperable
  - differing, mutually incompatible foundations (e.g., set theory, higher-order logic, constructive type theory, etc.),
  - library formats, and library structures,
- Consequence: Too much work is spent developing
  - basic libraries for mathematics in each system.
  - library organization features (e.g., distribution, browsing, search, change management) for each library format.
- Problem: All these investments bind resources that could be used to improve the core functionality of the systems and the scope of the libraries.
- Idea2 (QAF = QED reloaded): System and tool chain for all of formal maths!

OAF Architecture

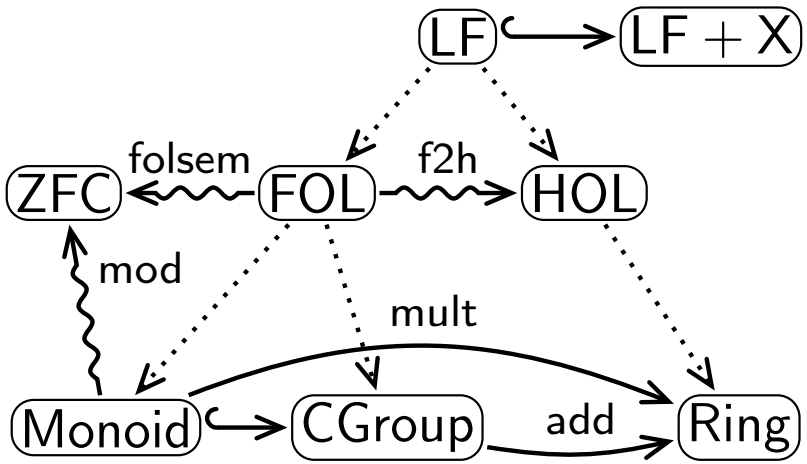
- Idea (OAF): Assemble all theorem prover libraries in a common synergy space
- Problem: Different systems have different, mutually incompatible logical/mathematical foundations (optimize different aspects)
- Observation: need a system with multiple foundations $\leadsto$ foundational pluralism
- **Definition 3.1** A foundation (of mathematics) consists of
  - a foundational language (aka. loigc, e.g. first-order logic or the CIC)
  - a foundational theory (e.g. axiomatic set theory)

  Idea1: treat logics as mathematical theories themselves (metalogical frameworks)

- Idea2: relate logics in a theory graph via logic transformations (LATIN)

Representing Logics and Foundations as Theories

- Logics and foundations represented as MMT theories (in the sample graph)

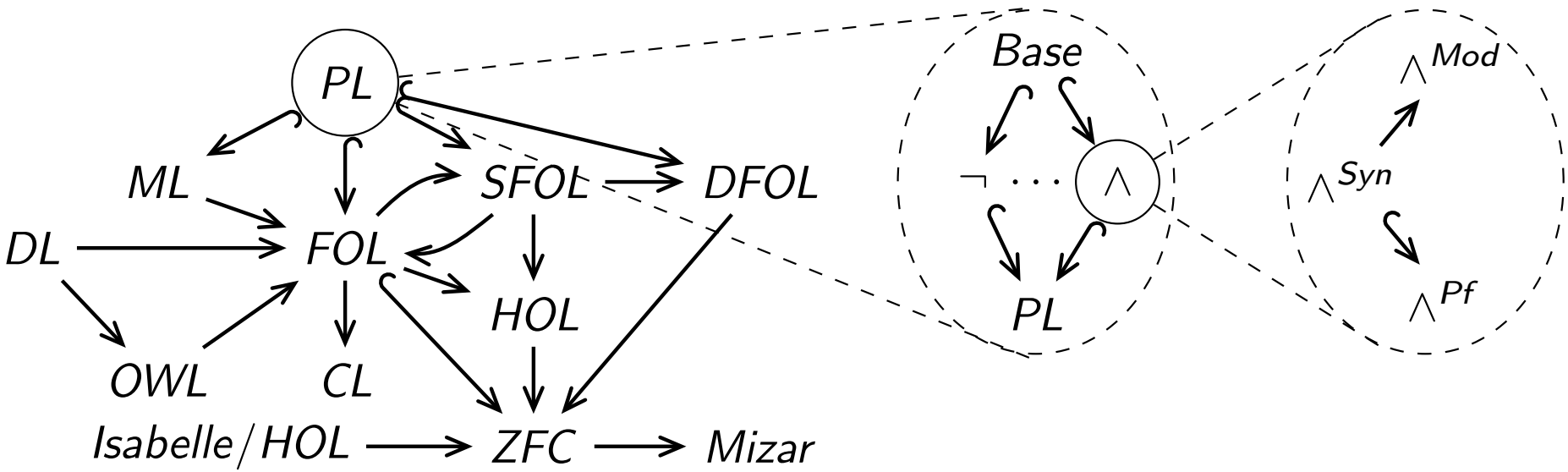


  Meta-relation between theories – special case of inclusion (meta*-level)

- Uniform Meaning Space: morphisms between formalizations in different logics become possible via meta-morphisms.
- **Remark 3.2** Semantics of logics as views into foundations, e.g., folsem.
- **Remark 3.3** Models represented as views into foundations
- **Example 3.4** mod $:= \{G \mapsto \mathbb{Z}, \circ \mapsto +, e \mapsto 0\}$ interprets Monoid in ZFC.

The LATIN Logic Atlas

- **Definition 3.5** The LATIN project (Logic Atlas and Integrator) develops a logic atlas, its home page is at `http://latin.omdoc.org`.
- Idea: Provide a standardized, well-documented set of theories for logical languages, logic morphisms as theory morphisms.



- Technically: Use MMT as a representation language logics-as-theories
- Integrate logic-based software systems via views.
- State: About 1000 modules (theories and morphisms) written in MMT/LF [RS09]

MMT a Module System for Mathematical Content

- MMT: Universal representation language for formal mathematical/logical content
- Implementation: MMT API with generic
  - module system for math libraries, logics, foundations
  - parsing + type reconstruction + simplification
  - IDE (web server + JEdit)
  - change management
- Continuous development since 2007 ($>$ 30000 lines of Scala code)
- Close relatives:
  - LF, Isabelle, Dedukti: but flexible choice of logical framework
  - Hets: but declarative logic definitions

Exports from Proof Assistants

- General Approach: Export library as MMT projects, store in MathHub
- Library Export Architecture: (this seems to work sustainably)
  - System-near export (e.g. to XML or JSON) as part of system code
  - aggregate, into OMDoc/MMT in MMT API system.
- Current state of the collection effort:
  - Mizar: set theoretical (initial export done (with Josef Urban))
  - HOL Family: HOL Light, HOL4, Isabelle, TPS (initial export done (Rabe/Kaliszyk))
  - Coq or Matita: type theoretical (Work with Sacerdoti Coen ongoing)
  - IMPS: heterogeneous method (Partial Export Done)
  - PVS: rich foundational language (Müller/Owre)
  - TPTP: mostly first-order ATP problems
  - Computer Algebra Signatures: GAP, Sage (Konovalov/Pfeiffer/Thierry)
  - Specware, OEIS, MetaMath, ... (experimental)

Search

- **Example 3.6 (Search in the MMT API/MathHub)**

Enter Java regular expressions to filter based on the URI of a declaration

Namespace

Theory

Name

Enter an expression over theory http://code.google.com/p/hol-light/source/browse/trunk

$x,y,p: x MOD p = y MOD p

Use $x,y,z:query to enter unification variables.

Search

type of MOD_EQ

$\vdash \forall m:\text{num} . \forall n:\text{num} . \forall p:\text{num} . \forall q:\text{num} . m = n + q * p \Longrightarrow m \,\text{MOD}\, p = n \,\text{MOD}\, p$

type of MOD_MULT_ADD

$\vdash \forall m:\text{num} . \forall n:\text{num} . \forall p:\text{num} . (m * n + p) \,\text{MOD}\, n = p \,\text{MOD}\, n$

Goal: Towards Library Integration

- Refactor exports to introduce modularity
- 2 options
  - systematically during export (e.g., one theory for every HOL type definition)
  - heuristic or interactive MMT API-based refactoring
- Collect correspondences between concepts in different libraries (heuristically or interactively)
- Relate isomorphic theories across languages
- Use partial morphisms to translate libraries

Take-Home Message: OAF

- There is a wealth of formal mathematics out there (diversity?)
- Unfortunately, it is segregated into 20+ silos (need foundational pluralism)
- System-specific part of the exporter must be part of the exporting system

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- System-specific part of the exporter must be part of the exporting system
- integration of heterogeneous libraries necessary (DLAI?)

4 The OEIS as a Mathematical Resource

4.1 The OEIS: Online Encyclopedia of Integer Sequences

OEIS: Open Encyclopedia of Integer Sequences

- **Definition 4.1** An intger sequence is a function $s: \mathbb{N} \to \mathbb{Z}$.
- Applications: Every parametric phenomenon that can be counted
- **Example 4.2** A000944: Number of polyhedra (or 3-connected simple planar graphs) with n nodes (0, 0, 0, 1, 2, 7, 34, 257, 2606,...)
- **Example 4.3** A001222: Number of prime divisors of n counted with multiplicity (0, 1, 1, 2, 1, 2, 1, 3, 2, 2, 1, 3, 1, 2, 2,...)
- **Example 4.4** A031214: First elements in all OEIS sequences (in order) (1, 1, 1, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,...)
- Intuition: If phenomena grow with the same sequence $\rightsquigarrow$ related?
- Idea: Collect many integer sequences (Neil Sloane 1965$\rightsquigarrow$ OEIS)
  - started as a book: *A Handbook of Integer Sequences* 1973 (2372 sequences)
  - online since 1994 (16.000 sequences $\rightsquigarrow$ `http://oeis.org`)
  - OEIS Foundation: 2009 (Creative Commons License)
  - today: $\sim$ 275.000 sequences

OEIS Data Representation

- One "record" per sequence with fields including
  - Identifier: A??????
  - start values (DB Key)
  - name (maybe with short explanation)
  - author
  - references to papers
  - program code (Mathematica, Pari, ...)
  - Formulae

  All in ASCII files keyed by one-letter line prefixes.

- **Example 4.5 (Fibonacci Numbers)**

```
%I A000045 M0692 N0256
%S A000045 0,1,1,2,3,5,8,13,21,34,55,89,144,233,377,610,987
%N A000045 Fibonacci numbers: F(n) = F(n−1) + F(n−2) with F(0) = 0 and F(1) = 1.
%D A000045 V. E. Hoggatt, Jr., Fibonacci and Lucas Numbers. Houghton, Boston, MA, 1969.
%F A000045 F(n) = ((1+sqrt(5))^n−(1−sqrt(5))^n)/(2^n∗sqrt(5))
%F A000045 G.f.: Sum_{n>=0} x^n ∗ Product_{k=1..n} (k + x)/(1 + k∗x). − _Paul D. Ha
%F A000045 This is a divisibility sequence; that is, if n divides m, then a(n) divides a(m)
%A A000045 _N. J. A. Sloane_, Apr 30 1991
```

4.2 OEIS Semantification

Parsing the OEIS format

- Formulae have no prescribed format (look good to the editors)
- But they are sufficiently regular (on average) to allow parsing
  - **infix operators**, e.g the + symbol in m+n.
  - **suffix operators**, e.g. the ! symbol in n!.
  - **prefix operators** (with or without brackets), e.g. sin in sin(x) or sin x.
  - **infix relation symbols**, e.g. the < symbol in x<2.
  - **binding operators**, e.g. the ∀ symbol in ∀x. x^2 > 0.

  and some OEIS idioms like G.f. or g.f. for "generating function".
- Problem: open-ended set of primitives, e.g. sqrt, ^, sum/Σ and prod/Π
- Ambiguity: ASCII formulae have multiple plausible readings, e.g.
  - implicit multiplication/application: a(x+y) or ln x
  - elided brackets/precedences: sin x or even sin x/y
- Delineating Formulae/Text:

> Note that ppzeta(s) = sum_{p prime} 1/(p^s−1) and
> ppzeta(s) = sum_{k=1}^{infinity} primezeta(k∗s).
> – Franklin T. Adams−Watters, Sep 11 2005.

The Generated OMDoc

```
<omdoc xmlns:dc="http://purl.org/dc/elements/1.1/">
  <theory id="A000045">
  <metadata>
    <dc:creator>N. J. A. Sloane</dc:creator>
    <dc:title>Fibonacci numbers</dc:title>
  </metadata>
  <symbol name="seq"/>
  <assertion>
    <!-- OpenMath for ∀n.seq(n) = (1+√5)^n−(1−√5)^n / 2^n√5 -->
    <OMBIND>
      <OMS cd="arith" name="forall"/>
      <OMBVAR> <OMV name="n"/> </OMBVAR>
      <OMA>
        <OMS cd="arith" name="equal"/>
        <OMA><OMS name="seq"/><OMV name="n"></OMA>
          ⋮
      </OMA>
    </OMBIND>
  </assertion>
```

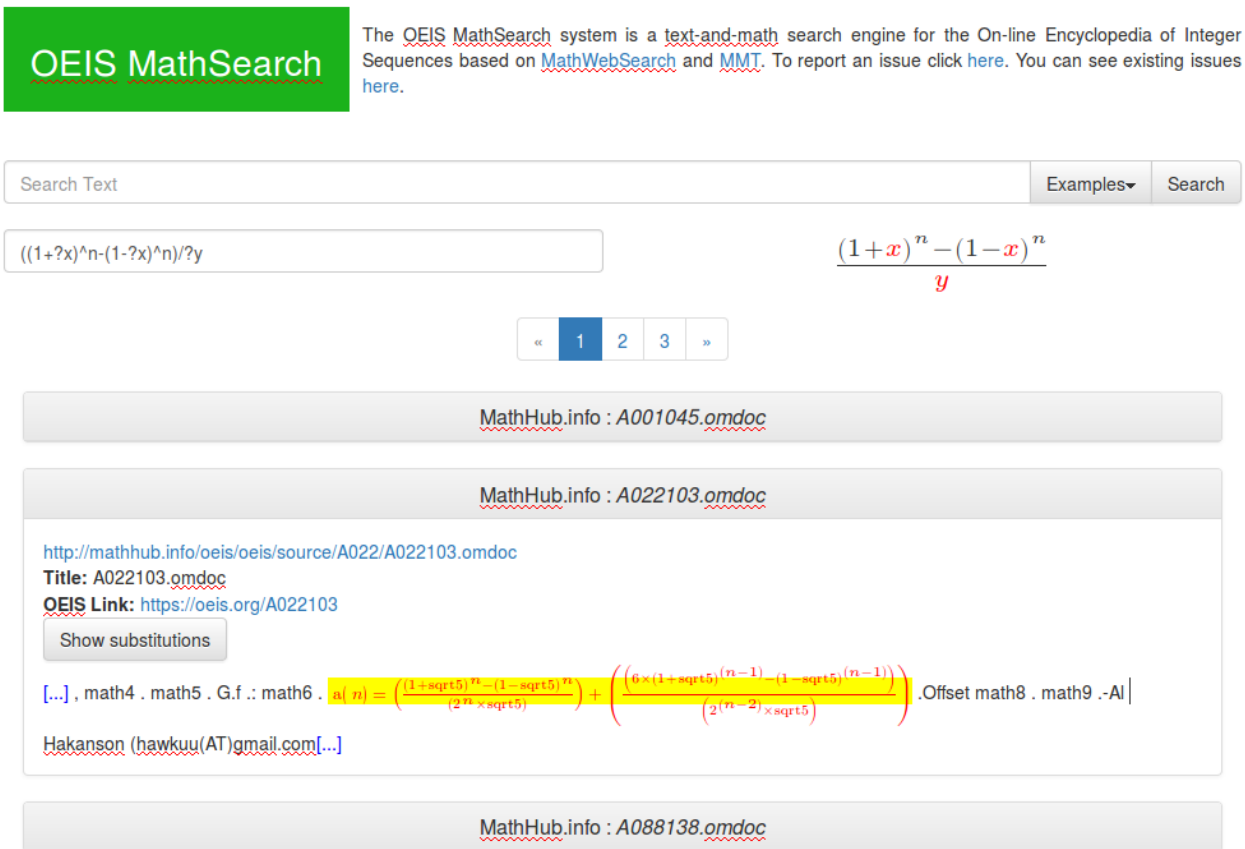
Implementation

- Implementation as an extension of the MMT System (2000 LoC)
- Formula parsing via the Scala PackRat framework (left recursive linear parsing)
- available at `https://svn.kwarc.info/repos/MMT/src/mmt-oeis/`
- OEIS corpus:
  - 223.866 formula lines, The formula parser succeeds on 201384 (or 90%).
  - Out of that, 196515 (or 97.6%) contain mathematical expressions.
  - remaining problems: connectives, formula delineation
- What does the 90% mean? $\leadsto$ parser accepts formula
- Manual Evaluation: 40 randomly selected parsed formulae evaluated $\leadsto$ 85% semantically correct
- Need to scale evaluation $\leadsto$ involve OEIS editors (see below)

4.3 Applications

Application: Math (Formula) Search

- We have a Math Search Engine: MathWebSearch (employ it)
  - Harvest Formulae ⇜⇝ convert OpenMath to MathML
  - index them in MWS (together with full text).
  - formula converter daemon (for user input)



- Demo: `http://oeissearch.mathweb.org/`

Application: Standardizing Input in OEIS

- Problem: 400 OEIS submissions per week (three out of 60 editors really active)
  quality of submissions often low (including syntax)

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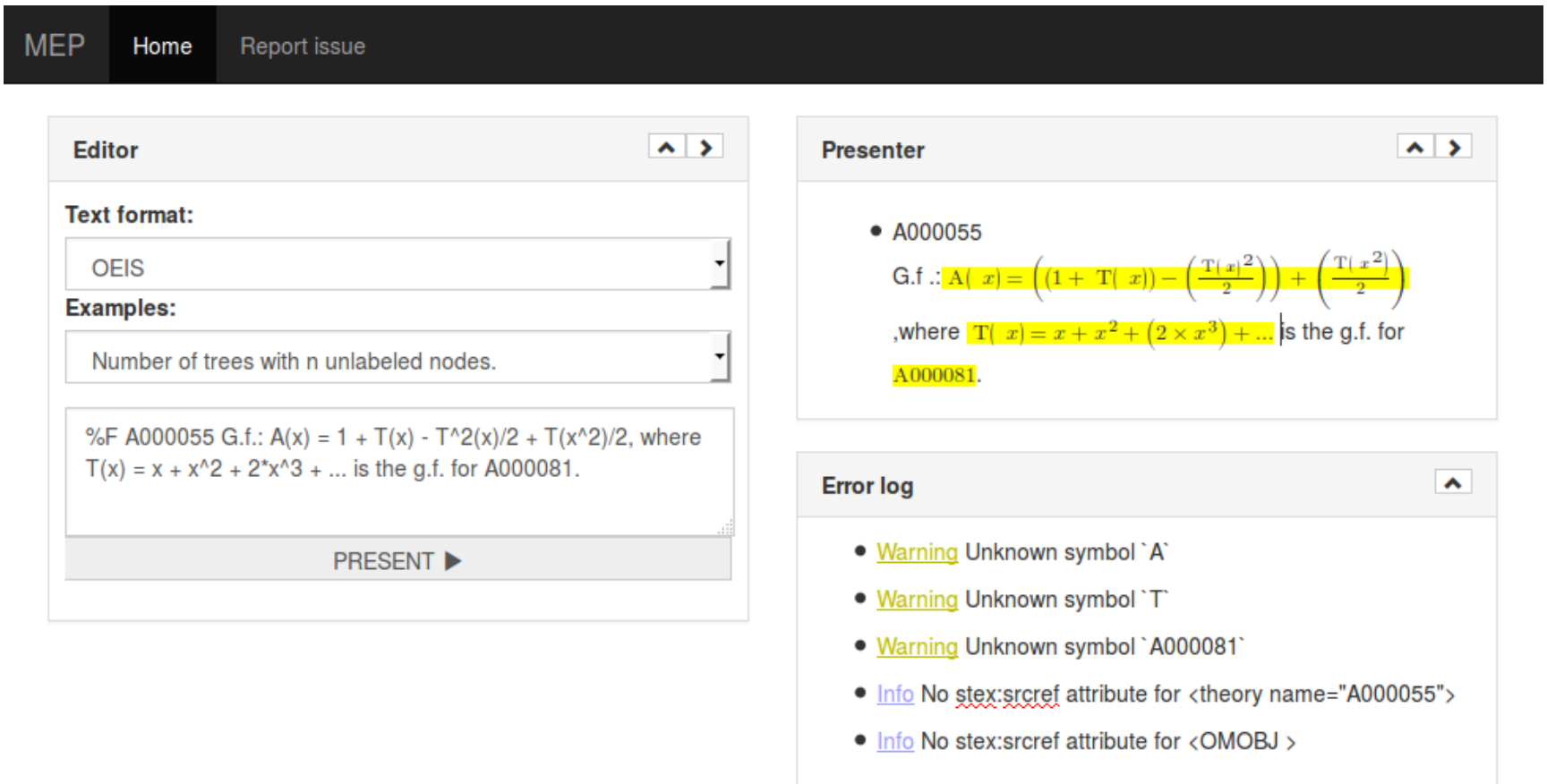
- Problem: 400 OEIS submissions per week (three out of 60 editors really active)
  quality of submissions often low (including syntax)
- Idea: Parse before submitting (use a normative parser)



- Demo: `http://ash.eecs.jacobs-university.de:9090/`

4.4 Finding Relations between Sequences

Relations between Sequences

- Understanding relations between sequences is a genuine mathematical concern.
- State of the Art: Matching initial segments of sequences.
- **Example 4.6** [Ste04] found 117 conjectures proves 100.
- Problem: Sampling limited data gives only conjectures. (need proof)
- **Example 4.7** $\lfloor \frac{2n}{log(2)} \rfloor$ and $\lceil \frac{2}{2^{1/n}-1} \rceil$ agree for 777451915729367 terms but are not equal [Slo12].
- Idea: use the formulae from the OEIS instead.
  - they are exact and peer-reviewed (relations found will be theorems)
  - we have about 50k generating functions (powerful, compact, structured representations)

Generating Functions for Integer Sequences

- **Definition 4.8** Let $s := (a_n)_{n\in\mathbb{N}}$ be an integer sequence, then we call $g_s(x) := \sum_{i=0}^{\infty} a_i x^i$ the ordinary generating function of s.
- **Example 4.9** The sequence $A000012 = 1, 1, 1, 1, 1, \ldots$ can be represented as $1 + x + x^2 + \ldots = \frac{1}{1-x}$
- represent an infinite sequence finitely (cf. Kolmogorov complexity)
- There are other generating functions: exponential generating functions, Lambert series, Bell series, and Dirichlet series. (use only ordinary ones for now)
- Operations on Generating Functions: induce to operations on the sequences.
  - constant factor: $c \cdot g_s = g_{c\cdot s}$.
  - shift: $x^n \cdot g_s(x) = g_s(x + n)$.
  - ... partial fraction decomposition, differentiation, integration, ...
- Idea: systematically search for relations on the generating functions in the OEIS induced by such operations

Relation Finding Experiment

- Experiment: search for relations on $\sim 50\,000$ OEIS generating functions
  - Method 1: const, shift, sort (sanity check; expect known relations)
  - Method 2: . . . partial fraction decomposition, differentiation, integration, . . .
  - Method 3: See Enxhell's B.Sc thesis [Luz16]

  Implementation: import parsed equations into MMT, normalize/transform by Sage, hash, compare.

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  Implementation: import parsed equations into MMT, normalize/transform by Sage, hash, compare.
- **Example 4.13 (from Method 1)** $A001478(n) = -A000027(n)$. ($\pm\mathrm{Id}$ on $\mathbb{N}$)
- **Example 4.14 (from Method 2)** accepted in `https://oeis.org/A001787`:

$$A001787(n) = \frac{n}{6} A007283(n)$$

- **Example 4.15 (from Method 2)** accepted in `https://oeis.org/A037532`:

$$A037532(n) = \frac{5}{57} A049347(n-1) + \frac{3}{57} A049347(n) + \frac{29}{171} A000420(n) - \frac{2}{9}$$

Relation Finding Experiment

- Experiment: search for relations on $\sim 50\,000$ OEIS generating functions
  - Method 1: const, shift, sort (sanity check; expect known relations)
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  - Method 3: See Enxhell's B.Sc thesis [Luz16]

  Implementation: import parsed equations into MMT, normalize/transform by Sage, hash, compare.
- **Example 4.16 (from Method 1)** $A001478(n) = -A000027(n)$. ($\pm\mathrm{Id}$ on $\mathbb{N}$)
- **Example 4.17 (from Method 2)** accepted in `https://oeis.org/A001787`:

$$A001787(n) = \frac{n}{6} A007283(n)$$

- **Example 4.18 (from Method 2)** accepted in `https://oeis.org/A037532`:

$$A037532(n) = \frac{5}{57} A049347(n-1) + \frac{3}{57} A049347(n) + \frac{29}{171} A000420(n) - \frac{2}{9}$$

- two out of three randomly picked OEIS submissions were accepted by Neil Sloane (third one not deemed to be interesting enough)
- OEIS acceptance prompted immediate human submission of trivial corollaries

Relation Finding Experiment: Overall Results

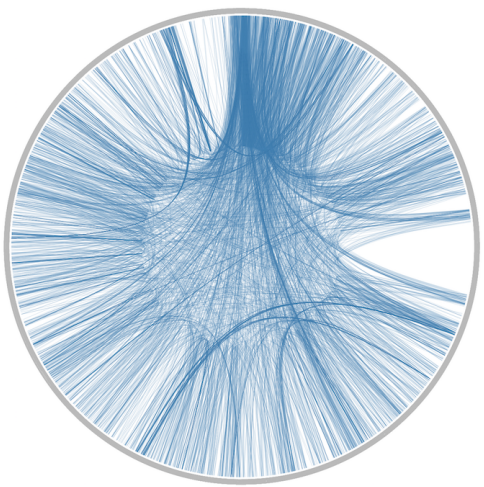
- Results: before recent parser enhancements.

Parsed Generating Functions	43 005
SageMath verified Generating Functions	16 065
Parsed Ordinary Generating Functions	35 953
SageMath verified Ordinary Generating Functions	13 400
Method 1 relations	4 859
Sequences in Method 1 relations	853
Method 2 relations	297 284 646
Method 2 relations without normalization	66 427

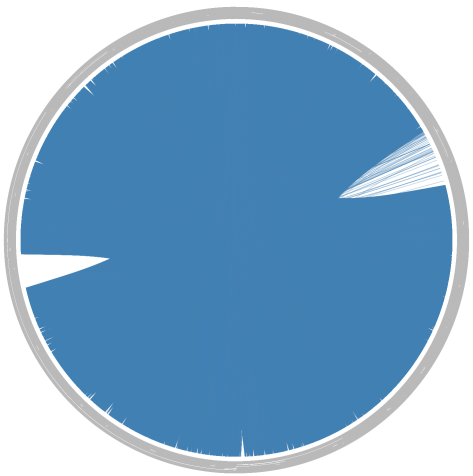
Caveat: $G = A + B + C$ is counted 4^3 times (trivial variants)

- Results:

current realations



one B.Sc.



Take-Home Message: OEIS

- OEIS is an interesting corpus (mostly Data $\widehat{=}$ facts about special individuals)
- OEIS grows steadily (6000 submitters, 300sub/week, 150 accepted by human editors))
- It is definitely not formal (but the GF are after parsing)
- induced GF database allows deriving new theorems

Take-Home Message: OEIS

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- OEIS grows steadily (6000 submitters, 300sub/week, 150 accepted by human editors))
- It is definitely not formal (but the GF are after parsing)
- induced GF database allows deriving new theorems
- Need a Theorem Appreciator for automated submission (DLAI?)

Take-Home Message agin (If I managed to get here)

- I only go GOFAI (Good Old-fashioned AI aka. Logic)
- My Domain of Application is Math (no e.g. protocol verification)
- no DLAI (applying Deep Learning to everything)
- BUT we have a lot of interesting Data
  - OAF: the Open Archive of Formalizations (`http://oaf.mathhub.info`)
  - arXMLiv preprints and ZBMath Abstracts (licensing problems)
  - OEIS: "Conjecturing relations between Sequences" (`https://github.com/eluzhnica/*`)
- Could use DLAI help (but not in ATP improvements)
- I am looking for good GOFAI Ph.D. students (maybe even DLAI)


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