Reinforming Autoformalizations: Combining Naproche and Informath

the semantics, converting Dedukti code to GF abstract syntax trees and ultimately to natural language. Since the grammar covers a wide variety of natural language structures (mixed with mathematical symbols), a given piece of Dedukti code can easily generate thousands of alternative informalizations. These alternatives are ranked by a scoring function that favours properties such as conciseness.

3 Case Study

We have started to reinformalize a recent Felix autoformalization of Munkres's *Topology* [6]. Felix, né Naproche-ZF [5], is a proof assistant in the lineage of Naproche [7]. It takes a controlled natural language as input and uses high-level proof automation backed by automated theorem provers. These features can make Felix formalizations more readable than traditional formal code. However, a Felix formalization can still turn out verbose, with a de Bruijn factor comparable to those of formalizations in other proof assistants. We aim to address this problem by embedding Felix in Informath, thereby gaining access to several mechanisms for making expressions more concise. In this way, we obtain an extension of Felix that is still controlled by a formal grammar and semantics, but comes closer to the ordinary language of mathematics.

To give an example of the effects of informalization, consider the original definition of Hausdorff spaces in Munkres's book [11, §17, p. 98]:

Definition 1. A topological space X is called a Hausdorff space if for each pair x_1, x_2 of distinct points of X, there exist neighborhoods U_1, and U_2 of x_1 and x_2, respectively, that are disjoint.

The Felix autoformalization, after distillation, gives us the following.

Definition 2. X is Hausdorff with respect to T iff T is a topology on X and for all x_1, x_2 such that $x_1 \in X$ and $x_2 \in X$ and $x_1 \neq x_2$ there exist U_1, U_2 such that U_1 is a neighborhood of x_1 in X with respect to T and U_2 is a neighborhood of x_2 in X with respect to T and U_1 is disjoint from U_2.

This is quite readable, but not entirely natural. In particular,

- it makes the topology T explicit, whereas the original leaves it implicit,
- it splits the condition that x_1 and x_2 are distinct points of X into three separate conjuncts.

A more concise informalization already within reach in Informath is

Definition 3. A topological space X is a Hausdorff space if, for all distinct $x_1, x_2 \in X$, there exist disjoint U_1 and U_2 such that U_1 is a neighborhood of x_1 and U_2 is a neighborhood of x_2.

We will also investigate how much work is needed to recover the exact original wording of this definition and of other passages in the book.

4 Evaluation

We will consider the following evaluation criteria:

- By how much can we reduce the size of the Felix formalization?
- How often can we recover the original wording?
- How readable are the informalizations in the remaining cases?

The first two criteria can be measured quantitatively, whereas the third will be assessed by human inspection.

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