Applying AI to automated theorem proving in loop theory

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Automated theorem provers are now widely used in mathematics. This is significant progress from a mere 25 years ago when they were viewed skeptically (at best) by many in the mathematical establishment. One of the (many) “next steps” in this progression is the integration of artificial intelligence with automated theorem provers in the actual practice of mathematics. In this talk, I will report on recent, modest progress in this program.

The mathematics in this work is focused mainly on the theory of loops. No prior knowledge of loop theory is required to understand this talk, as I will present a gentle bit of background. Basically loops can be thought of as "nonassociative groups." More precisely, a loop is a set together with three binary operations $*$, $\backslash$, and $/$ satisfying the following identities: $x \backslash (x * y) = x * (x \backslash y) = y$, and $(x / y) * y = (x * y) / y = x$; and with a 2-sided neutral element: $x * 1 = 1 * x = x$.

The theory of loops, then, is a generalization of the theory of groups. This generalization takes two main forms. First, since all groups can also be viewed as loops, the theory of loops includes the theory of groups as an appealing subtheory. This forces the theory into a leaner elegance and greatly simplifies certain notions from group theory. Moreover, and not surprisingly, the techniques involved in studying loops closely resemble the techniques of group theory. In fact, the theory of loops is intimately related to the theory of certain groups associated with loops. This explains, for instance, the rich history of involvement by some of the world's most eminent group theorists in developing the theory of loops.

From another perspective, loops can be viewed as universal algebras, as above. As such, loops lend themselves naturally to the full spectrum of techniques available to the universal algebraist, especially the powerful tools and techniques of computational mathematics, e.g., automated theorem provers.

The new, fresh aspect of this work involves artificial intelligence. Explicitly, we are using artificial intelligence to recognize theorems in various loop theories, for example Bruck loops. We use the automated theorem prover Prover9 to generate hundreds of thousands of theorems in the theory of Bruck loops, and we use Prover9 to generate thousands of formulas that are not theorems in the same theory. We then train an AI system to recognize the difference between them. The ultimate payoff is (will be) to use AI to indicate whether or not a given formula (explicitly, an open problem stated as a theorem in a particular theory, e.g., Bruck loops) is likely to be a theorem or not. One can then attack the formalae likely to be theorems with theorem provers, etc., and set the others aside.

This is a joint work with Bojan Tunguz, a prominent AI researcher.