Developing a Concept-Oriented Search Engine for Isabelle Based on Natural Language: Technical Challenges

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The ALEXANDRIA Project

- Expand the libraries and AFP with new mathematical results
- Build tools for managing large bodies of formal Mathematical Knowledge
  - **Intelligent Search**
  - Computer-aided Knowledge Discovery
- Create automated and semi-automated environments and tools to aid *working mathematicians*
  - **Intelligent Search**
  - Proof completion recommender systems
- Borrow ideas and techniques from Information Retrieval, Machine Learning and Natural Language Processing

Searching for Isabelle Facts – The Status Quo

- *find_theorems*: Limitations :

  1. Inexperienced users might have an idea of what is needed to complete proof

     BUT not enough experience with library organisation and naming conventions to construct effective *find_theorems* queries

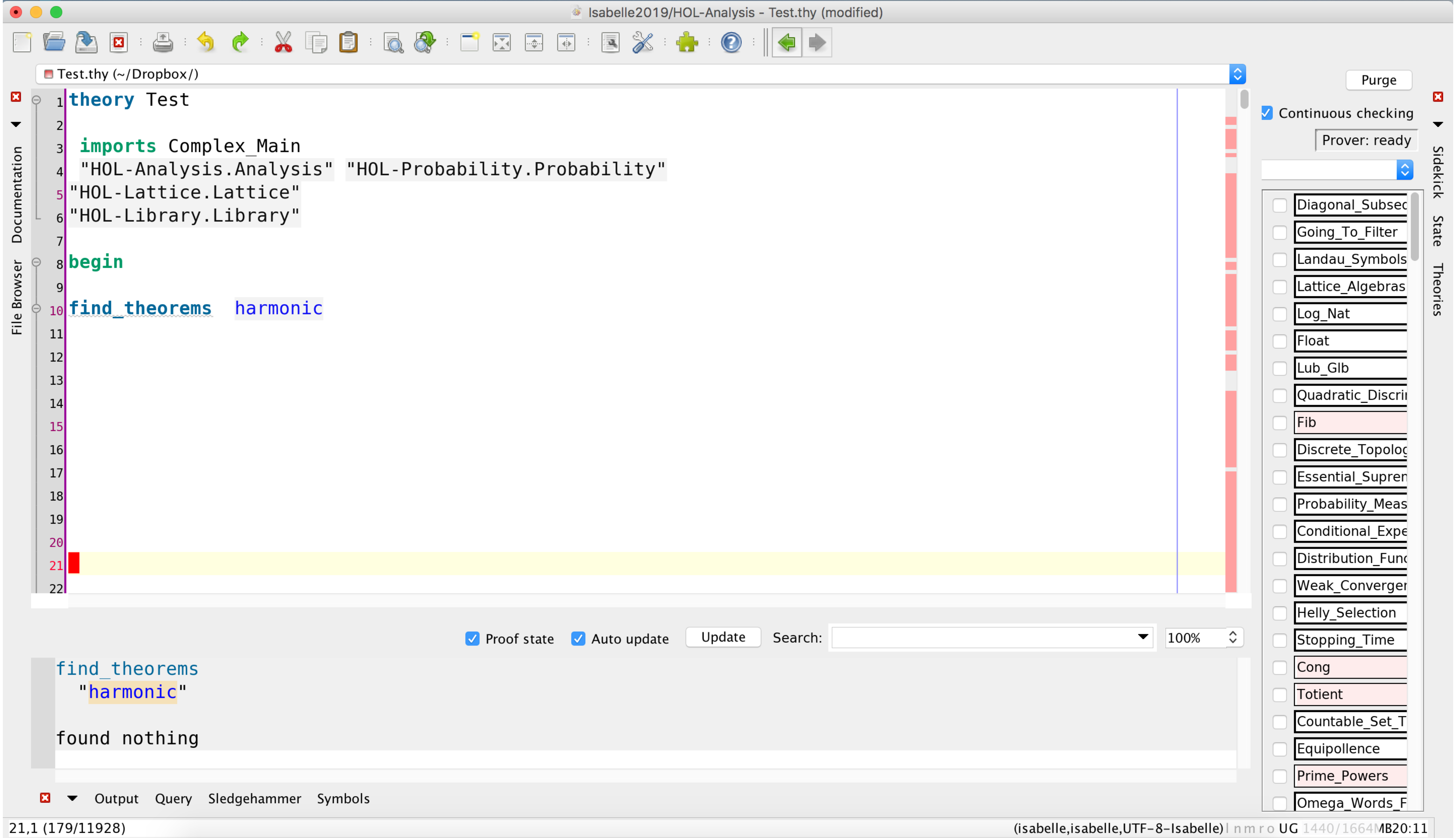
  2. Modern search users expect an experience akin to a google search box:

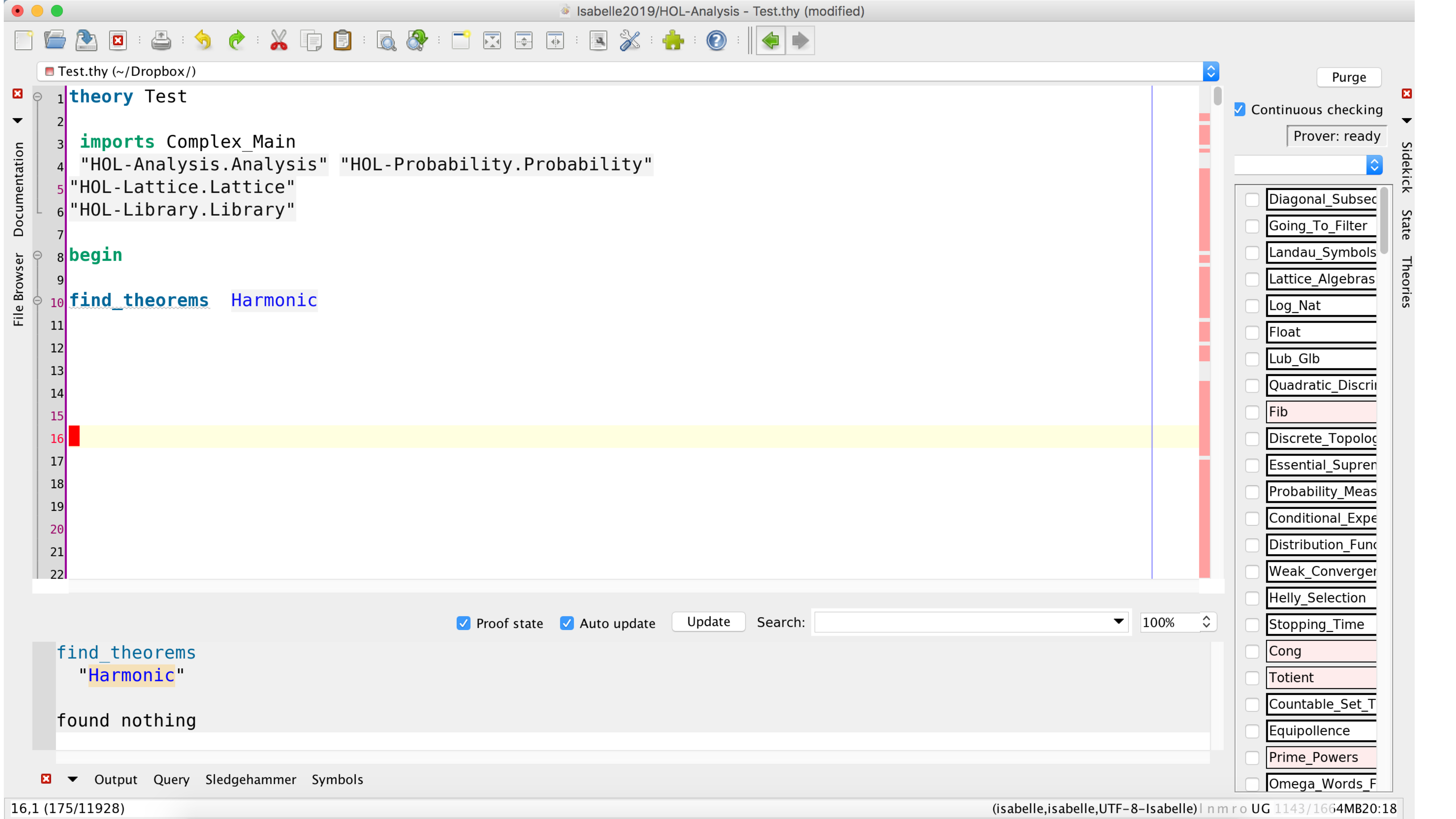
     - Input a “bag-of-words” natural language description of need

     - Quickly get back a list of results, ordered by relevance

  3. Mathematical knowledge can be organised in different ways. It is thus useful to have search results from the entire Isabelle libraries and AFP.

     NOT just the libraries currently loaded in the active session (“online” search). “Offline” search required.


Isabelle2019/HOL-Analysis - Test.thy (modified)
Test.thy (~/Dropbox/)
theory Test
imports Complex_Main
"HOL-Analysis.Analysis" "HOL-Probability.Probability"
"HOL-Lattice.Lattice"
"HOL-Library.Library"
begin
find_theorems harmonic
Purge
Continuous checking
Prover: ready
Diagonal_Subsec
Going_To_Filter
Landau_Symbols
Lattice_Algebras
Log_Nat
Float
Lub_Glb
Quadratic_Discri
Fib
Discrete_Topolog
Essential_Supren
Probability_Meas
Conditional_Expe
Distribution_Func
Weak_Converger
Helly_Selection
Stopping_Time
Cong
Totient
Countable_Set_T
Equipollence
Prime_Powers
Omega_Words_F
Sidekick
State
Theories
Documentation
File Browser
Proof state
Auto update
Update
Search:
100%
find_theorems
"harmonic"
found nothing
Output
Query
Sledgehammer
Symbols
21,1 (179/11928)
(isabelle,isabelle,UTF-8-Isabelle)
UG
MB20:11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theory Test
imports Complex_Main
"HOL-Analysis.Analysis" "HOL-Probability.Probability"
"HOL-Lattice.Lattice"
"HOL-Library.Library"
begin
find_theorems Harmonic
Purge
Continuous checking
Prover: ready
Diagonal_Subsec
Going_To_Filter
Landau_Symbols
Lattice_Algebras
Log_Nat
Float
Lub_Glb
Quadratic_Discri
Fib
Discrete_Topolog
Essential_Supren
Probability_Meas
Conditional_Expe
Distribution_Func
Weak_Converger
Helly_Selection
Stopping_Time
Cong
Totient
Countable_Set_T
Equipollence
Prime_Powers
Omega_Words_F
Sidekick
State
Theories
Documentation
File Browser
Proof state
Auto update
Update
Search:
100%
find_theorems
"Harmonic"
found nothing
Output
Query
Sledgehammer
Symbols
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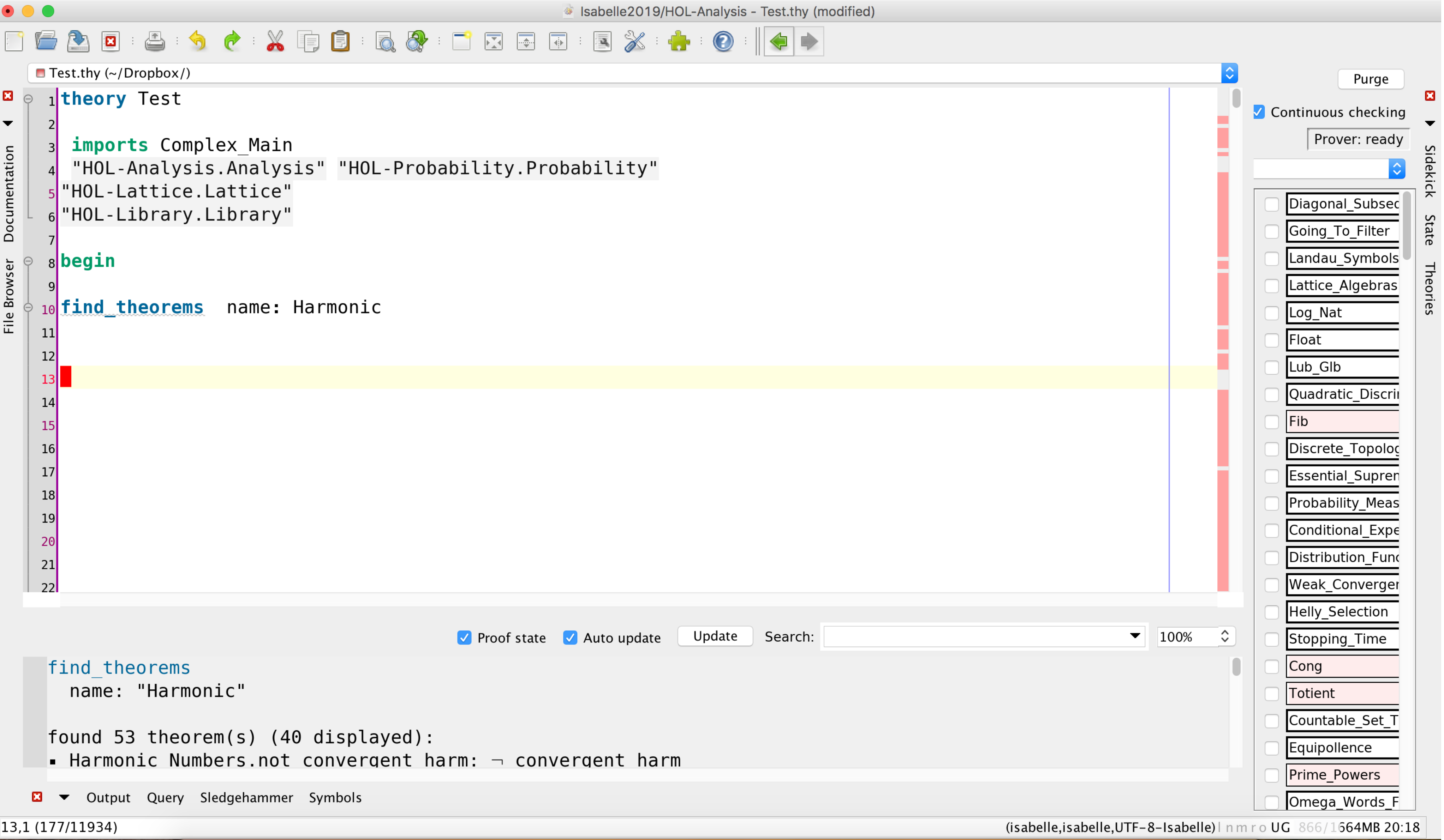
(isabelle,isabelle,UTF-8-Isabelle)
UG
4MB20:18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Test.thy (~/Dropbox/)
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imports Complex_Main
"HOL-Analysis.Analysis" "HOL-Probability.Probability"
"HOL-Lattice.Lattice"
"HOL-Library.Library"
begin
find_theorems name: Harmonic
Purge
Continuous checking
Prover: ready
Diagonal_Subsec
Going_To_Filter
Landau_Symbols
Lattice_Algebras
Log_Nat
Float
Lub_Glb
Quadratic_Discri
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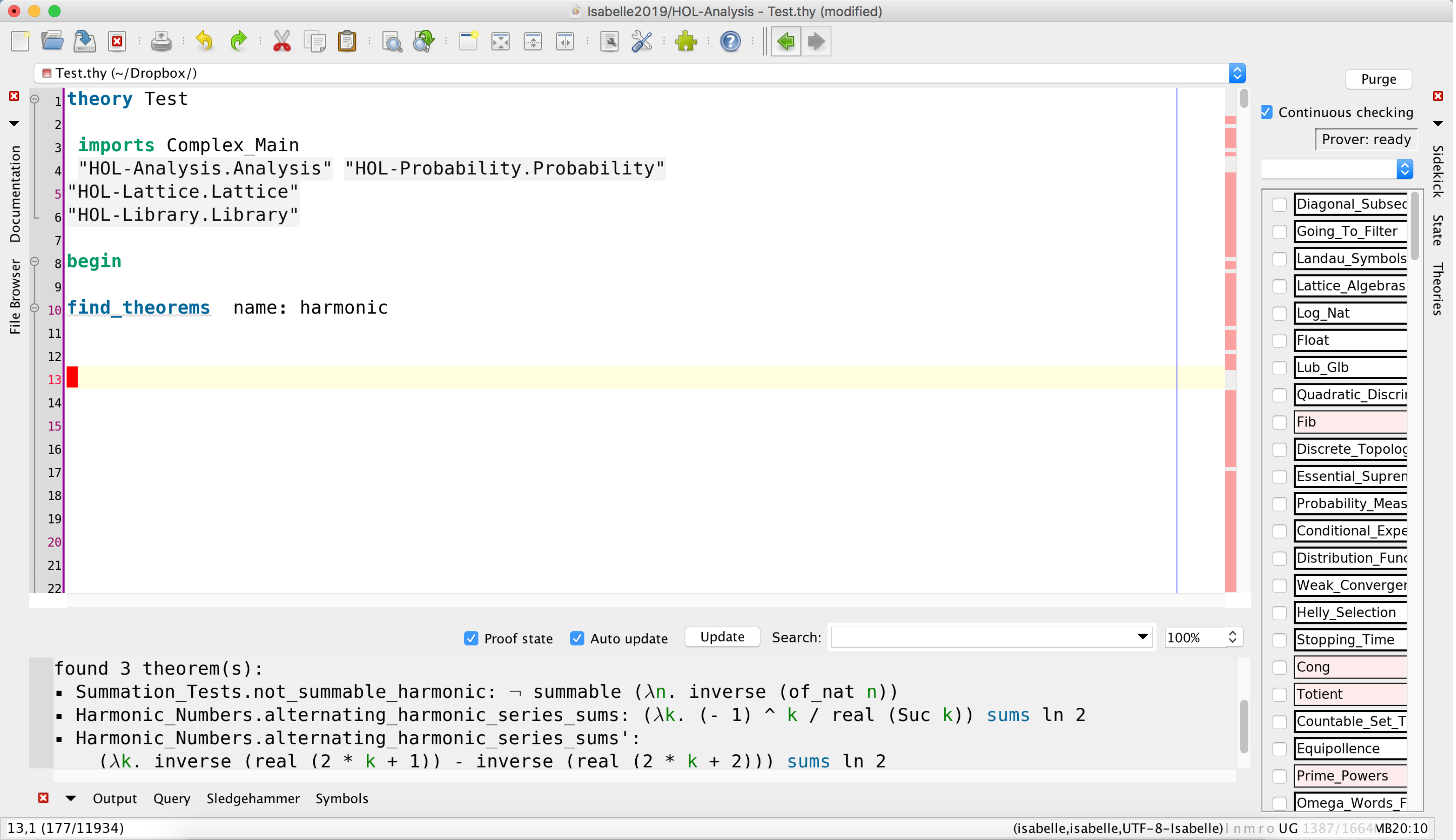
Totient
Countable_Set_T
Equipollence
Prime_Powers
Omega_Words_F
Sidekick
State
Theories
Documentation
File Browser
Proof state
Auto update
Update
Search:
100%
find_theorems
name: "Harmonic"
found 53 theorem(s) (40 displayed):
▪ Harmonic Numbers.not convergent harm: ¬ convergent harm
Output
Query
Sledgehammer
Symbols
13,1 (177/11934)
(isabelle,isabelle,UTF-8-Isabelle) nmro UG 866/1664MB 20:18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theory Test
imports Complex_Main
"HOL-Analysis.Analysis" "HOL-Probability.Probability"
"HOL-Lattice.Lattice"
"HOL-Library.Library"
begin
find_theorems name: harmonic
Purge
Continuous checking
Prover: ready
Diagonal_Subsec
Going_To_Filter
Landau_Symbols
Lattice_Algebras
Log_Nat
Float
Lub_Glb
Quadratic_Discri
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Discrete_Topolog
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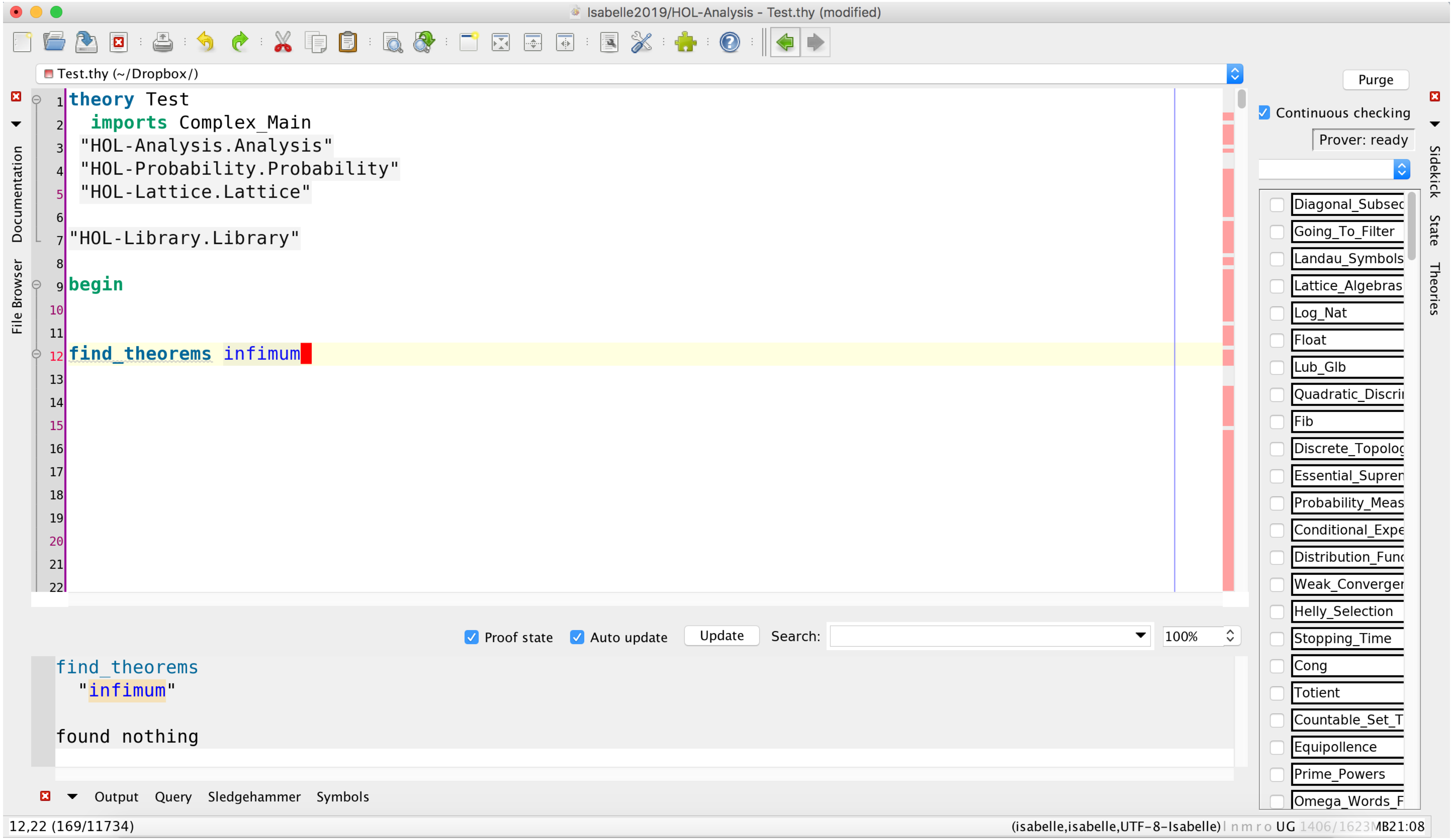
Totient
Countable_Set_T
Equipollence
Prime_Powers
Omega_Words_F
Sidekick
State
Theories
Documentation
File Browser
Proof state
Auto update
Update
Search:
100%
found 3 theorem(s):
▪ Summation_Tests.not_summable_harmonic: ¬ summable (λn. inverse (of_nat n))
▪ Harmonic_Numbers.alternating_harmonic_series_sums: (λk. (- 1) ^ k / real (Suc k)) sums ln 2
▪ Harmonic_Numbers.alternating_harmonic_series_sums':
(λk. inverse (real (2 * k + 1)) - inverse (real (2 * k + 2))) sums ln 2
Output
Query
Sledgehammer
Symbols
13,1 (177/11934)
(isabelle,isabelle,UTF-8-Isabelle) | n m r o UG 1387/1664MB 20:10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imports Complex_Main
"HOL-Analysis.Analysis"
"HOL-Probability.Probability"
"HOL-Lattice.Lattice"
"HOL-Library.Library"
begin
find_theorems infimum
Proof state
Auto update
Update
Search:
100%
find_theorems
"infimum"
found nothing
Output
Query
Sledgehammer
Symbols
12,22 (169/11734)
(isabelle,isabelle,UTF-8-Isabelle)
Purge
Continuous checking
Prover: ready
Diagonal_Subsec
Going_To_Filter
Landau_Symbols
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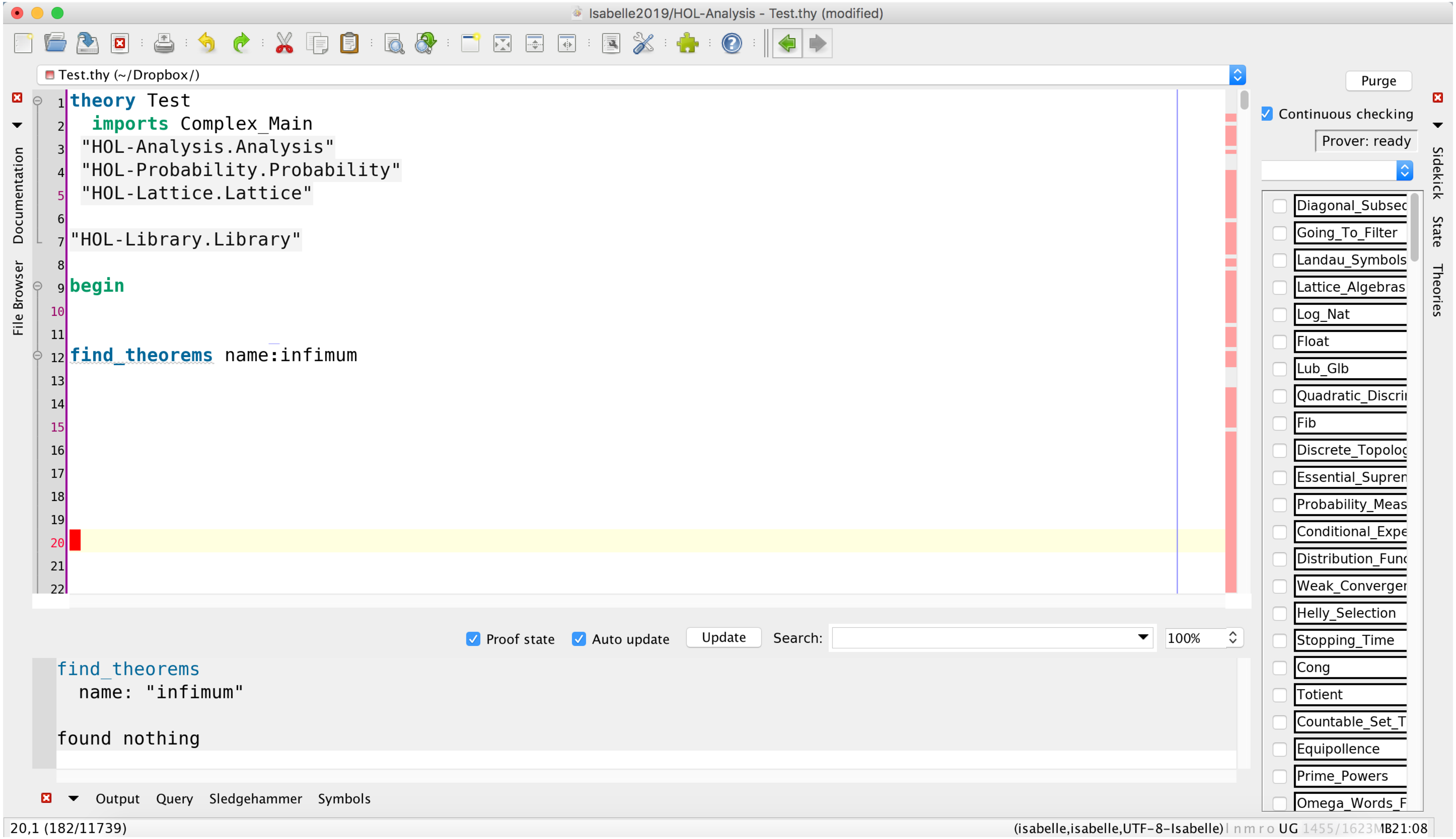
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imports Complex_Main
"HOL-Analysis.Analysis"
"HOL-Probability.Probability"
"HOL-Lattice.Lattice"
"HOL-Library.Library"
begin
find_theorems name:infimum
Purge
Continuous checking
Prover: ready
Diagonal_Subsec
Going_To_Filter
Landau_Symbols
Lattice_Algebras
Log_Nat
Float
Lub_Glb
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Omega_Words_F
Sidekick
State
Theories
Documentation
File Browser
Proof state
Auto update
Update
Search:
100%
find_theorems
name: "infimum"
found nothing
Output
Query
Sledgehammer
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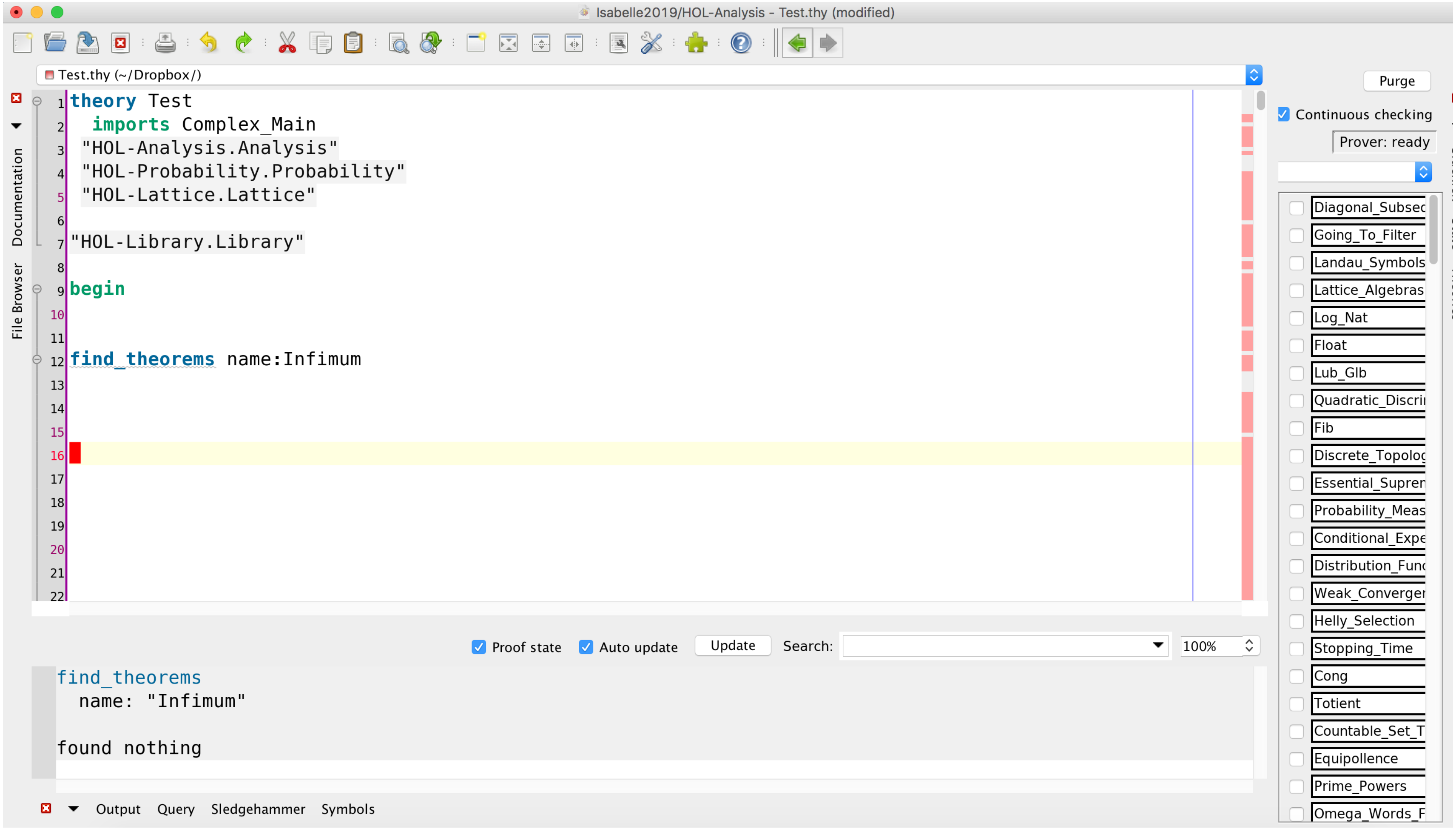
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"HOL-Probability.Probability"
"HOL-Lattice.Lattice"
"HOL-Library.Library"
begin
find_theorems name:Infimum
Documentation
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Continuous checking
Prover: ready
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Prime_Powers
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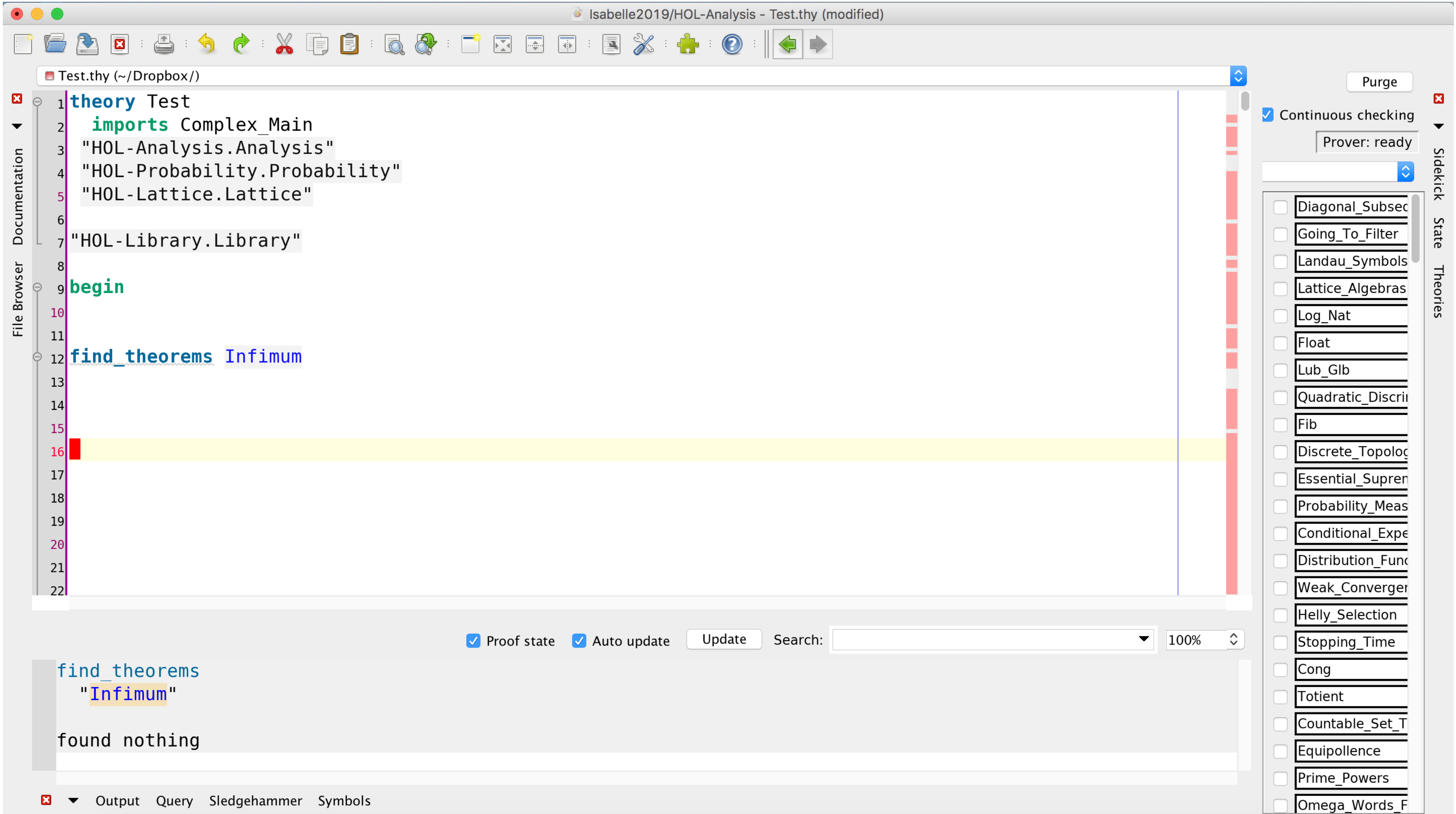
Proof state
Auto update
Update
Search:
100%
find_theorems
name: "Infimum"
found nothing
Output
Query
Sledgehammer
Symbols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Test.thy (~/Dropbox/)
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imports Complex_Main
"HOL-Analysis.Analysis"
"HOL-Probability.Probability"
"HOL-Lattice.Lattice"
"HOL-Library.Library"
begin
find_theorems Infimum
Purge
Continuous checking
Prover: ready
Diagonal_Subsec
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Theories
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File Browser
Proof state
Auto update
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Search:
100%
find_theorems
"Infimum"
found nothing
Output
Query
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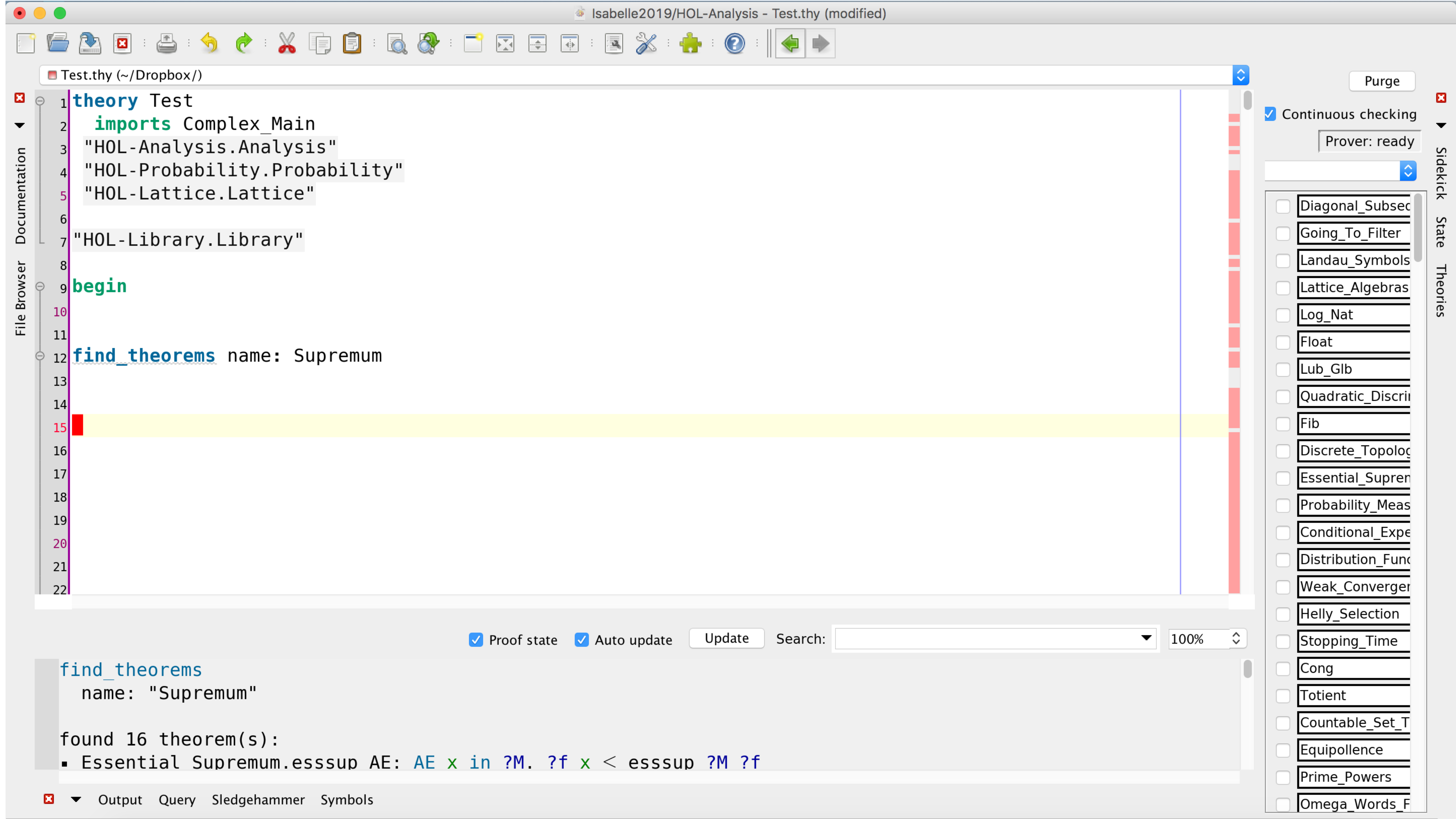
Symbols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theory Test
imports Complex_Main
"HOL-Analysis.Analysis"
"HOL-Probability.Probability"
"HOL-Lattice.Lattice"
"HOL-Library.Library"
begin
find_theorems name: Supremum
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Continuous checking
Prover: ready
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Prime_Powers
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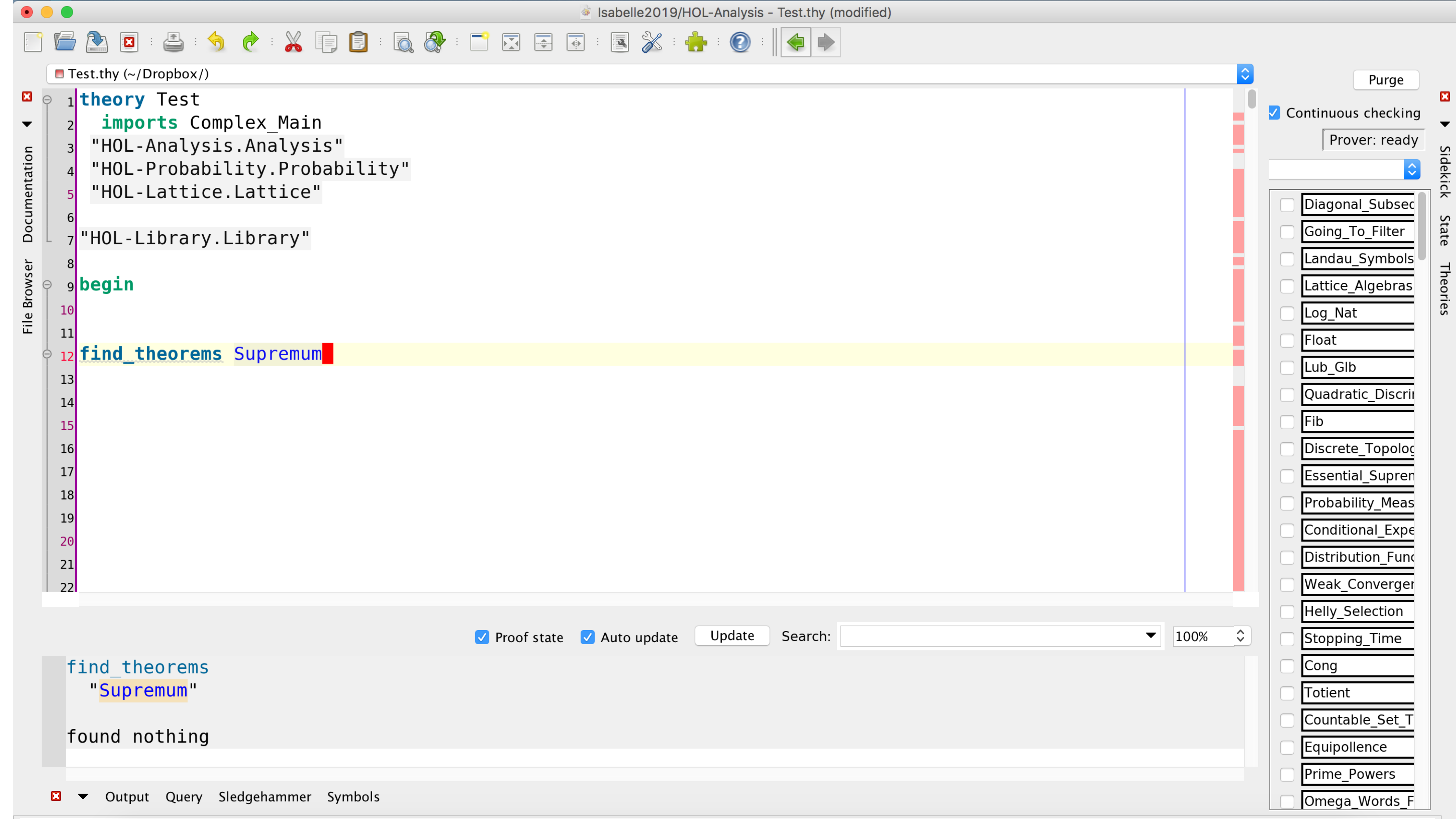
Documentation
File Browser
Sidekick
State
Theories
Proof state
Auto update
Update
Search:
100%
find_theorems
name: "Supremum"
found 16 theorem(s):
▪ Essential_Supremum.esssup_AE: AE x in ?M. ?f x ≤ esssup ?M ?f
Output
Query
Sledgehammer
Symbols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Test.thy (~/Dropbox/)
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"HOL-Analysis.Analysis"
"HOL-Probability.Probability"
"HOL-Lattice.Lattice"
"HOL-Library.Library"
begin
find_theorems Supremum
Proof state
Auto update
Update
Search:
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"Supremum"
found nothing
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Symbols
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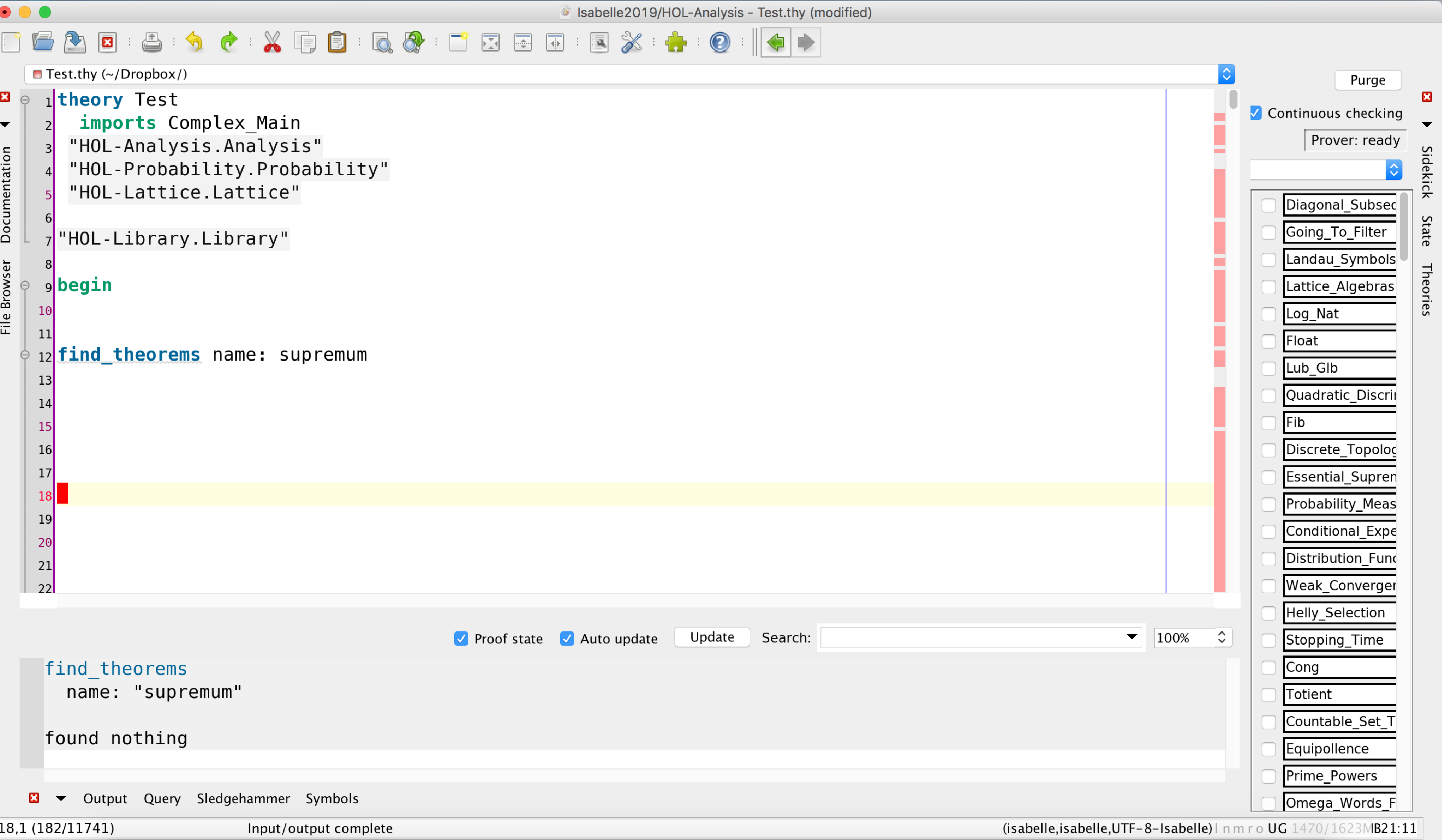
Theories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theory Test
imports Complex_Main
"HOL-Analysis.Analysis"
"HOL-Probability.Probability"
"HOL-Lattice.Lattice"
"HOL-Library.Library"
begin
find_theorems name: supremum
Purge
Continuous checking
Prover: ready
Diagonal_Subsec
Going_To_Filter
Landau_Symbols
Lattice_Algebras
Log_Nat
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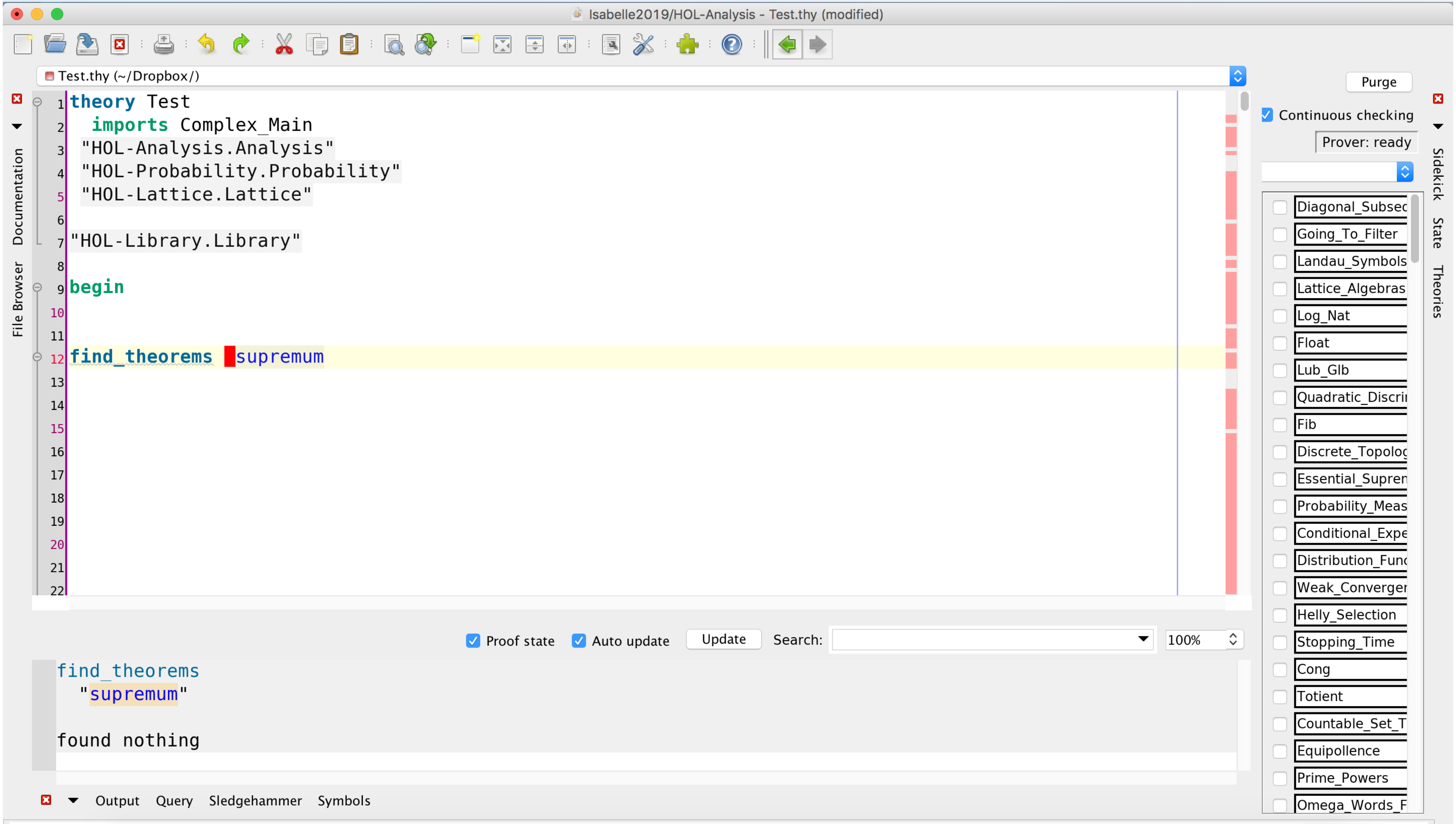
Equipollence
Prime_Powers
Omega_Words_F
Sidekick
State
Theories
Documentation
File Browser
Proof state
Auto update
Update
Search:
100%
find_theorems
name: "supremum"
found nothing
Output
Query
Sledgehammer
Symbols
18,1 (182/11741)
Input/output complete
(isabelle,isabelle,UTF-8-Isabelle)
UG
21:11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"HOL-Probability.Probability"
"HOL-Lattice.Lattice"
"HOL-Library.Library"
begin
find_theorems supremum
Proof state
Auto update
Update
Search:
100%
find_theorems
"supremum"
found nothing
Output
Query
Sledgehammer
Symbols
Documentation
File Browser
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Continuous checking
Prover: ready
Diagonal_Subsec
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Omega_Words_F
Sidekick
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Overview of Challenges

Challenge 1: Offline Indexing of Isabelle facts

- How do we extract from Isabelle scripts for effective indexing?
- We need a pre-computed and cached global index for fast search.

Challenge 2: Automatic modelling of formal mathematical knowledge using keywords and phrases

- Make the libraries accessible to all Isabelle users
- How do we make formally expressed mathematics searchable using natural language?

Challenge 3: Evaluating the effectiveness of Isabelle fact retrieval

- How do we make large-scale reliable measurements of retrieval performance for Isabelle libraries?

The SErAPIS Search Engine

- SErAPIS: **S**earch **E**ngine by the **A**lexandria **P**roject for **IS**abelle
- **Goal**: Develop and evaluate a *concept-oriented* search engine that:

1. enables efficient *offline* search – query entire Isabelle collection in seconds
2. allow Isabelle users to search libraries using a simple search box
3. support "conceptual search" rather than exact pattern matching
   - users express queries as natural language bag-of-words
   - queries can include phrases that refer to "mathematical concepts"
   - queries are flexible approximations to information needs, rather than rigid pattern matching rules
4. Results are ordered by relevance

What do we mean by Concept-Oriented?

1. "understand" the mathematical concepts/ideas behind a search. Associate closely related notions.

    - no need to specify information need explicitly in terms of patterns

2. A concrete unit of "mathematical concept":

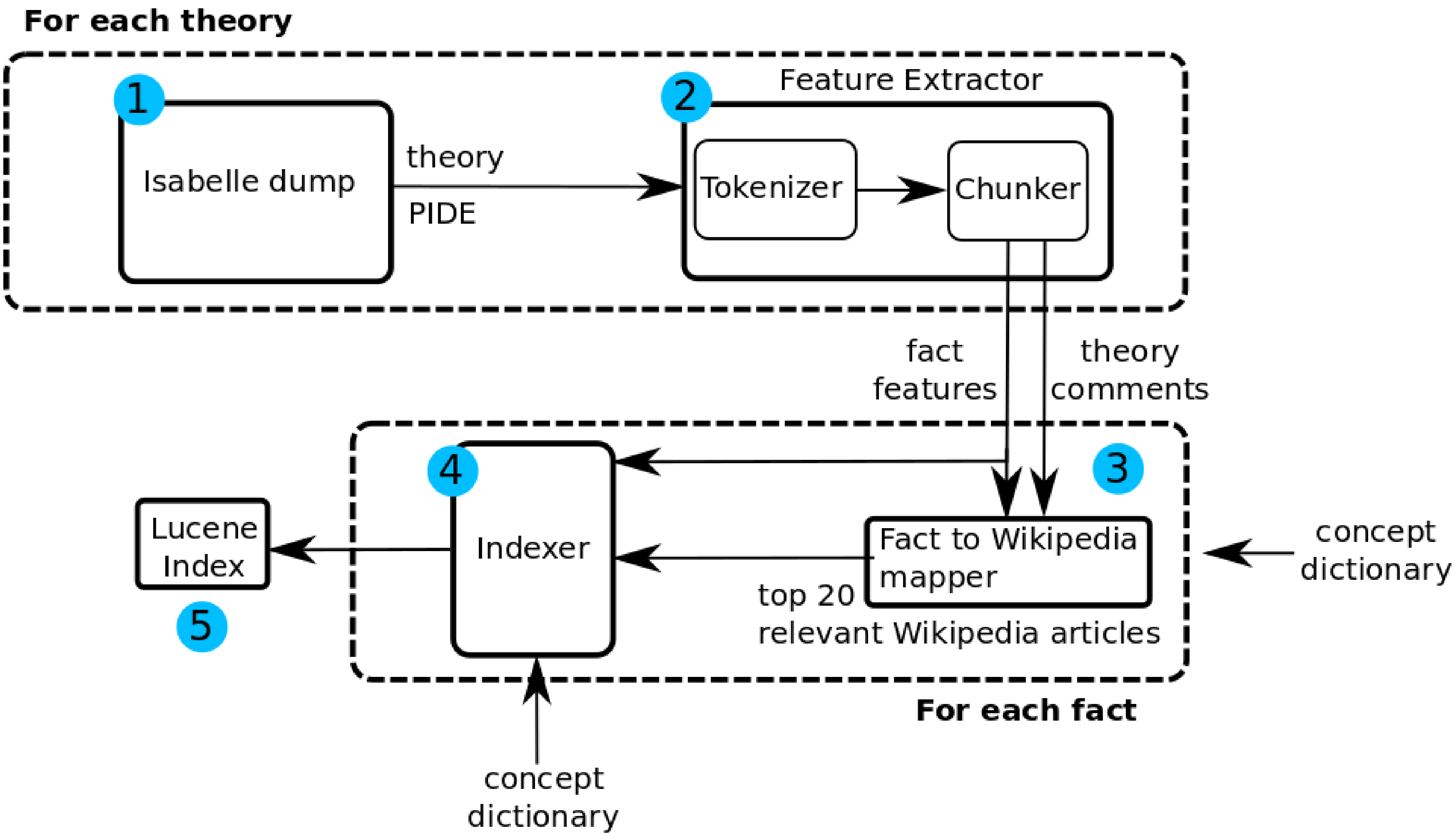
    - Words or phrases that refer to mathematical constructs, objects and ideas

    - Most are noun phrases pre-modified by adjectives

Let P be a ***parabolic subgroup*** *of $GL(n)$ with* ***Levi decomposition*** *$P = MN$, where N is the* ***unipotent radical****. Let π be an* ***irreducible representation*** *of $M(\mathbb{Z}_p)$ inflated to $P(\mathbb{Z})$.*

3. Dictionary of 1.23 million concept phrases extracted from subset of ArXiv

The SErAPIS Pipeline



Challenge 1: Offline Indexing of Isabelle Facts

- Isabelle users interact with theorem prover using Isabelle's rich syntax
  - – includes: outer syntax commands, structured Isar proofs, inner syntax terms
- Offline indexing: we need to extract information from:
  - – Isabelle syntax
  - – Internal state of the theorem prover
- Complicated for two reasons:
  1. Non-trivial to write an external parser of Isabelle's syntax
     (syntax is ambiguous and valid parse trees selected after type-checking)
  2. Useful information about Isabelle facts (e.g., types) in an Isabelle session must be retrieved from internal state of theorem prover.

Not easily achieved using external tools!

Feature Extraction

- Communication between prover and jEdit is message exchange
  - – Prover IDE (PIDE) messages update state of editor (e.g., syntax highlighting)
  - – PIDE messages generated after parsing and typing
- Information extraction through interpretation of PIDE messages
  - – Use *isabelle-dump* tool in simulated sessions of Isabelle theories
  - – BUT our methods can be applied on live Isabelle sessions
  - – Output is an XML stream of commands (at all levels)
- Tokenise and chunk PIDE command blocks belonging to facts
  - – Build a feature extractor on top of PIDE tokeniser/chunker output

PIDE Example

HOL-Number_Theory/Gauss.thy

```
lemma finite_B: "finite B"
  by (auto simp add: B_def finite_A)
```


```
<accepted>
  <running>
    <finished>
      <keyword1 kind="command">
        <entity ref="40626" def_offset="19441"
def_file="~~/src/Pure/Pure.thy" def_id="2" kind="command"
def_line="524" name="lemma" def_end_offset="19446">
          <text>
            lemma
          </text>
        </entity>
      </keyword1>
      <entity def="13291686" kind="fact"
name="Gauss.GAUSS.finite_B">
        <entity def="13291698" kind="fact" name="local.finite_B">
          <text>
            finite_B
          </text>
        </entity>
      </entity>
      <delimiter>
        <no_completion>
          <text>
            :
          </text>
```

Tokeniser Example

HOL-Number_Theory/Gauss.thy

```
lemma finite_B: "finite B"
  by (auto simp add: B_def finite_A)
```

```
<command 1> 'lemma'
  <text>'lemma'
<fact ::fact meta=local.finite_B> 'finite_B'
<delimiter> ':'
<proposition delimited=true antiquotes=false meta=null>
  <text>""
  <text>""
<command 1> 'by'
  <text>'by'
<method meta=null>
  <delimiter> '('
  <operator operator> 'auto'
  <command 4 method_modifier> 'simp'
  <command 4 method_modifier> 'add'
  <delimiter> ':'
  <fact ::fact meta=local.B_def> 'B_def'
  <fact ::fact meta=local.finite_A> 'finite_A'
  <delimiter> ')'
<command 1> 'lemma'
  <text>'lemma'
<fact ::fact meta=local.finite_C> 'finite_C'
<delimiter> ':'
<proposition delimited=true antiquotes=false meta=null>
  <text>""
  <text>""
<command 1> 'by'
.
.
.
```

Chunker Example

HOL-Number_Theory/Gauss.thy

```
lemma finite_B: "finite B"
  by (auto simp add: B_def finite_A)
```

```
===========
Chunk 19
================
<command 1> 'lemma'
  <text>'lemma'

<fact ::fact meta=local.finite_B> 'finite_B'

<delimiter> ':'

<proposition delimited=true antiquotes=false meta=null>
  <text>'"'
  <function type::{typing::{  meta='Int.int'   meta='Set.set'   meta='fun'
meta='HOL.bool' }}>> finite
  <function type::{typing::{  meta='Int.int'   meta='Set.set' }}>> B
  <text>'"'

<command 1> 'by'
  <text>'by'

<method meta=null>
  <delimiter> '('
  <operator operator> 'auto'
  <command 4 method_modifier> 'simp'
  <command 4 method_modifier> 'add'
  <delimiter> ':'
  <fact ::fact meta=local.B_def> 'B_def'
  <fact ::fact meta=local.finite_A> 'finite_A'
  <delimiter> ')'
```

Extracted Features

General Features

	Feature	Kind	Description
1	name	String	The name of the fact
2	kind	String	The kind of the fact: theorem, lemma, definition or axiom.
3	theory_key	String	Identifier for the source theory in Library_Theory format.
4	theory_name	String	The name of the source theory, produced from its filename.
5	comments	Text	Comments above the fact in the theory file.
6	incomments	Text	Comments appearing inside the fact's body.

Fact body Features

	Feature	Kind	Description
7	commandvec	Vector	Inner syntax of Isar commands and their frequency.
8	opvec	Vector	Operators that appear in the body of the fact and their frequency.
9	constvec	Vector	Constants that appear in the body of the fact and their frequency.
10	refvec	Edges	Other facts referenced in the body of the fact and the frequency of their evocation.
11	typevec	Vector	Isabelle types used in the body of the fact and their frequency.

Proof Block Features

12	Feature	Kind	Description
13	proofblocks	Integer	The number of "proof" blocks in the fact's body.
14	byblocks	Integer	The number of "by" blocks in the fact's body.
15	proof_commandvec	Vector	Commands used in the proof block of the fact and the frequency of their evokation.
16	proof_methodvec	Vector	Methods used with the "by" command and their frequency.
17	proof_opvec	Vector	Operators used in the proof block and their frequency.
18	proof_constvec	Vector	Constants used in the proof block and their frequency.
19	proof_refvec	Edges	Other facts referenced in the proof block and the frequecy by which they are used.
20	proof_typevec	Vector	Isabelle types used in the proof of the fact and their frequency.

have "$P \vee Q$" $\langle proof \rangle$
then show "R"
proof
  assume "P"
  $\vdots$
  show "R" $\langle proof \rangle$
next
  assume "Q"
  $\vdots$
  show "R" $\langle proof \rangle$
qed

Challenge 2: Automatic modelling of formal mathematical knowledge

- Mathematical knowledge almost exclusively in Isabelle’s formal language

- How do we model formal mathematical knowledge?
  - Maybe map keywords and special phrases to Isabelle facts?

- Mathematical knowledge almost exclusively in Isabelle’s formal language
  - How to map natural language to Isabelle facts is not straight-forward

- A viable solution must not only perform well but be applicable at scale
  - Thousands of facts in the Isabelle libraries and AFP

Fact Representations From Wikipedia

- **Our approach**: Assign word and concept term vectors to facts from Wikipedia Mathematics articles

- Mapping Isabelle facts to keywords and concepts from Wikipedia:

  - Allows us to model mathematical knowledge such that:

    1. We can use established techniques in AI, Information Retrieval and Natural Language Processing for knowledge representation

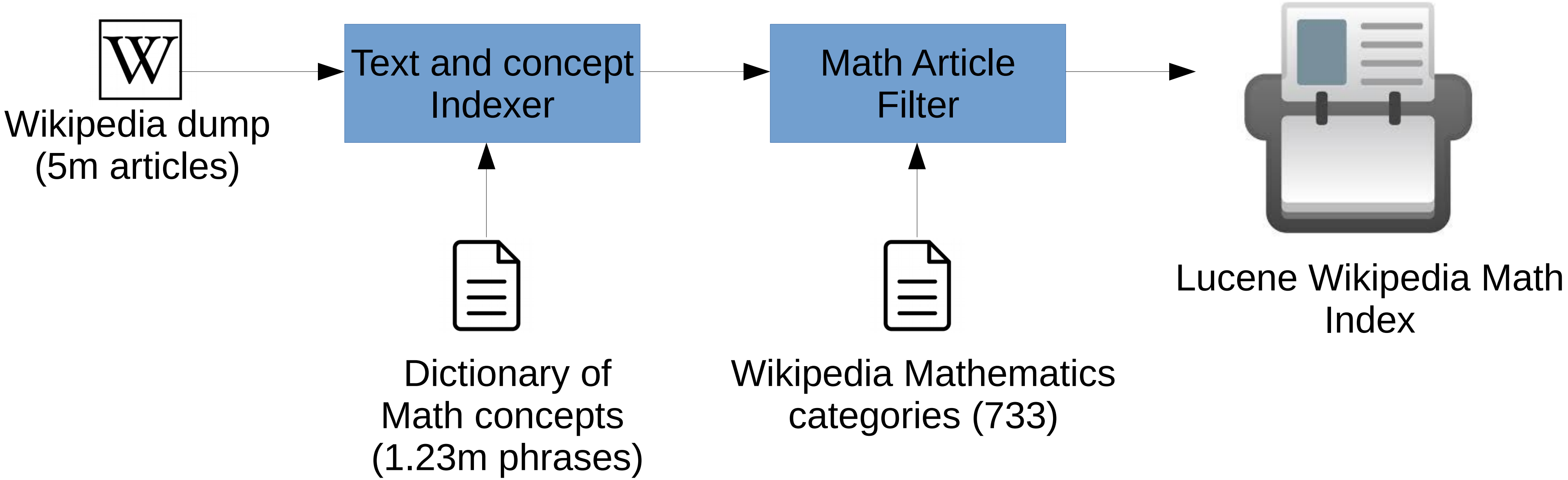
       e.g., Vector Space Model, Jaccard coefficient, cosine similarity, LSI

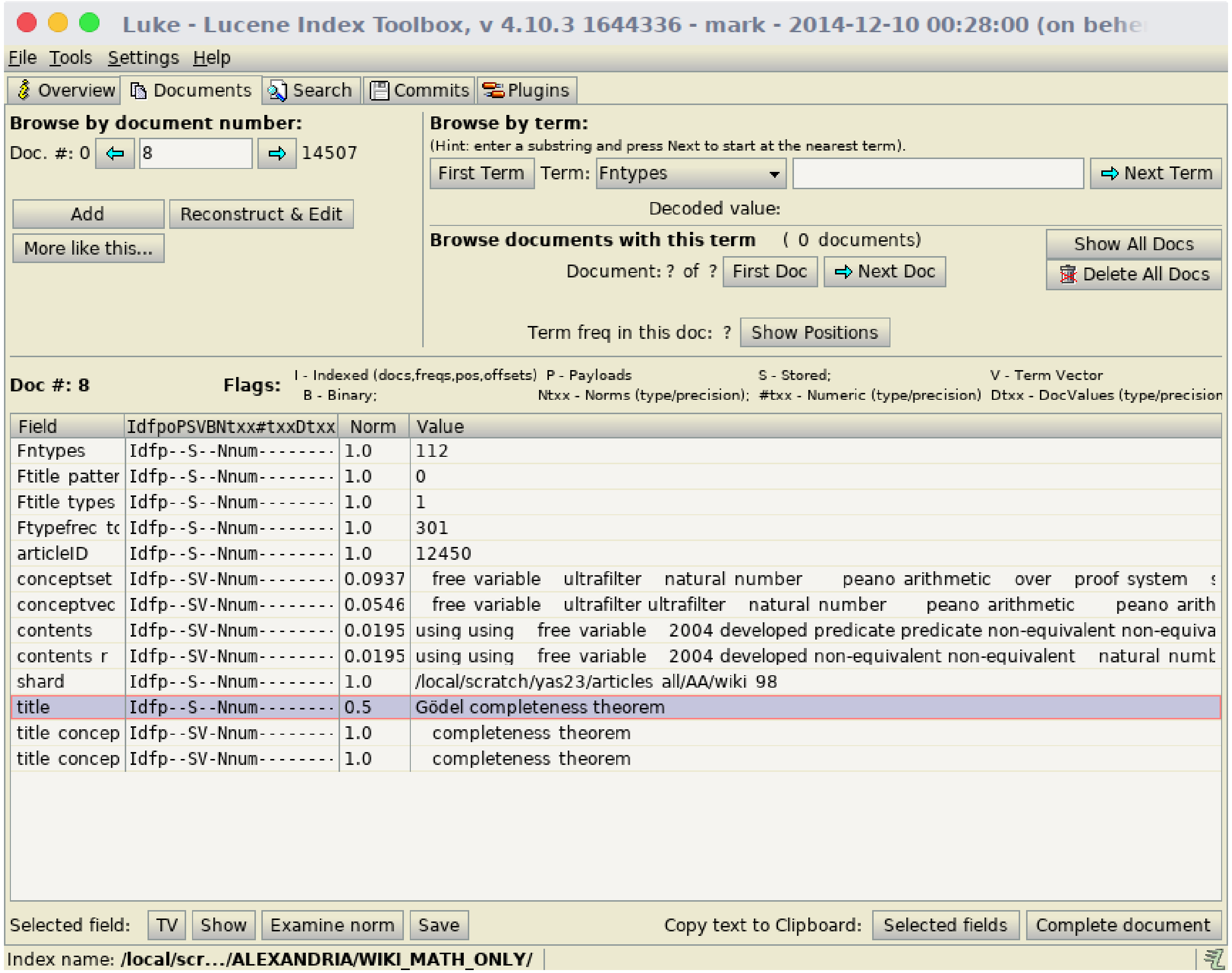
    2. We can model mathematical knowledge for large-scale retrieval.

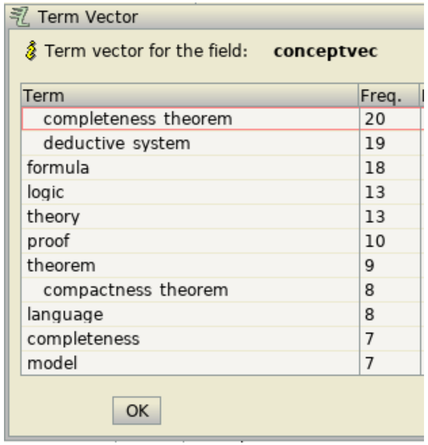
       – Thousands of facts in the Isabelle libraries and AFP

Mapping Facts to Wikipedia Articles - I

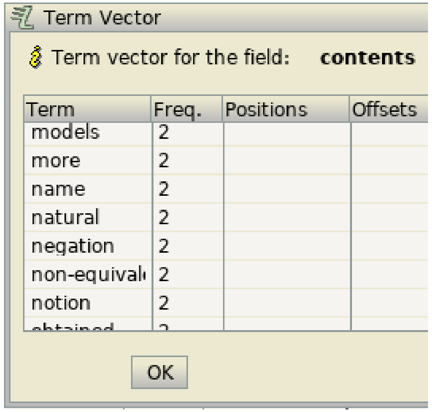
Step 1. Index (keywords and concepts) Wikipedia maths articles







tf model of concepts



tf model of words

Mapping Facts to Wikipedia Articles - II

Question: How do we map Isabelle facts to Wikipedia articles?

Step 2. Perform one Wikipedia index search per fact using query built from:

– Keywords and concepts from a fact’s name

– Keywords and concepts from comments around a fact

– Keywords and concepts from the source theory (background model)

Mapping Facts to Wikipedia Articles - III

FACT

– Keywords and concepts from a fact’s name

– Keywords and concepts from comments near to or in the body of a fact

– Keywords and concepts from source theory

ARTICLE

1. Title words
2. Article body words
3. Title concepts
4. Article concepts

Mapping Facts to Wikipedia Articles - IV

Cauchy_Schwarz_ineq
(HOL-Analysis/Inner_Product.thy)

Rank	Title
1	Cauchy–Schwarz inequality
2	Augustin-Louis Cauchy
3	Cauchy–Riemann equations
4	Cauchy sequence
5	Schwarz list
6	Cauchy momentum equation
7	Cauchy–Kowalevski theorem
8	Cauchy surface
9	Cauchy product
10	Albert Schwarz
11	Schwarz lemma
12	Binet–Cauchy identity
13	Cauchy theorem (group theory)
14	Cauchy–Rassias stability
15	Schwarz reflection principle
16	Schwarz–Ahlfors–Pick theorem
17	Abstract additive Schwarz method
18	Schwarz minimal surface
19	Schwarz triangle function
20	Cauchy theorem

meet_dual
(HOL-Algebra/Lattice.thy)

Rank	Title
1	Join and meet
2	Langlands dual group
3	Petrie dual
4	Lattice (order)
5	De Groot dual
6	Reductive dual pair
7	Complete lattice
8	Heyting algebra
9	Free lattice
10	F-algebra
11	Boolean algebra (structure)
12	Capelli identity
13	Skew lattice
14	Closure operator
15	0,1-simple lattice
16	Comparison of topologies
17	Fixed-point combinator
18	Distributive lattice
19	Semimodular lattice
20	Birkhoff representation theorem

Generating Representations for Facts

Step 3. Generate description for fact from the 20 most relevant articles:

- Build a distributional profile for each fact and the source theory from the 20 top-ranking Wikipedia articles

	Term Vector for Fact	Concept Vector for Fact
Method 1:	Sum up top 20 article term vectors	Sum up top 20 article concept vectors
Method 2:	Select 100 important words from top 20 articles using TF-IDF metric	Select 100 important concept phrases from top 20 articles using TF-IDF metric
Method 3:	Find the set that maximises the overlap of words between the top-20 articles using the Jaccard coefficient	Find the set that maximises the overlap of concepts between the top-20 articles using the Jaccard coefficient

Preliminary Evaluation - I

- Conducted over the Isabelle library only, did not include the AFP.
- Carefully constructed 25 queries to simulate a user describing a fact.

1. Came up with an information need and an example fact that satisfies it.
2. Wrote down keywords that describe it that do not exactly match its name to test concept associations e.g. “summability”, “zero”, “criterion” instead of “summable”, “null”, “test”.
3. Selected concept phrases from our dictionary that are topically related to the example fact

ID	Query Keywords	Query Concepts	# Relevant Facts
1	disk, norm, function, differentiable, derivative, bound	“derivative function”, “disk”, “bound”, “differentiability property”	35
2	borel, measure, basis, box	“borel measure”, “basis”	20
3	summability, criterion, test, norm, less, comparison	“test”, “comparison”, “summability condition”, “summability”, “norm”	22
4	multiply, less, positive	“multiply element”, “positive number”, “multiply”	6
5	summation, test, geometric, series	“summation”, “summability condition”, “geometric series”, “summable series”	22

Preliminary Evaluation - II

- Retrieval Models

  - Three methods presented earlier

  - Baseline (model 4): keywords only (no concept phrases)

- Lucene query generation done consistently across methods

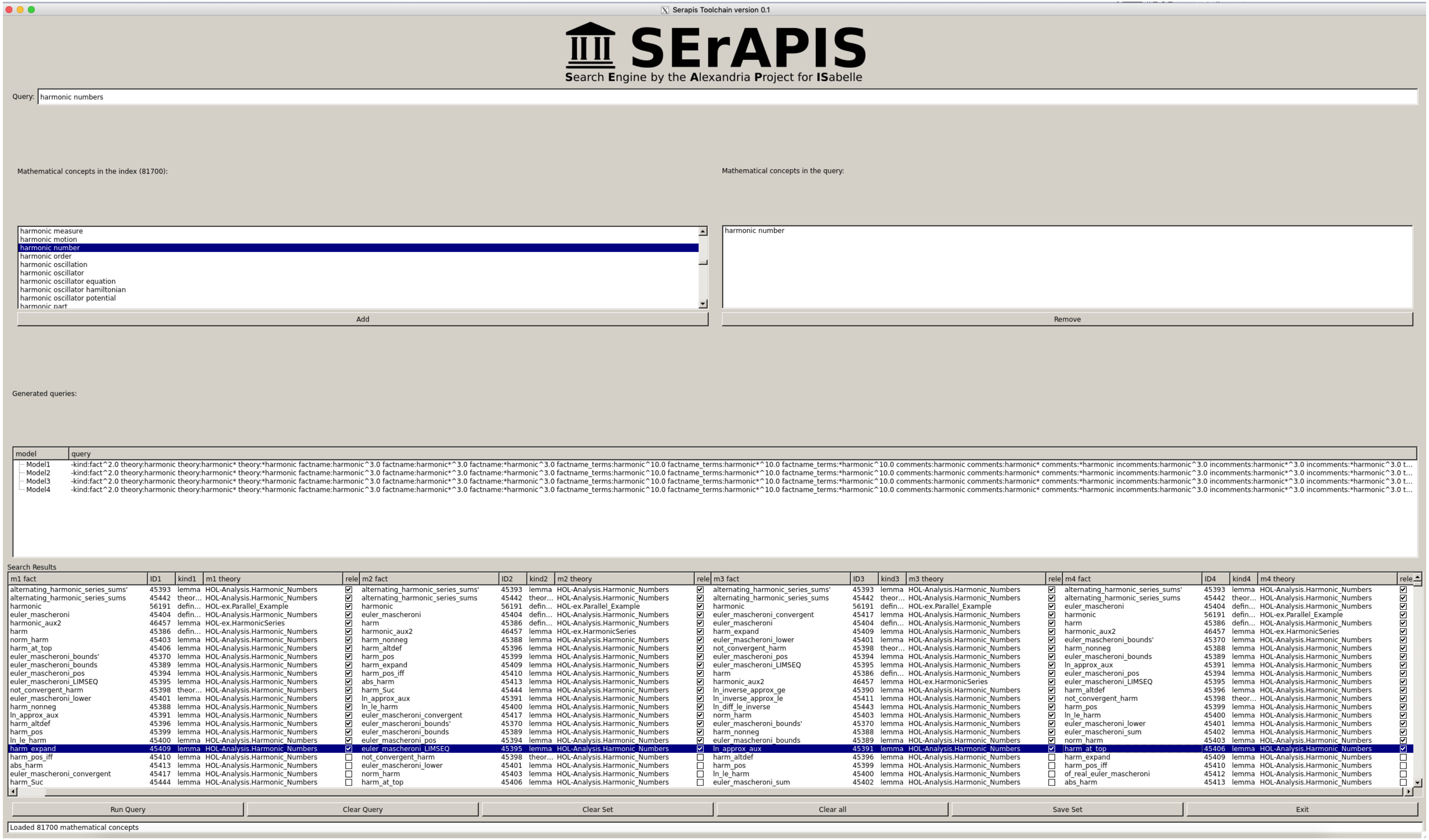
- Relevance judgements

  - Produced manually by Angeliki for all methods. Judged for relevance only the **first 20 results** for each method.

    Must contain main notion to be judged as relevant. If contains only secondary notion judged as irrelevant -judged by case.

  - Recorded using the SErAPIS desktop user interface

  - Pooled relevance judgements from all methods for evaluation



Serapis Toolchain version 0.1
SErAPIS
Search Engine by the Alexandria Project for ISabelle
Query: harmonic numbers
Mathematical concepts in the index (81700):
harmonic measure
harmonic motion
harmonic number
harmonic order
harmonic oscillation
harmonic oscillator
harmonic oscillator equation
harmonic oscillator hamiltonian
harmonic oscillator potential
Add
Mathematical concepts in the query:
harmonic number
Remove
Generated queries:
model query
Model1 -kind:fact^2.0 theory:harmonic theory:harmonic* theory:*harmonic factname:harmonic^3.0 factname:harmonic*^3.0 factname:*harmonic^3.0 factname_terms:harmonic^10.0 factname_terms:harmonic*^10.0 factname_terms:*harmonic^10.0 comments:harmonic comments:harmonic* comments:*harmonic incomments:harmonic^3.0 incomments:harmonic*^3.0 incomments:*harmonic^3.0 t...
Model2 -kind:fact^2.0 theory:harmonic theory:harmonic* theory:*harmonic factname:harmonic^3.0 factname:harmonic*^3.0 factname:*harmonic^3.0 factname_terms:harmonic^10.0 factname_terms:harmonic*^10.0 factname_terms:*harmonic^10.0 comments:harmonic comments:harmonic* comments:*harmonic incomments:harmonic^3.0 incomments:harmonic*^3.0 incomments:*harmonic^3.0 t...
Model3 -kind:fact^2.0 theory:harmonic theory:harmonic* theory:*harmonic factname:harmonic^3.0 factname:harmonic*^3.0 factname:*harmonic^3.0 factname_terms:harmonic^10.0 factname_terms:harmonic*^10.0 factname_terms:*harmonic^10.0 comments:harmonic comments:harmonic* comments:*harmonic incomments:harmonic^3.0 incomments:harmonic*^3.0 incomments:*harmonic^3.0 t...
Model4 -kind:fact^2.0 theory:harmonic theory:harmonic* theory:*harmonic factname:harmonic^3.0 factname:harmonic*^3.0 factname:*harmonic^3.0 factname_terms:harmonic^10.0 factname_terms:harmonic*^10.0 factname_terms:*harmonic^10.0 comments:harmonic comments:harmonic* comments:*harmonic incomments:harmonic^3.0 incomments:harmonic*^3.0 incomments:*harmonic^3.0 t...
Search Results
m1 fact | ID1 | kind1 | m1 theory | rele | m2 fact | ID2 | kind2 | m2 theory | rele | m3 fact | ID3 | kind3 | m3 theory | rele | m4 fact | ID4 | kind4 | m4 theory | rele
alternating_harmonic_series_sums' 45393 lemma HOL-Analysis.Harmonic_Numbers | alternating_harmonic_series_sums' 45393 lemma HOL-Analysis.Harmonic_Numbers | alternating_harmonic_series_sums' 45393 lemma HOL-Analysis.Harmonic_Numbers | alternating_harmonic_series_sums' 45393 lemma HOL-Analysis.Harmonic_Numbers
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norm_harm 45403 lemma HOL-Analysis.Harmonic_Numbers | harm_nonneg 45388 lemma HOL-Analysis.Harmonic_Numbers | euler_mascheroni_lower 45401 lemma HOL-Analysis.Harmonic_Numbers | euler_mascheroni_bounds' 45370 lemma HOL-Analysis.Harmonic_Numbers
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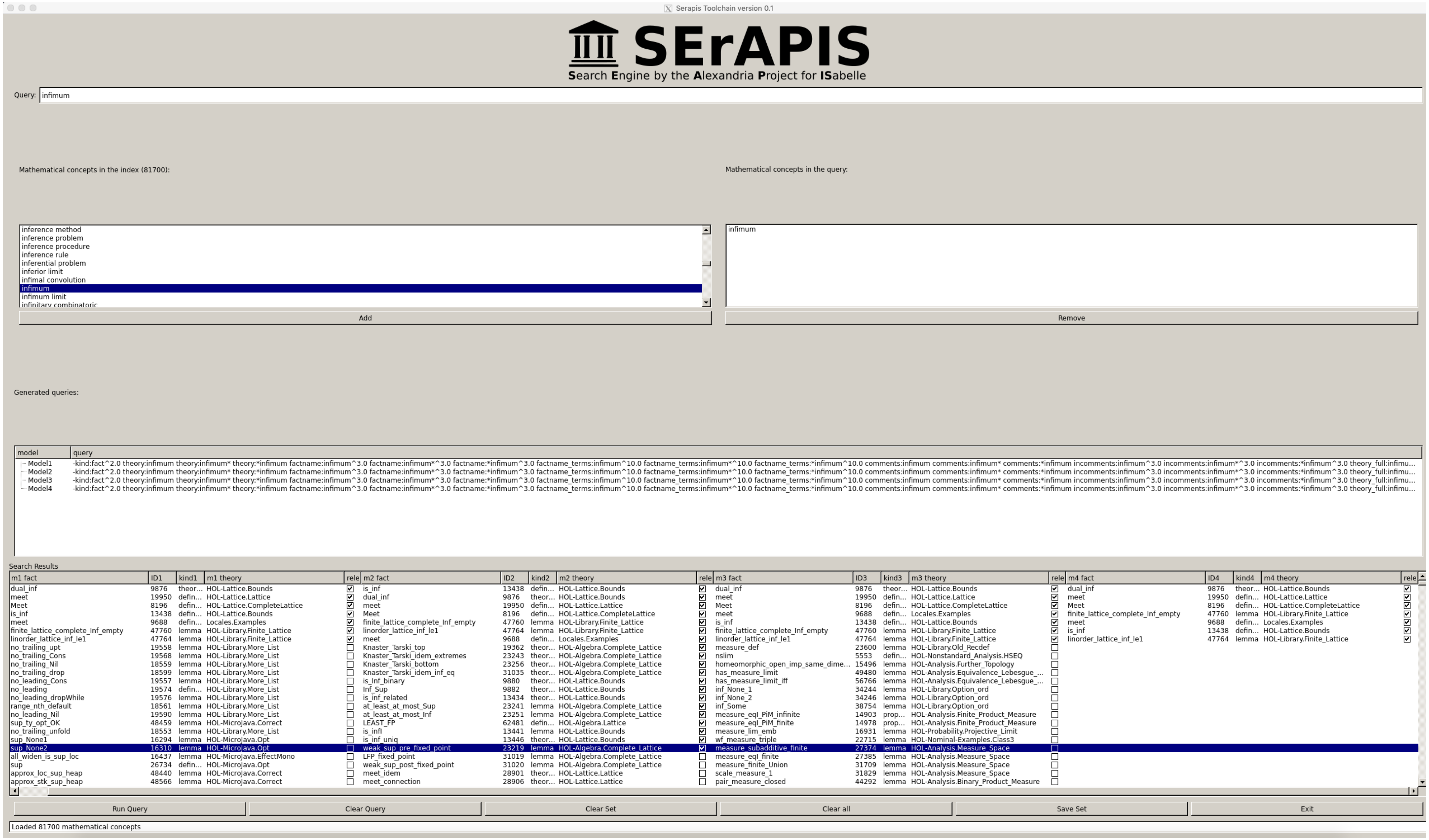
harm_Suc 45444 lemma HOL-Analysis.Harmonic_Numbers | harm_at_top 45406 lemma HOL-Analysis.Harmonic_Numbers | euler_mascheroni_sum 45402 lemma HOL-Analysis.Harmonic_Numbers | abs_harm 45413 lemma HOL-Analysis.Harmonic_Numbers
Run Query
Clear Query
Clear Set
Clear all
Save Set
Exit
Loaded 81700 mathematical concepts


Serapis Toolchain version 0.1
SErAPIS
Search Engine by the Alexandria Project for ISabelle
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Mathematical concepts in the index (81700):
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inference problem
inference procedure
inference rule
inferential problem
inferior limit
infimal convolution
infimum
infimum limit
infinitary combinatoric
Add
Mathematical concepts in the query:
infimum
Remove
Generated queries:
model query
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Model2 -kind:fact^2.0 theory:infimum theory:infimum* theory:*infimum factname:infimum^3.0 factname:infimum*^3.0 factname:*infimum^3.0 factname_terms:infimum^10.0 factname_terms:infimum*^10.0 factname_terms:*infimum^10.0 comments:infimum comments:infimum* comments:*infimum incomments:infimum^3.0 incomments:infimum*^3.0 incomments:*infimum^3.0 theory_full:infimu...
Model3 -kind:fact^2.0 theory:infimum theory:infimum* theory:*infimum factname:infimum^3.0 factname:infimum*^3.0 factname:*infimum^3.0 factname_terms:infimum^10.0 factname_terms:infimum*^10.0 factname_terms:*infimum^10.0 comments:infimum comments:infimum* comments:*infimum incomments:infimum^3.0 incomments:infimum*^3.0 incomments:*infimum^3.0 theory_full:infimu...
Model4 -kind:fact^2.0 theory:infimum theory:infimum* theory:*infimum factname:infimum^3.0 factname:infimum*^3.0 factname:*infimum^3.0 factname_terms:infimum^10.0 factname_terms:infimum*^10.0 factname_terms:*infimum^10.0 comments:infimum comments:infimum* comments:*infimum incomments:infimum^3.0 incomments:infimum*^3.0 incomments:*infimum^3.0 theory_full:infimu...
Search Results
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meet 9688 defin... Locales.Examples finite_lattice_complete_Inf_empty 47760 lemma HOL-Library.Finite_Lattice is_inf 13438 defin... HOL-Lattice.Bounds meet 9688 defin... Locales.Examples
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linorder_lattice_inf_le1 47764 lemma HOL-Library.Finite_Lattice meet 9688 defin... Locales.Examples linorder_lattice_inf_le1 47764 lemma HOL-Library.Finite_Lattice linorder_lattice_inf_le1 47764 lemma HOL-Library.Finite_Lattice
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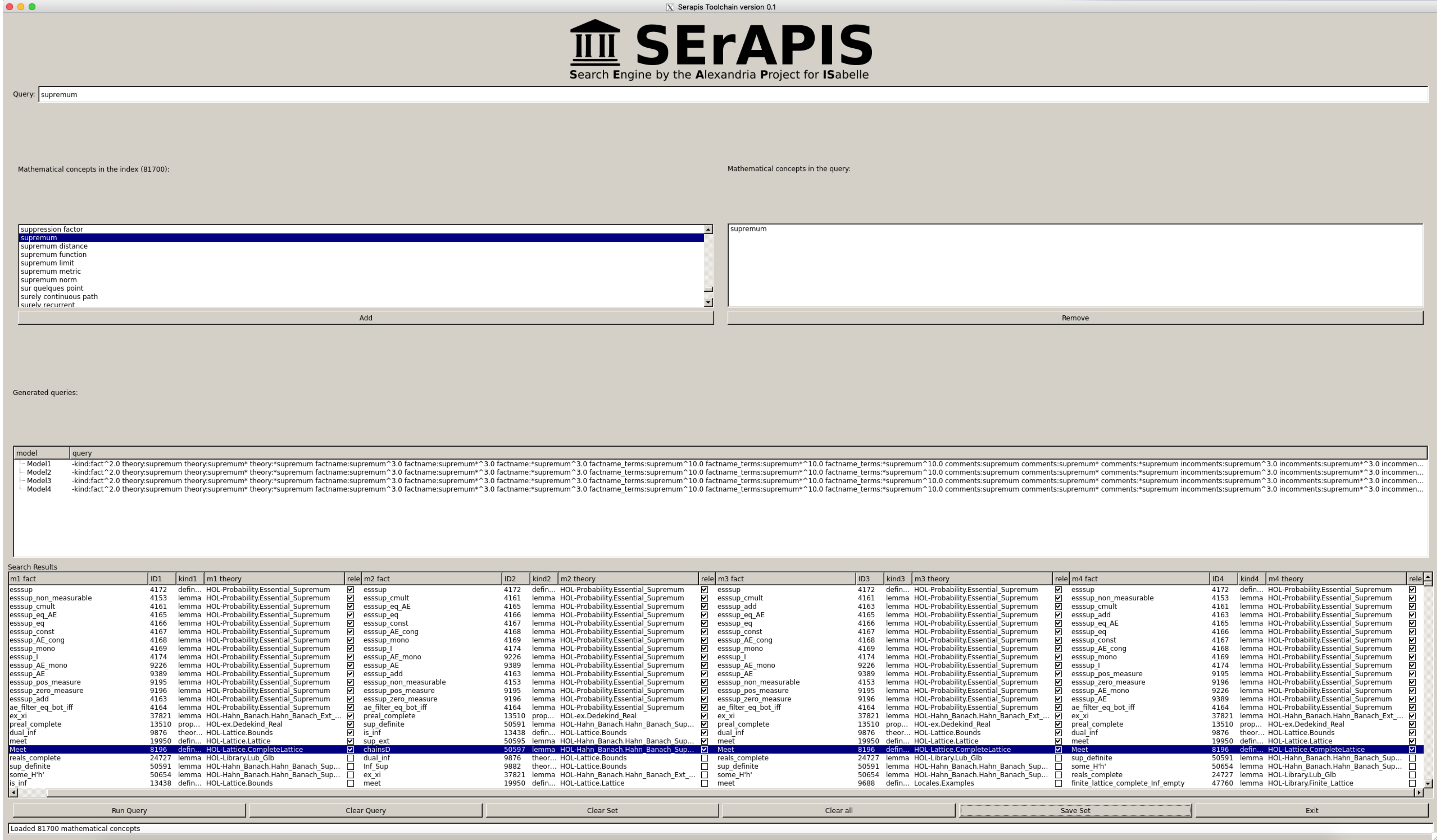
approx_stk_sup_heap 48566 lemma HOL-MicroJava.Correct meet_connection 28906 theor... HOL-Lattice.Lattice pair_measure_closed 44292 lemma HOL-Analysis.Binary_Product_Measure
Run Query
Clear Query
Clear Set
Clear all
Save Set
Exit
Loaded 81700 mathematical concepts



Serapis Toolchain version 0.1
SErAPIS
Search Engine by the Alexandria Project for ISabelle
Query: supremum
Mathematical concepts in the index (81700):
suppression factor
supremum
supremum distance
supremum function
supremum limit
supremum metric
supremum norm
sur quelques point
surely continuous path
surely recurrent
Add
Mathematical concepts in the query:
supremum
Remove
Generated queries:
model | query
Model1 -kind:fact^2.0 theory:supremum theory:supremum* theory:*supremum factname:supremum^3.0 factname:supremum*^3.0 factname:*supremum^3.0 factname_terms:supremum^10.0 factname_terms:supremum*^10.0 factname_terms:*supremum^10.0 comments:supremum comments:supremum* comments:*supremum incomments:supremum^3.0 incomments:supremum*^3.0 incommen...
Model2 -kind:fact^2.0 theory:supremum theory:supremum* theory:*supremum factname:supremum^3.0 factname:supremum*^3.0 factname:*supremum^3.0 factname_terms:supremum^10.0 factname_terms:supremum*^10.0 factname_terms:*supremum^10.0 comments:supremum comments:supremum* comments:*supremum incomments:supremum^3.0 incomments:supremum*^3.0 incommen...
Model3 -kind:fact^2.0 theory:supremum theory:supremum* theory:*supremum factname:supremum^3.0 factname:supremum*^3.0 factname:*supremum^3.0 factname_terms:supremum^10.0 factname_terms:supremum*^10.0 factname_terms:*supremum^10.0 comments:supremum comments:supremum* comments:*supremum incomments:supremum^3.0 incomments:supremum*^3.0 incommen...
Model4 -kind:fact^2.0 theory:supremum theory:supremum* theory:*supremum factname:supremum^3.0 factname:supremum*^3.0 factname:*supremum^3.0 factname_terms:supremum^10.0 factname_terms:supremum*^10.0 factname_terms:*supremum^10.0 comments:supremum comments:supremum* comments:*supremum incomments:supremum^3.0 incomments:supremum*^3.0 incommen...
Search Results

m1 fact	ID1	kind1	m1 theory	rele	m2 fact	ID2	kind2	m2 theory	rele	m3 fact	ID3	kind3	m3 theory	rele	m4 fact	ID4	kind4	m4 theory	rele
esssup	4172	defin...	HOL-Probability.Essential_Supremum	☑	esssup	4172	defin...	HOL-Probability.Essential_Supremum	☑	esssup	4172	defin...	HOL-Probability.Essential_Supremum	☑	esssup	4172	defin...	HOL-Probability.Essential_Supremum	☑
esssup_non_measurable	4153	lemma	HOL-Probability.Essential_Supremum	☑	esssup_cmult	4161	lemma	HOL-Probability.Essential_Supremum	☑	esssup_cmult	4161	lemma	HOL-Probability.Essential_Supremum	☑	esssup_non_measurable	4153	lemma	HOL-Probability.Essential_Supremum	☑
esssup_cmult	4161	lemma	HOL-Probability.Essential_Supremum	☑	esssup_eq_AE	4165	lemma	HOL-Probability.Essential_Supremum	☑	esssup_add	4163	lemma	HOL-Probability.Essential_Supremum	☑	esssup_cmult	4161	lemma	HOL-Probability.Essential_Supremum	☑
esssup_eq_AE	4165	lemma	HOL-Probability.Essential_Supremum	☑	esssup_eq	4166	lemma	HOL-Probability.Essential_Supremum	☑	esssup_eq_AE	4165	lemma	HOL-Probability.Essential_Supremum	☑	esssup_add	4163	lemma	HOL-Probability.Essential_Supremum	☑
esssup_eq	4166	lemma	HOL-Probability.Essential_Supremum	☑	esssup_const	4167	lemma	HOL-Probability.Essential_Supremum	☑	esssup_eq	4166	lemma	HOL-Probability.Essential_Supremum	☑	esssup_eq_AE	4165	lemma	HOL-Probability.Essential_Supremum	☑
esssup_const	4167	lemma	HOL-Probability.Essential_Supremum	☑	esssup_AE_cong	4168	lemma	HOL-Probability.Essential_Supremum	☑	esssup_const	4167	lemma	HOL-Probability.Essential_Supremum	☑	esssup_eq	4166	lemma	HOL-Probability.Essential_Supremum	☑
esssup_AE_cong	4168	lemma	HOL-Probability.Essential_Supremum	☑	esssup_mono	4169	lemma	HOL-Probability.Essential_Supremum	☑	esssup_AE_cong	4168	lemma	HOL-Probability.Essential_Supremum	☑	esssup_const	4167	lemma	HOL-Probability.Essential_Supremum	☑
esssup_mono	4169	lemma	HOL-Probability.Essential_Supremum	☑	esssup_I	4174	lemma	HOL-Probability.Essential_Supremum	☑	esssup_mono	4169	lemma	HOL-Probability.Essential_Supremum	☑	esssup_AE_cong	4168	lemma	HOL-Probability.Essential_Supremum	☑
esssup_I	4174	lemma	HOL-Probability.Essential_Supremum	☑	esssup_AE_mono	9226	lemma	HOL-Probability.Essential_Supremum	☑	esssup_I	4174	lemma	HOL-Probability.Essential_Supremum	☑	esssup_mono	4169	lemma	HOL-Probability.Essential_Supremum	☑
esssup_AE_mono	9226	lemma	HOL-Probability.Essential_Supremum	☑	esssup_AE	9389	lemma	HOL-Probability.Essential_Supremum	☑	esssup_AE_mono	9226	lemma	HOL-Probability.Essential_Supremum	☑	esssup_I	4174	lemma	HOL-Probability.Essential_Supremum	☑
esssup_AE	9389	lemma	HOL-Probability.Essential_Supremum	☑	esssup_add	4163	lemma	HOL-Probability.Essential_Supremum	☑	esssup_AE	9389	lemma	HOL-Probability.Essential_Supremum	☑	esssup_pos_measure	9195	lemma	HOL-Probability.Essential_Supremum	☑
esssup_pos_measure	9195	lemma	HOL-Probability.Essential_Supremum	☑	esssup_non_measurable	4153	lemma	HOL-Probability.Essential_Supremum	☑	esssup_non_measurable	4153	lemma	HOL-Probability.Essential_Supremum	☑	esssup_zero_measure	9196	lemma	HOL-Probability.Essential_Supremum	☑
esssup_zero_measure	9196	lemma	HOL-Probability.Essential_Supremum	☑	esssup_pos_measure	9195	lemma	HOL-Probability.Essential_Supremum	☑	esssup_pos_measure	9195	lemma	HOL-Probability.Essential_Supremum	☑	esssup_AE_mono	9226	lemma	HOL-Probability.Essential_Supremum	☑
esssup_add	4163	lemma	HOL-Probability.Essential_Supremum	☑	esssup_zero_measure	9196	lemma	HOL-Probability.Essential_Supremum	☑	esssup_zero_measure	9196	lemma	HOL-Probability.Essential_Supremum	☑	esssup_AE	9389	lemma	HOL-Probability.Essential_Supremum	☑
ae_filter_eq_bot_iff	4164	lemma	HOL-Probability.Essential_Supremum	☑	ae_filter_eq_bot_iff	4164	lemma	HOL-Probability.Essential_Supremum	☑	ae_filter_eq_bot_iff	4164	lemma	HOL-Probability.Essential_Supremum	☑	ae_filter_eq_bot_iff	4164	lemma	HOL-Probability.Essential_Supremum	☑
ex_xi	37821	lemma	HOL-Hahn_Banach.Hahn_Banach_Ext_...	☑	preal_complete	13510	prop...	HOL-ex.Dedekind_Real	☑	ex_xi	37821	lemma	HOL-Hahn_Banach.Hahn_Banach_Ext_...	☑	ex_xi	37821	lemma	HOL-Hahn_Banach.Hahn_Banach_Ext_...	☑
preal_complete	13510	prop...	HOL-ex.Dedekind_Real	☑	sup_definite	50591	lemma	HOL-Hahn_Banach.Hahn_Banach_Sup...	☑	preal_complete	13510	prop...	HOL-ex.Dedekind_Real	☑	preal_complete	13510	prop...	HOL-ex.Dedekind_Real	☑
dual_inf	9876	theor...	HOL-Lattice.Bounds	☑	is_inf	13438	defin...	HOL-Lattice.Bounds	☑	dual_inf	9876	theor...	HOL-Lattice.Bounds	☑	dual_inf	9876	theor...	HOL-Lattice.Bounds	☑
meet	19950	defin...	HOL-Lattice.Lattice	☑	sup_ext	50595	lemma	HOL-Hahn_Banach.Hahn_Banach_Sup...	☑	meet	19950	defin...	HOL-Lattice.Lattice	☑	meet	19950	defin...	HOL-Lattice.Lattice	☑
Meet	8196	defin...	HOL-Lattice.CompleteLattice	☑	chainsD	50597	lemma	HOL-Hahn_Banach.Hahn_Banach_Sup...	☑	Meet	8196	defin...	HOL-Lattice.CompleteLattice	☑	Meet	8196	defin...	HOL-Lattice.CompleteLattice	☑
reals_complete	24727	lemma	HOL-Library.Lub_Glb	☐	dual_inf	9876	theor...	HOL-Lattice.Bounds	☐	reals_complete	24727	lemma	HOL-Library.Lub_Glb	☐	sup_definite	50591	lemma	HOL-Hahn_Banach.Hahn_Banach_Sup...	☐
sup_definite	50591	lemma	HOL-Hahn_Banach.Hahn_Banach_Sup...	☐	Inf_Sup	9882	theor...	HOL-Lattice.Bounds	☐	sup_definite	50591	lemma	HOL-Hahn_Banach.Hahn_Banach_Sup...	☐	some_H'h'	50654	lemma	HOL-Hahn_Banach.Hahn_Banach_Sup...	☐
some_H'h'	50654	lemma	HOL-Hahn_Banach.Hahn_Banach_Sup...	☐	ex_xi	37821	lemma	HOL-Hahn_Banach.Hahn_Banach_Ext_...	☐	some_H'h'	50654	lemma	HOL-Hahn_Banach.Hahn_Banach_Sup...	☐	reals_complete	24727	lemma	HOL-Library.Lub_Glb	☐
is_inf	13438	defin...	HOL-Lattice.Bounds	☐	meet	19950	defin...	HOL-Lattice.Lattice	☐	meet	9688	defin...	Locales.Examples	☐	finite_lattice_complete_Inf_empty	47760	lemma	HOL-Library.Finite_Lattice	☐

Run Query
Clear Query
Clear Set
Clear all
Save Set
Exit
Loaded 81700 mathematical concepts

Preliminary Evaluation - III

- Results

	Model 1	Model 2	Model 3	Model 4
MAP	.775	.659	.731	.688
Model 1	-	$>$	$>$	$>$
Model 2	$<$	-	$\approx$	$\approx$
Model 3	$<$	$\approx$	-	$\approx$
Model 4	$<$	$\approx$	$\approx$	-

- Performance measured in terms of Mean Average Precision (MAP)
- X > Y : difference statistically significant at $\alpha = 0.05$
- Significance tested using the paired Permutation (non-parametric) test

Challenge 3: Evaluating Effectiveness of Isabelle Fact Retrieval

1. No baseline to compare our methods against

   – Results from *find_theorems* are unranked

   AND

   – depend on the libraries loaded by the user

2. There is no large-scale test collection of Isabelle facts

   - Need realistic queries from working mathematicians

   - Thousands of facts to judge relevance against

Large-scale Evaluation

- Plan: build a data set for large-scale Isabelle search research and evaluation

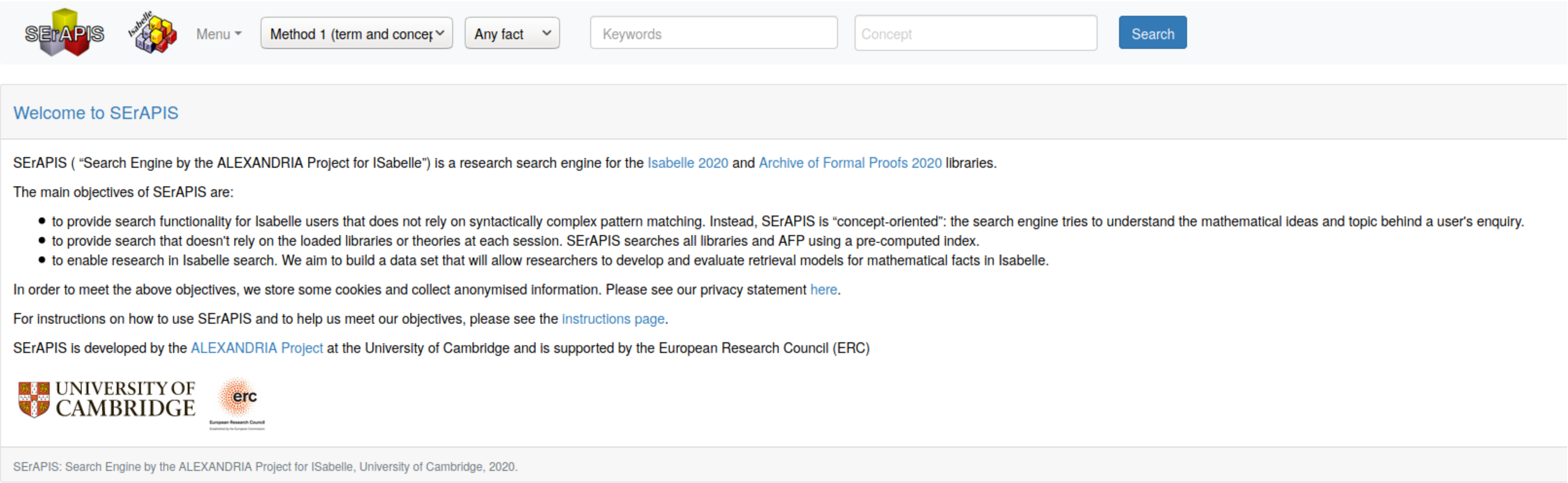
- We want to make SErAPIS available online for the Isabelle community:

  - Isabelle users can benefit from concept-oriented Isabelle search

  - We collect real-life queries and relevance decisions anonymously

Compile a large (anonymised) search data set for public release

SErAPIS Online Isabelle Search Engine



Demo

Ongoing and Future Work

1. Identify and make searchable proof idioms.

```
show "P(n)"
proof (induction n)
  case 0
  ⋮
  show ?case  ⟨proof⟩
next
  case (Suc n)
  ⋮
  show ?case  ⟨proof⟩
qed
```

```
have "P ∨ Q"  ⟨proof⟩
then show "R"
proof
  assume "P"
  ⋮
  show "R"  ⟨proof⟩
next
  assume "Q"
  ⋮
  show "R"  ⟨proof⟩
qed
```

```
show "P ⟷ Q"
proof
  assume "P"
  ⋮
  show "Q"  ⟨proof⟩
next
  assume "Q"
  ⋮
  show "P"  ⟨proof⟩
qed
```

```
show "¬ P"
proof
  assume "P"
  ⋮
  show "False"  ⟨proof⟩
qed
```

```
show "P"
proof (rule ccontr)
  assume "¬P"
  ⋮
  show "False"  ⟨proof⟩
qed
```

2. Support formula search for matching propositions (statement and proofs).

3. Deep indexing of libraries for recommending next steps in interactive proofs.

- Integrate SErAPIS to Isabelle and offer relevant suggestions in real-time.

Thank you for your time.

For more details see:

- Stathopoulos, Koutsoukou-Argyraki and Paulson: *SErAPIS: A Concept-Oriented Search Engine for the Isabelle Libraries Based on Natural Language*, to appear in the Informal proceedings of the Isabelle 2020 Workshop affiliated to ICJAR 2020, (in Virtual Space), June 30, 2020. https://files.sketis.net/Isabelle_Workshop_2020/Isabelle_2020_paper_4.pdf

- Stathopoulos, Koutsoukou-Argyraki and Paulson: *Developing a Concept-Oriented Search Engine for Isabelle Based on Natural Language : Technical Challenges*, to appear in the informal proceedings of the 5th Conference on Artificial Intelligence and Theorem Proving (AITP 2020), Aussois, France, Mar. 22-27, POSTPONED TO Sept. 13-18, 2020. http://aitp-conference.org/2020/abstract/paper_9.pdf

Questions?