Learning Clause Deletion Heuristics with Reinforcement Learning

(with a twist)

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Goal: Use ML in order to build better SAT solvers to tackle *industrial-scale* problems (e.g., SATCOMP).

Highlight

1. Preamble on SAT Heuristics
2. Earlier Efforts
3. Challenges of Gaining Wall-Clock Improvement on SAT
4. Problem Statement
5. Redemption *(a.k.a. the twist)*

Preamble on SAT Heuristics

Backtracking search algorithms for SAT (DPLL) gradually extend a partial assignment by selecting a new variable to branch on at each decision level. The partial assignment is extended until it becomes a satisfying assignment, or until a *conflict* is reached.

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Conflict-driven clause learning (CDCL) combines backtracking search with *clause learning*. While DPLL simply backtracks out of conflicts, CDCL "analyzes" the conflict by performing a couple of resolution steps and "learns" a clause which is added to the list of clauses in order to cut off large parts of the search space and thereby speeds up the search process.

Preamble on SAT Heuristics (cont.)

1. **Clause Deletion:** Audemard and Simon in 2009 devised a new score to identify clauses that are likely to be used more frequently in the future. The *Literals Block Distance (LBD)* of a clause is the number of distinct decision levels of its literals at the time of the conflict analysis.
   Very aggressive clause deletion where half of the learnt clauses are removed every ~ 20000 conflicts, gives a significant boost in performance.
2. **Branching Heuristic:** VSIDS has been the dominant heuristic here, where an *activity* score is kept for each variable. Upon each conflict the activity of variables involved in the conflict are *bumped*, and every variable's activity is periodically *decayed* exponentially.

Earlier Efforts

QBF: Lederman, et al. *"Learning Heuristics for Quantified Boolean Formulas through Reinforcement Learning."*, ICLR 2019.

SAT:

	NeuroSAT	NeuroCore	GQSAT	"deep" WalkSAT
Training Alg.	Supervised	Supervised	DQN	REINFORCE
GNN?	✔	✔	✔	✔
Target Heuristic	–	Branching	Branching	Branching
Completeness	✖	✔	✔	✔
Problem Size	(40, –)[1]	Scheduling Problems	(300, 1000)	rand(50, 213)
CDCL-based?	✖	✔	✔	✖

[1] (# variables, # clauses)

Challenges of Gaining Wall-Clock Improvement on SAT

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  - ***[Solution] RL can 1. better capture the dynamic nature of the solver and 2. it allows for training the heuristic directly towards optimizing the desired metric (e.g., number of decisions, running time, etc.)***

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    - Generalizability: formula should preserve some "structural properties" of the real industrial instances
  - ***[Solution 1] - simplify original formula f by attaching a partial assignment m to it:***
    1. $(SAT?, m) \leftarrow$ ***Solve*** f ***with vanilla*** `Glucose`
    2. ***if*** $SAT?$***:*** $m|_r \leftarrow$ ***random subset of*** m
    3. ***if*** $\neg SAT?$***:*** $m|_r \leftarrow$ ***random assignment to subset of*** $vars(f)$
    4. $f_{simp} = f \wedge \bigwedge_{u \in m|_r} u$

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  - ***[Solution] Either crop the problem graph or send delta updates or possibly partition the graph and use parallelism.***

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  - ***[Solution] Less frequent queries to the model, e.g., 1. Periodic Refocusing; 2. Clause Reduction***
- Action space explosion (Specifically for Clause Deletion)
  - Clause Reduction receives a list of $N(\sim 2000)$ clauses and returns a binary vector in $\{0,1\}^N$
  - The naive idea of deciding keep/drop per clause makes for an exponential space
  - In policy gradient algorithms that means huge "injected" randomness and huge variance in gradient estimates
  - ***[Solution] Threshold Policy: On every step, the policy outputs a single real positive number. All clauses with LBD score greater or equal to this threshold are deleted.***

Refined Goal: Use *RL* in order to extract *versatile* heuristics for *complete* solvers to tackle *industrial-scale* problems (e.g., SATCOMP).

Problem Statement

The environment $\mathcal{E}$ is a *Markov Decision Process (MDP)* with states $\mathcal{S}$, action space $\mathcal{A}$, and rewards per time step $r_t \in \mathbb{R}$. In our setting:

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- An *episode* is the result of the interaction of the agent with $\mathcal{E}$ while solving a formula f
  - *Complete*: Solver solved f successfully
  - *Incomplete*: Solver was aborted due to some *termination criteria*

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- The *reward* is a *deterministic* operation counter acting as a surrogate for solver's performance.

(Negative) Results

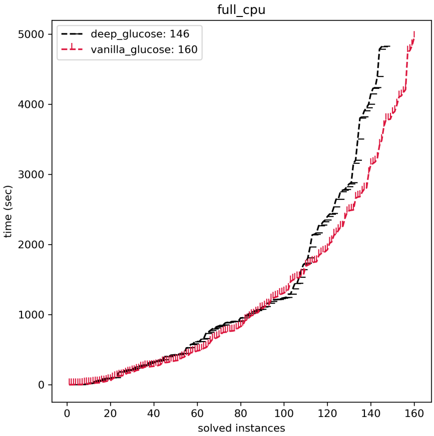

Figure 1: We trained on simplified formulas from past few SAT Competitions. The result is compared with `Glucose` on SAT competition 2018:

Culprits (conjectures)

- Clause deletion is a less direct method to change solver's behaviour.
- LBD is already too good and the ceiling is not high enough.
- The cost of querying a "better policy" is not justified by its benefits.

The Twist – `Neuro#`[2]

We used our framework to target another problem:

1. Target another domain with smaller problem sizes: #SAT
   - At each step the model only sees a subset of the entire graph (component) making GNNs more feasible.

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4. Go distribution-specific!
   - Target a specific family of problems
   - Use a high-level generative procedure to produce problems of varying sizes (small problems for training)
   - *No an unreasonable assumption in industry.*

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Neuro# – Results

Cellular:

GridWorld:

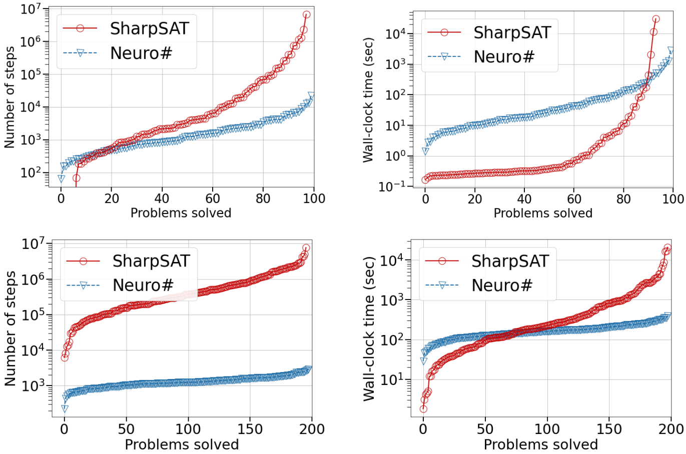

Figure 2: Trained on instances of size $6k/25k$ (Cellular) and $300/1k$ (Grid). Results are tested on instances of size $25k/102k$ (Cellular) and $2k/6k$ (Grid).

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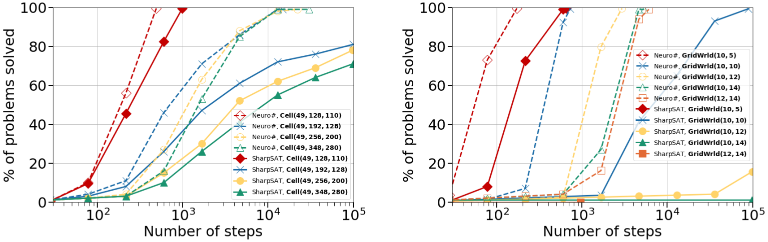

Cellular GridWorld

Figure 3: Neuro# generalizes well to larger problems. Compare the robustness of Neuro# vs. SharpSAT as the problem sizes increase. Solid and dashed lines correspond to SharpSAT and Neuro#, respectively. All episodes are capped at 100k steps.

Q&A

Thanks!