Arithmetical Mini-Games

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Motivation

Can we teach arithmetic to a computer?

- few basic concepts
- gradual difficulty
- easy to evaluate progress

Computation mini-game: problem

$$2 \times (2 + 3) \times 4 \ ?$$

Computation mini-game: problem space

Term of the form:

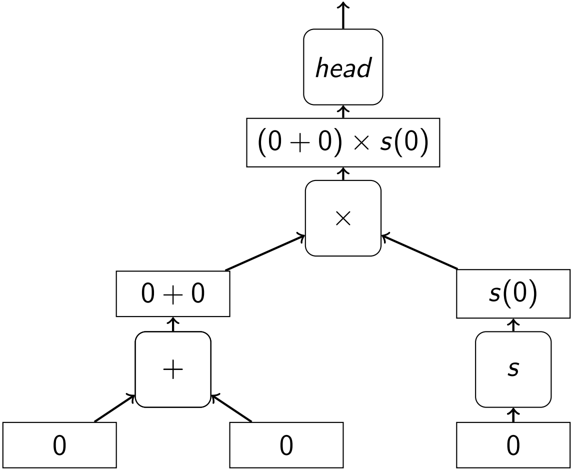
$$n_1 \ *_1 \ n_2 \ *_2 \ \dots \ n_K \ *_K \ n_{K+1}$$

where $*_i \in \{+, x\}$ and $n_i < N$ and arbitrary parentheses.

Unary numbers:

$$3 = s(s(s(0)))$$

Computation mini-game: recursive neural network



Computation mini-game: results

N | K	1	2	3	4	5	6	7	8
1	1.00	1.00	1.00	0.99	0.98	0.96	0.95	0.94
2	1.00	1.00	1.00	0.99	0.98	0.96	0.93	0.90
3	1.00	1.00	0.99	0.95	0.86	0.80	0.78	0.73
4	1.00	1.00	0.98	0.90	0.82	0.78	0.73	0.70
5	0.76	0.70	0.66	0.63	0.55	0.51	0.51	0.50
6	0.58	0.56	0.53	0.52	0.45	0.48	0.43	0.45
7	0.51	0.49	0.44	0.43	0.41	0.38	0.41	0.41
8	0.44	0.42	0.39	0.41	0.43	0.39	0.40	0.40

Computation mini-game: inspection

$$\begin{aligned} f(4) &= [0.0, 0.23, 1.0, 0.0] = 4 \\ f(5) &= [0.91, 0.0, 0.0, 0.04] = 1 \\ f(4+5) &= [1.0, 0.0, 1.0, 0.0] = 5 \\ f(4+(2+3)) &= [1.0, 0.0, 0.0, 1.0] = 9 \end{aligned}$$

Computation mini-game: inspection

$$
\begin{aligned}
f(4) &= [0.0, 0.23, 1.0, 0.0] = 4 \\
f(5) &= [0.91, 0.0, 0.0, 0.04] = 1 \\
f(4+5) &= [1.0, 0.0, 1.0, 0.0] = 5 \\
f(4+(2+3)) &= [1.0, 0.0, 0.0, 1.0] = 9
\end{aligned}
$$

$s(1+1)$?

Computation mini-game: improvement?

N | K	1	2	3	4	5	6	7	8
1	1.00	1.00	0.99	0.94	0.90	0.89	0.85	0.84
2	1.00	1.00	0.99	0.96	0.94	0.90	0.88	0.85
3	1.00	1.00	0.96	0.89	0.83	0.75	0.70	0.66
4	1.00	1.00	0.93	0.83	0.76	0.67	0.66	0.67
5	1.00	0.98	0.86	0.73	0.63	0.56	0.54	0.51
6	1.00	0.90	0.72	0.60	0.51	0.48	0.49	0.46
7	0.99	0.78	0.60	0.47	0.43	0.40	0.37	0.38
8	0.98	0.72	0.57	0.48	0.42	0.43	0.40	0.42

Computation mini-game: improvement inspection

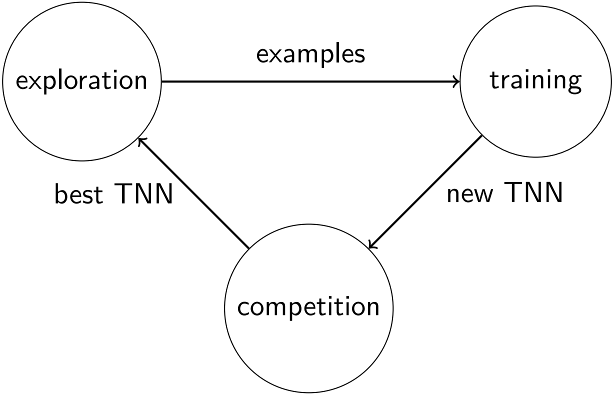
$$f(5) = [1.0, 0.0, 1.0, 0.0] = 5$$
$$f(6) = [0.02, 1.0, 1.0, 0.02] = 6$$
$$f(7) = [1.0, 1.0, 1.0, 0.0] = 7$$
$$f(8) = [0.0, 0.0, 0.0, 0.87] = 8$$

Proof mini-game: problem

$$x + 0 = x \tag{1}$$
$$x + s(y) = s(x + y) \tag{2}$$
$$x \times 0 \to 0 \tag{3}$$
$$x \times s(y) = x \times y + x \tag{4}$$

$$2 \times 2 \to 2 \times 1 + 2 \to \dots$$

Proof mini-game: overview of the solution

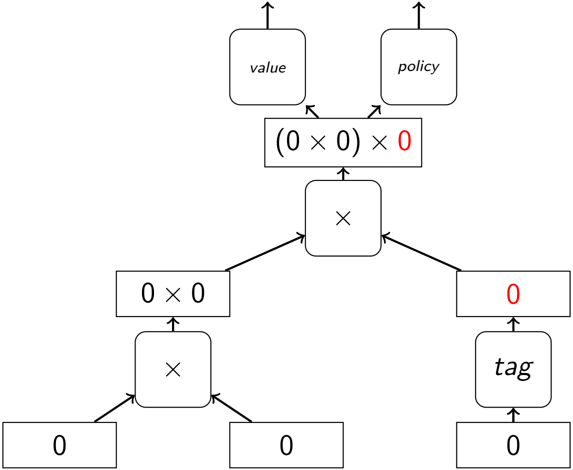


Proof mini-game: proof search (exploration,competition)

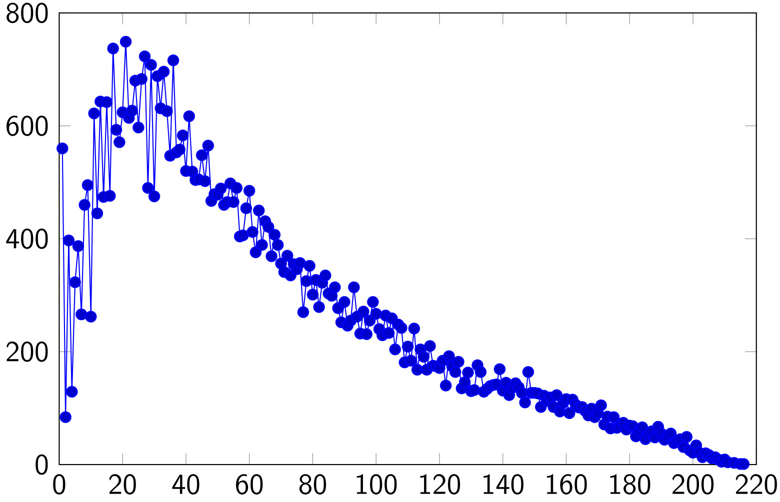
State: $2 \times 1 + 2$
Move:
  $2 \times 1 + 2 \to s(2 \times 1 + 1)$
  $2 \times 1 + 2 \to 2 \times 1 + 2 \to (2 \times 0 + 2) + 2$
Win condition: 4

Proof search (MCTS):
  1600 simulations, 0.99 decay, abstract time limit, Dirichlet noise

Proof mini-game: training tree neural network



Proof mini-game: distribution of training targets



Proof mini-game: target selection

> 15000 targets

First filter:

- Level 1: 400 easiest targets
- Level 2: 800 easiest targets
- ...

Second filter:

Competition: 100 random targets
Exploration: 100 self-assign targets

Proof mini-game: results

N | K	1	2	3	4
1	0.75	0.69	0.59	0.44
2	0.62	0.48	0.35	0.34
3	0.59	0.58	0.59	0.47
4	0.75	0.72	0.63	0.71

Take away

Choose the training set and the size of embeddings carefully!

Next challenges

Compute: $(2 + i) \times (2 - i)$

Solve: $x \times 2 = 4$

Prove: $\forall x.\ 0 + x = x$