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Using Machine Learning to Minimize User Intervention in Theorem Proving based Dynamic Fault Tree Analysis

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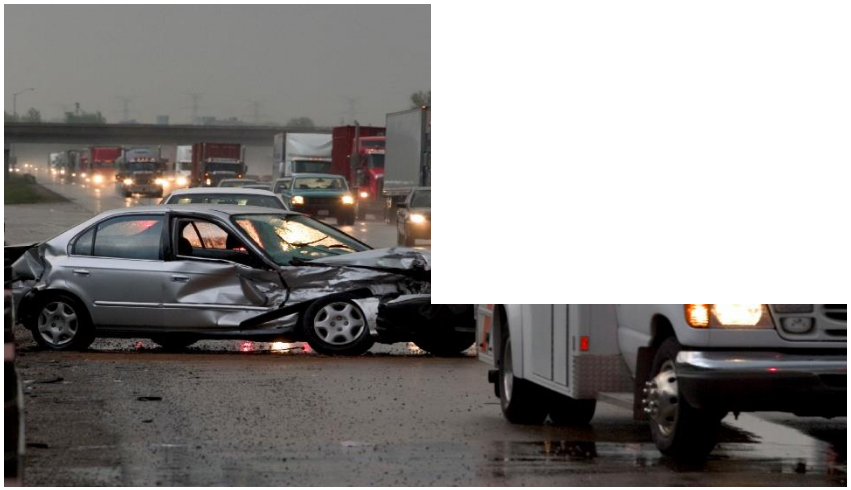
Outline

- Introduction
- Dynamic Fault Trees
- Proposed Methodology
- Preliminary Results
- Conclusion and Future Work

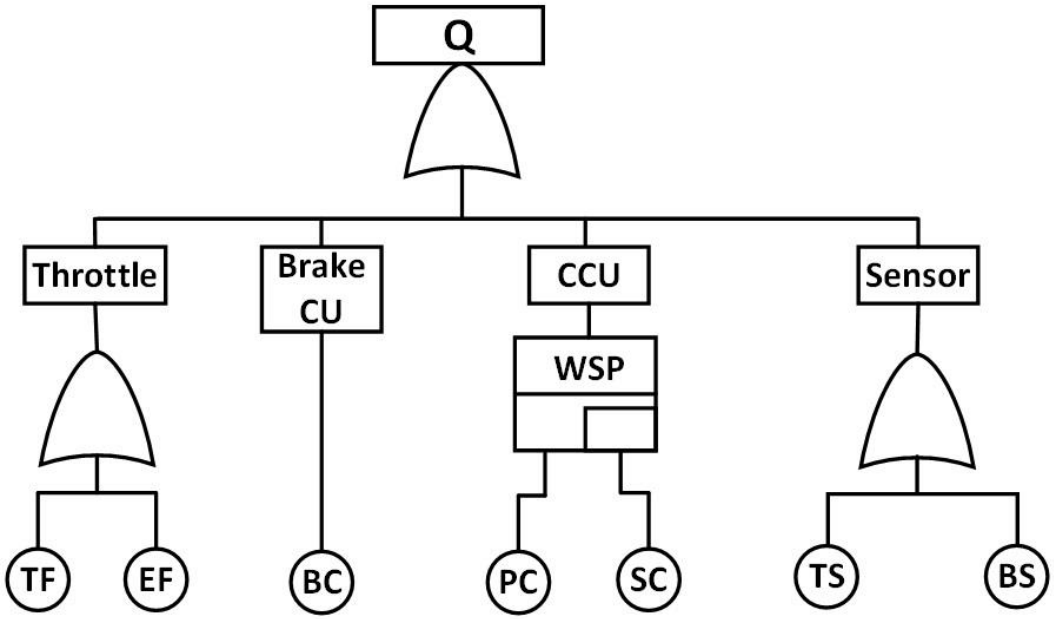
Failure Analysis



Analyze the effect of components faults on the system failure

Fault Trees

- Graphical representation of faults in the system
- Critical top event which will cause system failure
- The conditions are modeled using fault tree gates



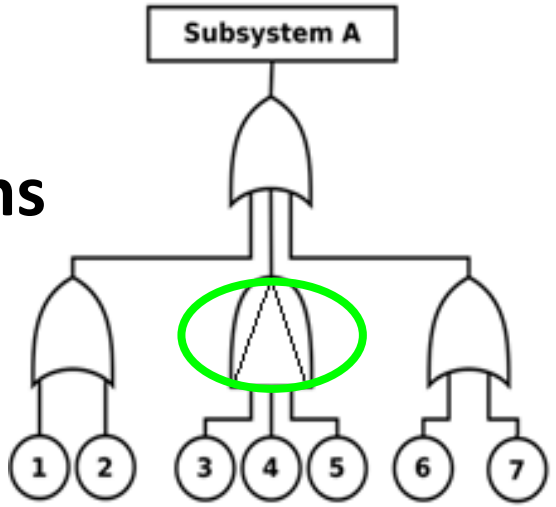
Fault Trees

Fault Trees

Static Fault Trees
SFTs

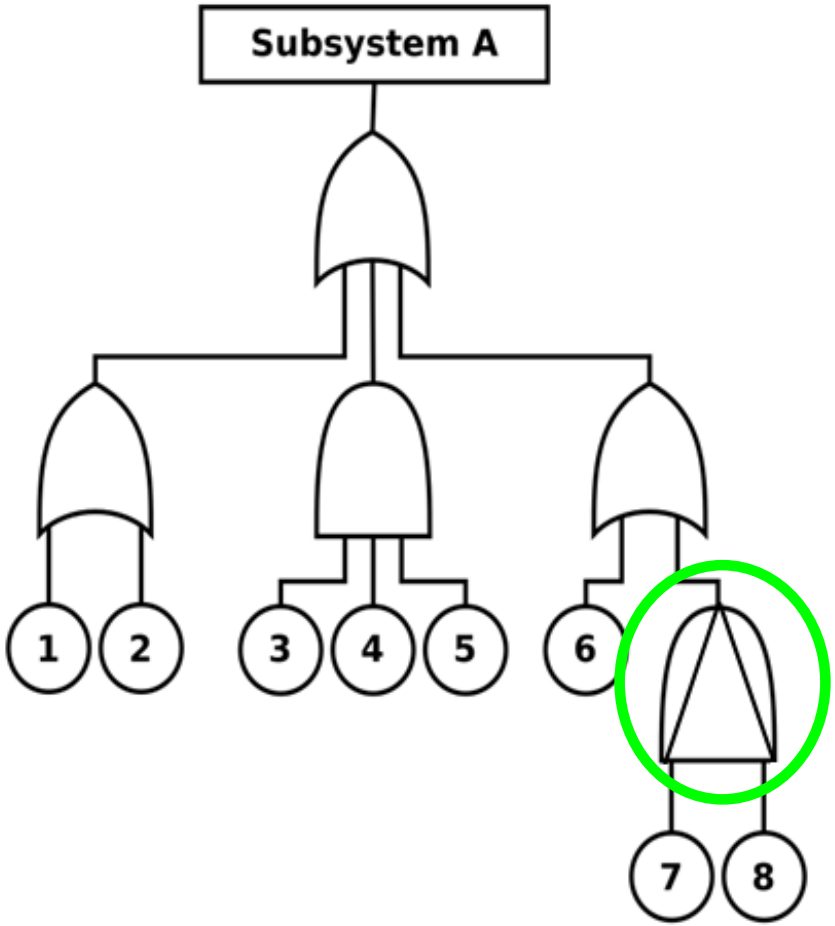
Dynamic Fault Trees
DFTs

Failure dependencies in real systems

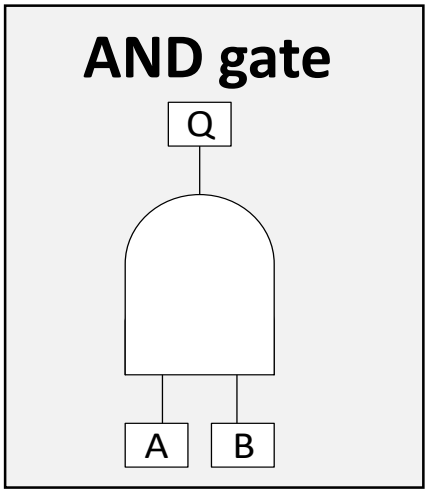


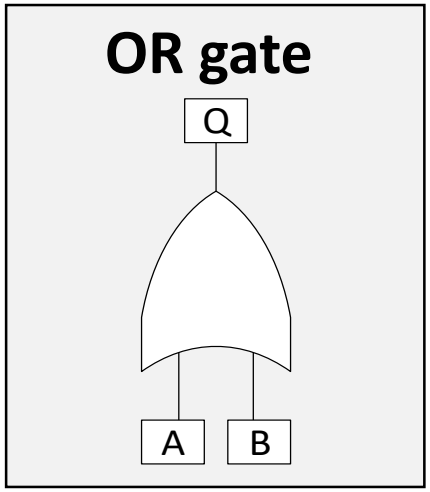
Dynamic Fault Trees

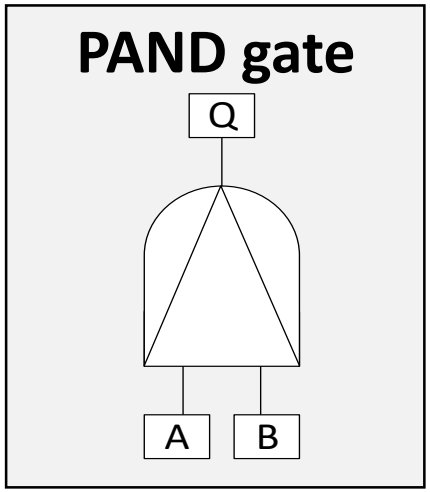
- Critical top event which will cause system failure
- The conditions are modeled using DFT and SFT gates
- DFTs capture the failure dependency using DFT gates (e.g. Priority-And gate)

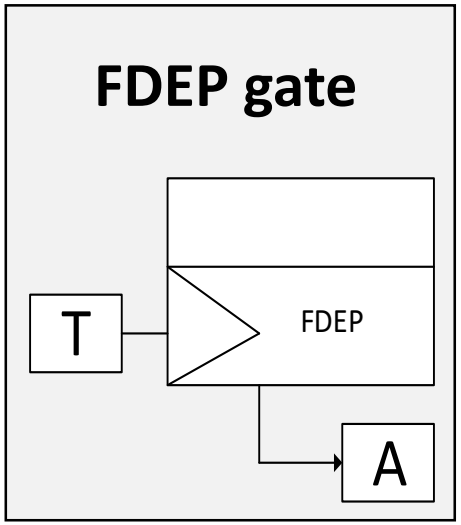


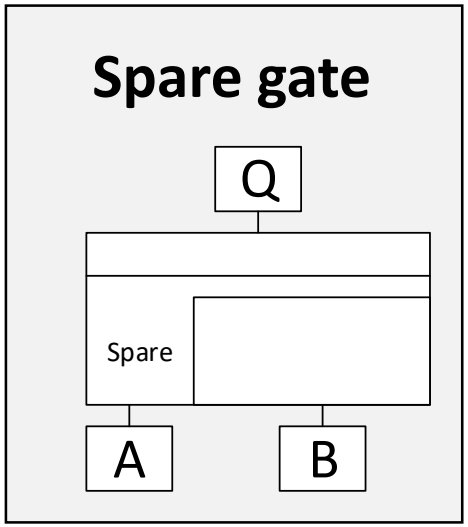
Dynamic Fault Trees Gates



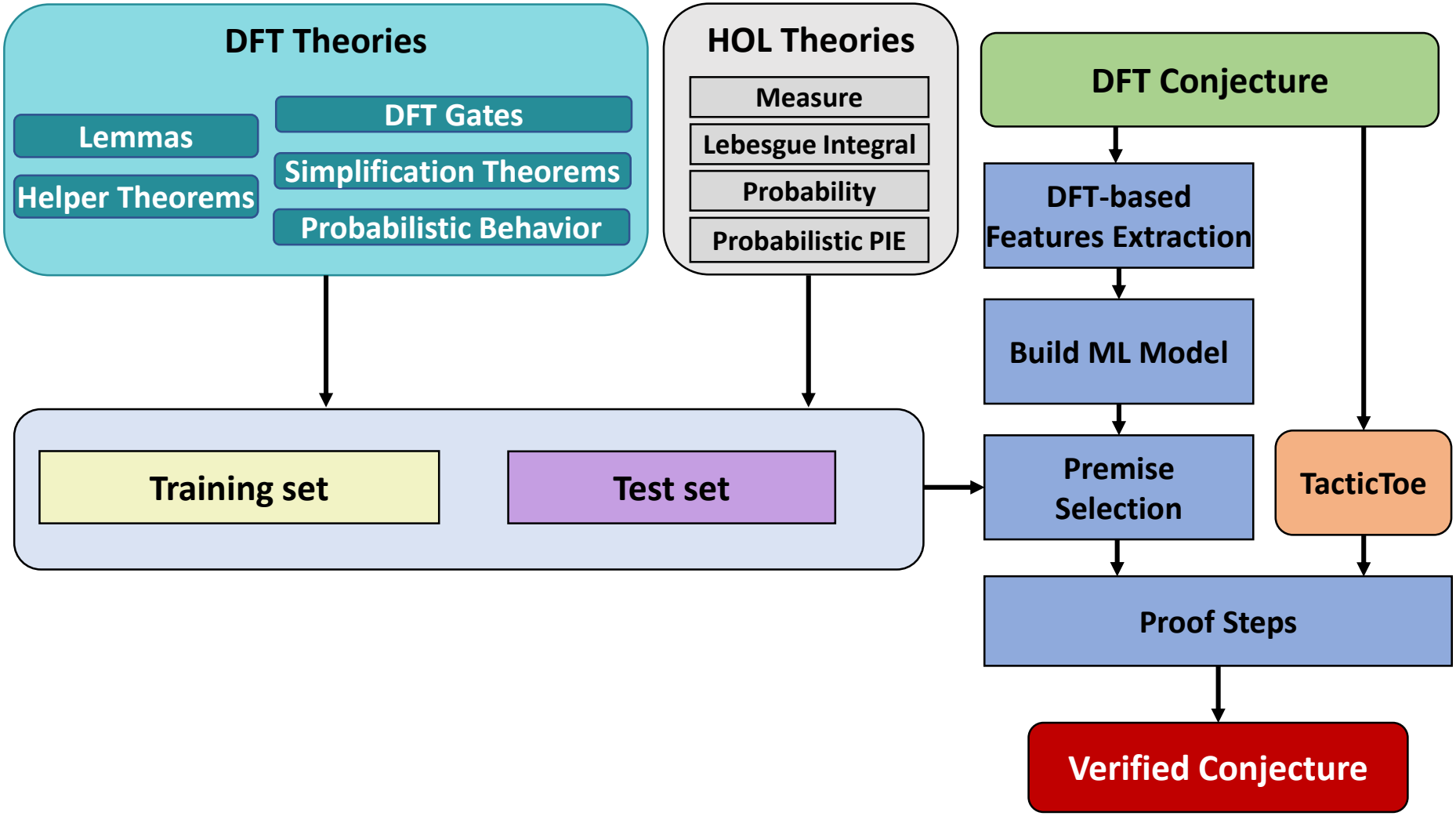








Ultimate Goal (HOL4)



Work done

- Formalization of DFT in HOL4 Theorem Prover

- Y. Elderhalli, O. Hasan, W. Ahmad and S. Tahar. "Formal Dynamic Fault Trees Analysis using an Integration of Theorem Proving and Model Checking". *In NASA Formal Methods (NFM-2018).*

- Y. Elderhalli, W. Ahmad, O. Hasan and S. Tahar "Probabilistic Analysis of Dynamic Fault Trees using HOL Theorem Proving", *In Journal of Applied Logic, 2019 [to appear]*

✓ Sound

✗ Interactive (not Automated)

The current libraries only support the analysis of Dynamic Fault Trees interactively

Dynamic Fault Trees

- Visualization of the cause of failure of the top event based on the basic events
- Dynamic gates in addition to the static gates
  - AND gate
  - OR gate
  - Priority AND gate
  - Functional Dependency gate
  - Spare gate
- Algebraic representation used in the DFT analysis

Dynamic Fault Trees Operators

- DFT temporal operators based on the time of failure:
  - **AND**

$$d(A\,.\,B) = \max(d(A), d(B))$$

  - **OR**

$$d(A + B) = \min(d(A), d(B))$$

  - **Simultaneous**

$$d(A\,\Delta\,B) = \begin{cases} d(A) & if\ d(A) = d(B) \\ +\infty & if\ d(A) \neq d(B) \end{cases}$$

Dynamic Fault Trees Operators

- **Before**

$$d(A \triangleleft B) = \begin{cases} d(A) & if\ d(A) < d(B) \\ +\infty & if\ d(A) \geq d(B) \end{cases}$$

- **Inclusive Before**

$$d(A \trianglelefteq B) = \begin{cases} d(A) & if\ d(A) \leq d(B) \\ +\infty & if\ d(A) > d(B) \end{cases}$$

Probabilistic Behavior of Gates

$$\mathcal{P}r\ \{A \cdot B\}\ (t) = F_A(t) \times F_B(t)$$

$$\mathcal{P}r\ \{A + B\}\ (t) = F_A(t)\ +\ F_B(t)\ -\ F_A(t) \times F_B(t)$$

$$\mathcal{P}r\ \{B.(A \lhd B)\}\ (t) = \int_0^t f_B(u) F_A(u)\ du$$

$$\mathcal{P}r\ \{Q_{spare}\}\ (t) = \int_0^t \left(\int_v^t f_{B_a}(u,v) du \right) f_A(v) dv\ +\ \int_0^t f_A(u) F_{B_d}(u) du$$

Quantitative Analysis

- The probability of the top event can be expressed using the probabilistic Principle of Inclusion Exclusion (PIE)

$$\begin{aligned}\mathcal{P}r\{TE\} &= \mathcal{P}r\{CSS_1 + CSS_2 + \ldots + CSS_m\} \\ &= \sum_{1\le i\le m} \mathcal{P}r\{CSS_i\} - \sum_{1\le i<j\le m} \mathcal{P}r\{CSS_i \cdot CSS_j\} \\ &\quad + \sum_{1\le i<j<k\le m} \mathcal{P}r\{CSS_i \cdot CSS_j \cdot CSS_k\} \\ &\quad + \ldots + (-1)^{m-1}\mathcal{P}r\{CSS_1 \cdot CSS_2 \cdot \ \ldots \ \cdot CSS_m\}\end{aligned}$$

Verification of Probabilistic Behavior of PAND

- The probabilistic failure behavior of the PAND

Theorem. Prob PAND

$$\vdash \forall\, X\ Y\ p\ f_Y\ t.\ \text{rv\_gt0\_ninfty}\ [X;Y] \wedge\ 0 \leq t\ \wedge \text{prob\_space}\ p \wedge$$

$$\text{indep\_var}\ p\ \text{lborel}\ X\ \text{lborel}\ Y \wedge \text{distributed}\ p\ \text{lborel}\ Y\ f_Y \wedge 0 \leq f_Y \wedge$$

$$\text{measurable\_CDF}\ \ (\text{real} \circ (\text{CDF}\ p\ \ (\text{real} \circ X)\ t)) \wedge$$

$$\text{cont\_CDF}\ (\text{real} \circ (\text{CDF}\ p\ (\text{real} \circ X)\ t)) \Longrightarrow$$

$$(\text{prob}\ p\ (\text{DFT\_event}\ p\ (Y\ .\ (X \lhd Y)\ t) = \int_0^t f_Y(y) \times F_X(y)\ dy)$$

Defines a density function for Y

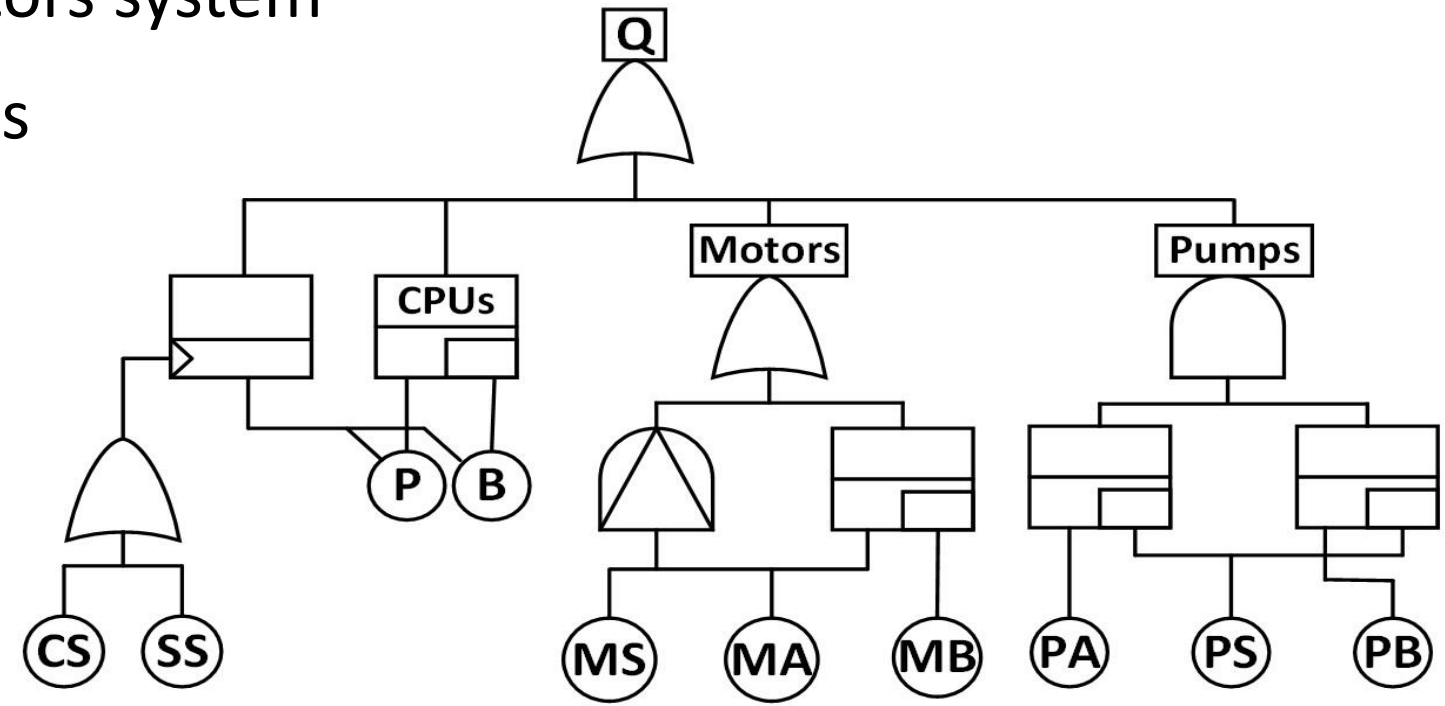
Algebraic Simplification Theorems

- Theorems needed to reduce the expression of the top event of the DFT (structure function)
- Many simplification theorems exist[1]:
  - Commutativity $$A \,\Delta\, B = B \,\Delta\, A$$
  - Associativity $$A + (B + C) = (A + B) + C$$
  - Distributivity $$A.(B + C) = A.B + A.C$$

1- [G. Merle , "Algebraic modelling of Dynamic Fault Trees, Contribution to Qualitative and Quantitative Analysis", PhD thesis, ENS, France, 2010].

The Cardiac Assist System

- It consists of:
  - Pumps system
  - Motors system
  - CPUs



The Cardiac Assist System

- A reduced structure function is obtained to conduct both qualitative and quantitative analyses.

Theorem. *Reduced cardiac assist system*

$$\vdash \forall CS\ SS\ MA\ MS\ MB\ P\ B\ PA\ PB\ PS.$$

$$(\forall\ s.\ \text{ALL\_DISTINCT}\ [MA\ s;\ MS\ s;\ PA\ s;\ PB\ s;\ PS\ s]) \Longrightarrow$$

$$((\text{shared\_spare}\ PA\ PB\ PS\ PS)\ .\ (\text{shared\_spare}\ PB\ PA\ PS\ PS) +$$

$$(\text{PAND}\ MS\ MA) + (\text{HSP}\ MA\ MB\ ) +$$

$$(\text{HSP}\ (\text{FDEP}\ ((CS + SS)\ P)\ (\text{FDEP}\ ((CS + SS)\ B)) =$$

$$CS + SS + (MA\ .\ (MS \lhd MA)) + MA\ .\ MB + P.B + PA\ .\ PB.\ PS)$$

The Cardiac Assist System

Theorem. *Reduced cardiac assist system*

$$\vdash \forall CS\ SS\ MA\ MS\ MB\ P\ B\ PA\ PB\ PS.$$

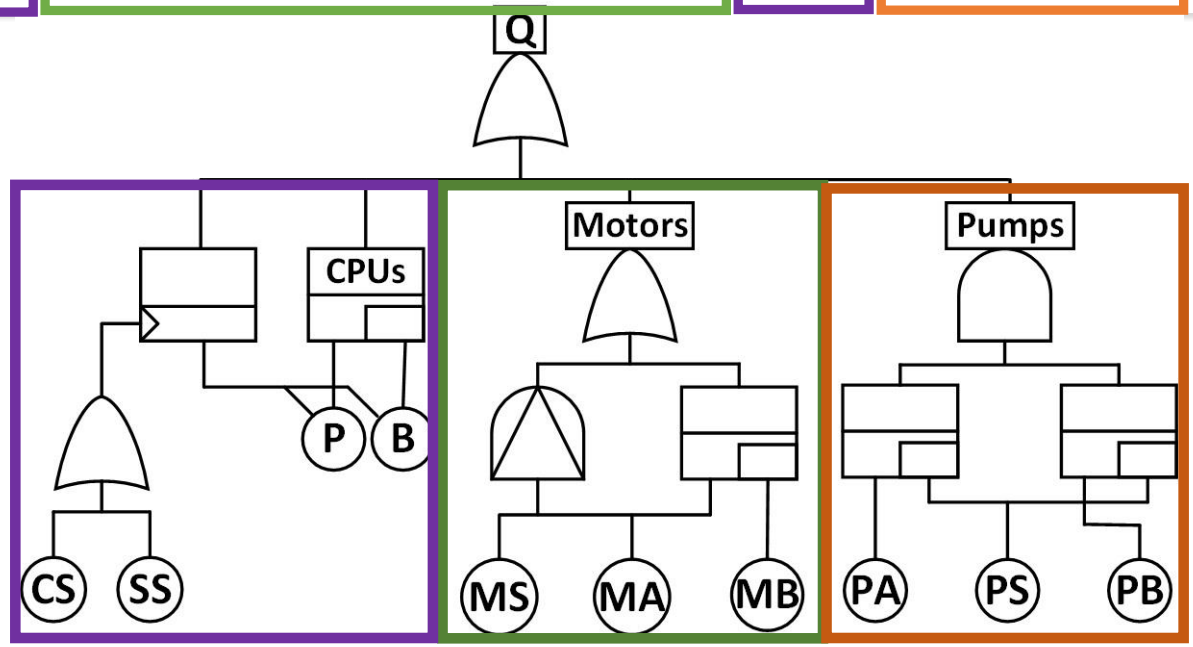
$$(\forall\, s.\ \mathrm{ALL\_DISTINCT}\ [MA\ s;\ MS\ s;\ PA\ s;\ PB\ s;\ PS\ s]) \Longrightarrow$$

$$((\mathrm{shared\_spare}\ PA\ PB\ PS\ PS)\ .\ (\mathrm{shared\_spare}\ PB\ PA\ PS\ PS)\ \text{--}$$

$$(\mathrm{PAND}\ MS\ MA) + (\mathrm{HSP}\ MA\ MB\ ) +$$

$$(\mathrm{HSP}\ (\mathrm{FDEP}\ ((CS + SS)\ P)\ (\mathrm{FDEP}\ ((CS + SS)\ B)) =$$

$$CS + SS + (MA\ .\ (MS \lhd MA)) + MA\ .\ MB\ \text{--}\ P.B + PA\ .\ PB.\ PS)$$



The Cardiac Assist System

Lemma. *Cardiac assist system union_list*

⊢ ∀PA PB PS MS MA MB CS SS P B p t.

DFT_event p

(CS + SS + (MA . (MS ◁ MA)) + MA . MB + P.B + PA . PB. PS) t =

union_list

[DFT_event p CS t; DFT_event p SS t;

DFT_event p (MA . (MS ◁ MA)) t;

DFT_event p (MA. MB) t;

DFT_event p (P. B) t; DFT_event p (PA. PB. PS) t]

The Cardiac Assist System

Theorem. Prob Cardiac assist system

$\vdash \forall CS\ SS\ MA\ MS\ MB\ P\ B\ PA\ PB\ PS\ \ p\ t\ f_{MA}.$

$0 \leq t \wedge\ prob\_space\ p \wedge$

$ALL\_DISTINCT\_RV\ [CS; SS; MA; MS; MB; P; B; PA; PB; PS]\ p\ t \wedge$

$indep\_vars\_sets\ [CS; SS; MA; MS; MB; P; B; PA; PB; PS]\ p\ t \wedge$

$distributed\ p\ lborel\ MA\ f_{MA} \wedge\ 0 \leq f_{MA} \wedge$

$cont\_CDF\ F_{MS} \wedge measurable\_CDF\ F_{MS} \Longrightarrow$

F_{MS} is continuous and measurable

The Cardiac Assist System

(prob p

(DFT_event p

(CS + SS + (MA . (MS ◅ MA)) + MA . MB + P.B + PA . PB. PS) t) =

$F_{CS}(t) + F_{SS}(t) + \int_0^t f_{MA}(y) \times F_{MS}(y)dy + F_{MA}(t) \times F_{MB}(t) + F_P(t) \times F_B(t) +$

$F_{PA}(t) \times F_{PB}(t) \times F_{PS}(t) - \cdots + \cdots -$

$F_{CS}(t) \times F_{SS}(t) \times (\int_0^t f_{MA}(y) \times F_{MS}(y)dy) \times F_{MA}(t) \times F_{MB}(t) \times F_P(t) \times F_B(t) \times$

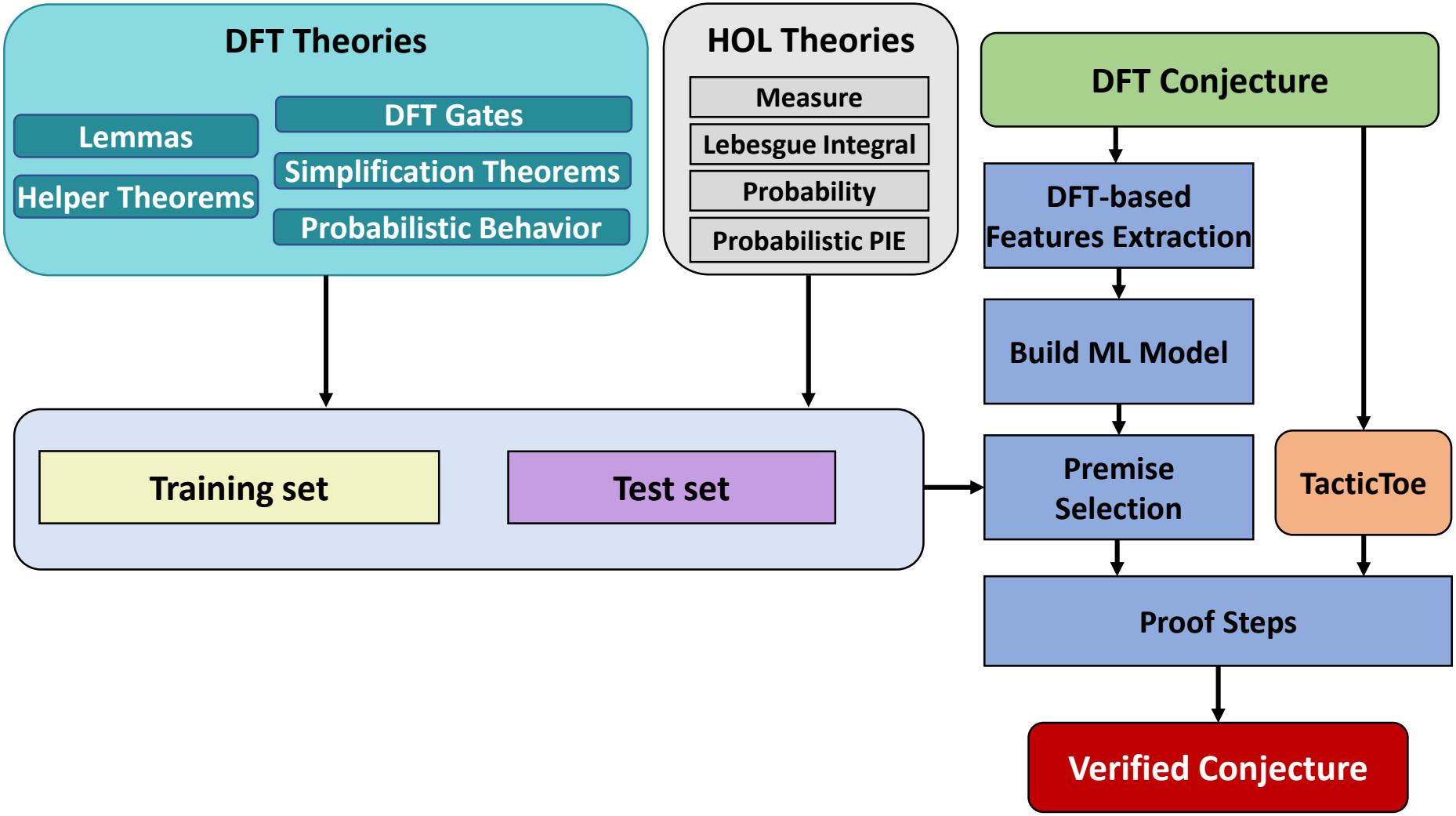
$F_{PA}(t) \times F_{PB}(t) \times F_{PS}(t)$

The result of applying PIE is 63 (2^6-1) elements

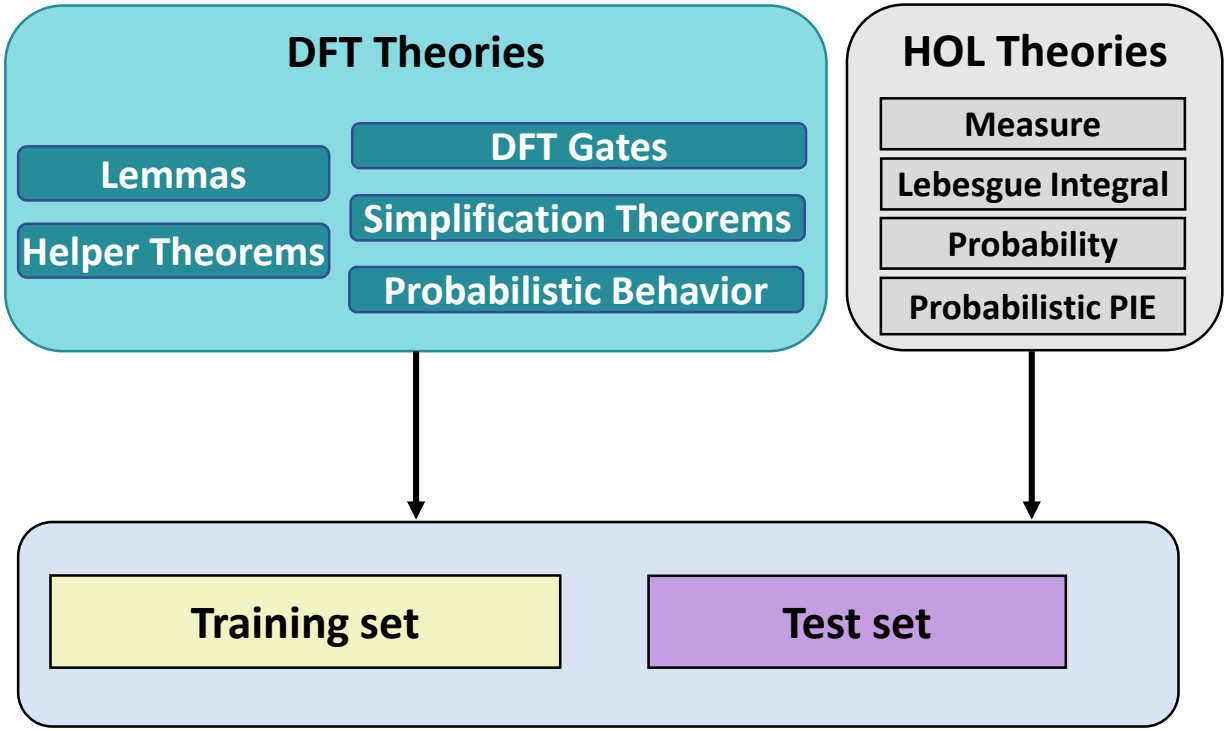
Formalization Summary

- DFT gates and simplification theorems
- Probabilistic behavior of DFT gates
- Utilizing the probabilistic PIE in the quantitative analysis leads to having many subgoals
- **Intermediate lemmas are verified that follow the same pattern**

Proposed Methodology



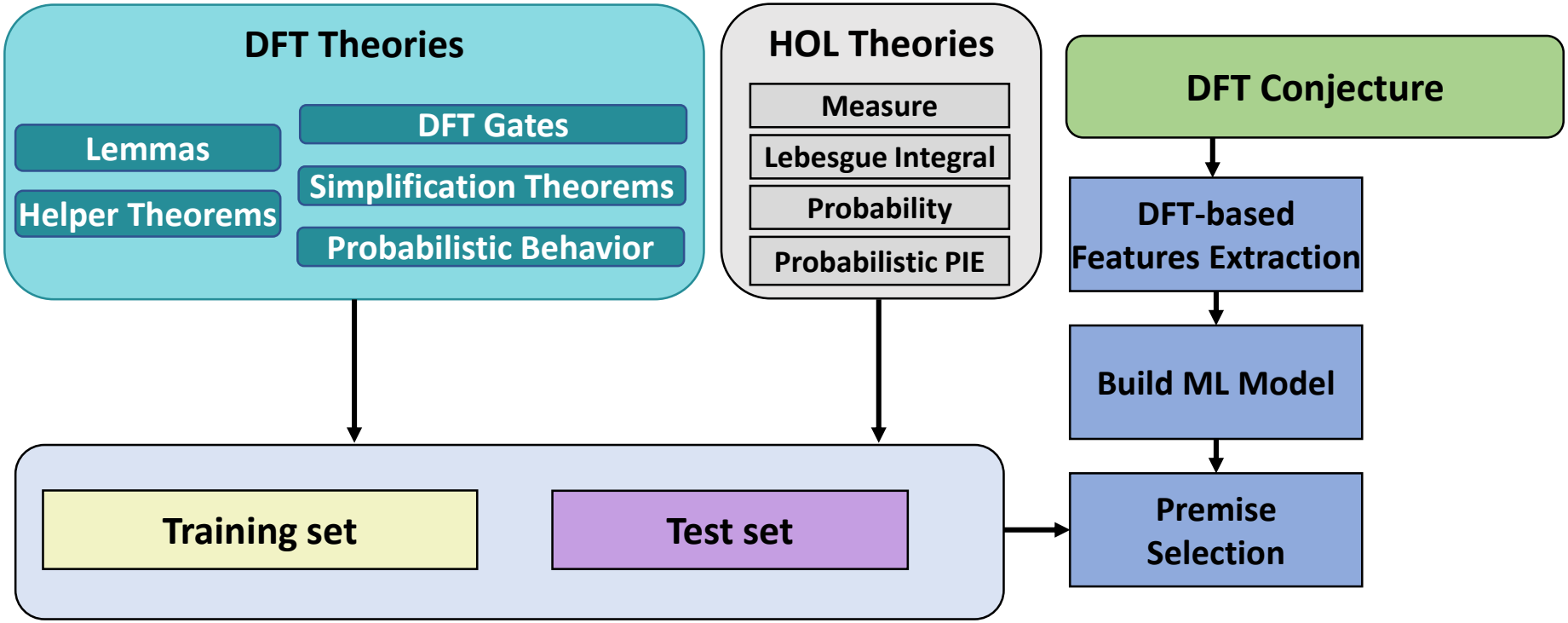
Proposed Methodology



- Divide the existing theories into training and test sets, similar to Holstep[1] , based on certain features, such as input statements

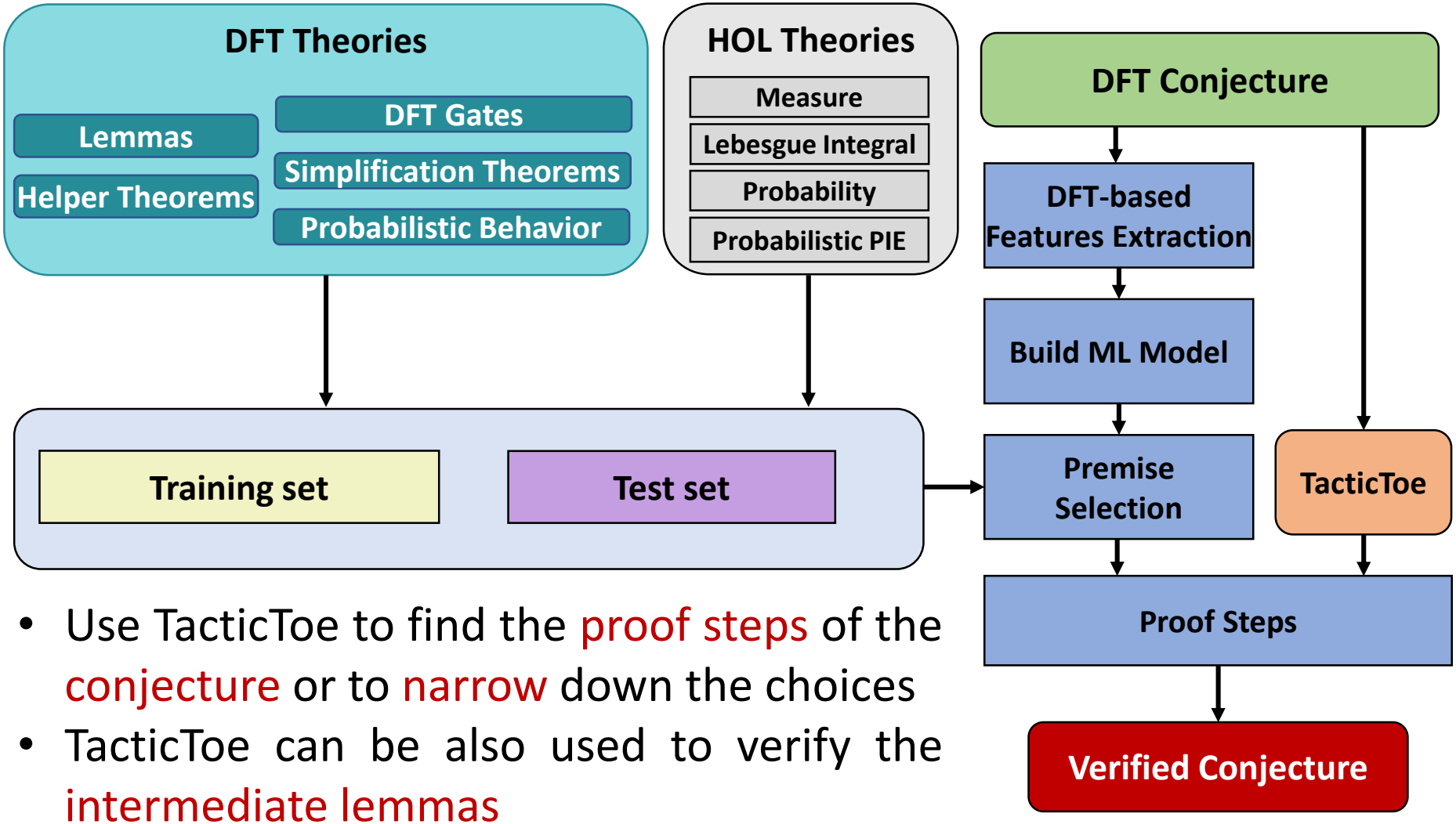
1. C. Kaliszyk, F. Chollet, and C. Szegedy. "*Holstep: A machine learning dataset for higher-order logic theorem proving*", 2017.

Proposed Methodology



- For DFT conjectures, features are extracted to build ML models such as neural networks
- The ML models will be used to find the suitable premises

Proposed Methodology



- Use TacticToe to find the proof steps of the conjecture or to narrow down the choices
- TacticToe can be also used to verify the intermediate lemmas

TacticToe

- TacticToe[1] is used to record part of DFT theories
- TacticToe is tested with a small subset of intermediate lemmas
- Proof steps were determined for small lemmas not complex theorems

$$\forall A1\ A2\ A3\ A4\ A5\ A6\ p.\ prob\_space\ p \wedge A1 \in events\ p \wedge A2 \in events\ p \wedge A3 \in events\ p \wedge A4 \in events\ p \wedge A5 \in events\ p \wedge A6 \in events\ p \Longrightarrow A1 \cap A2 \cap A3 \cap A4 \cap A5 \cap A6 \in events\ p$$

```
RW_TAC bossLib.std_ss [] THEN (MATCH_MP_TAC o REWRITE_RULE
[subsets_def] o Q.SPEC `(p_space p, events p)`) ALGEBRA_INTER
THEN RW_TAC bossLib.std_ss [] THENL [RW_TAC bossLib.std_ss
[EVENTS_ALGEBRA], RW_TAC bossLib.std_ss [EVENTS_INTER]]);
```

1- In collaboration with Cezary Kaliszyk

Generic Lemmas

- Verifying generic lemmas that can facilitate the learning process[1]
- The extreal addition associativity is used with the PIE and other lemmas to reach the final form of the probability of CAS

$$\forall L.\ (\neg \text{MEM PosInf } L) \bigvee (\neg \text{MEM NegInf } L) \Longrightarrow$$
$$(\text{FOLDR } (\lambda a\ b.\ a + b)\ 0\ L = \text{FOLDL } (\lambda a\ b.\ a+b)\ 0\ L)$$

$$\forall A1\ A2\ A3\ A4\ A5\ A6\ A7\ A8\ A9\ A10.$$
$$A1 \neq \text{PosInf} \bigwedge A2 \neq \text{PosInf} \bigwedge A3 \neq \text{PosInf} \bigwedge A4 \neq \text{PosInf} \bigwedge$$
$$A5 \neq \text{PosInf} \bigwedge A6 \neq \text{PosInf} \bigwedge A7 \neq \text{PosInf} \bigwedge A8 \neq \text{PosInf} \bigwedge$$
$$A9 \neq \text{PosInf} \bigwedge A10 \neq \text{PosInf} \Longrightarrow$$
$$(A1 + (A2 + (A3 + (A4 + (A5 + (A6 + (A7 + (A8 + (A9 + A10)))))))) =$$
$$A1 + A2 + A3 + A4 + A5 + A6 + A7 + A8 + A9 + A10)$$

1- In collaboration with Cezary Kaliszyk

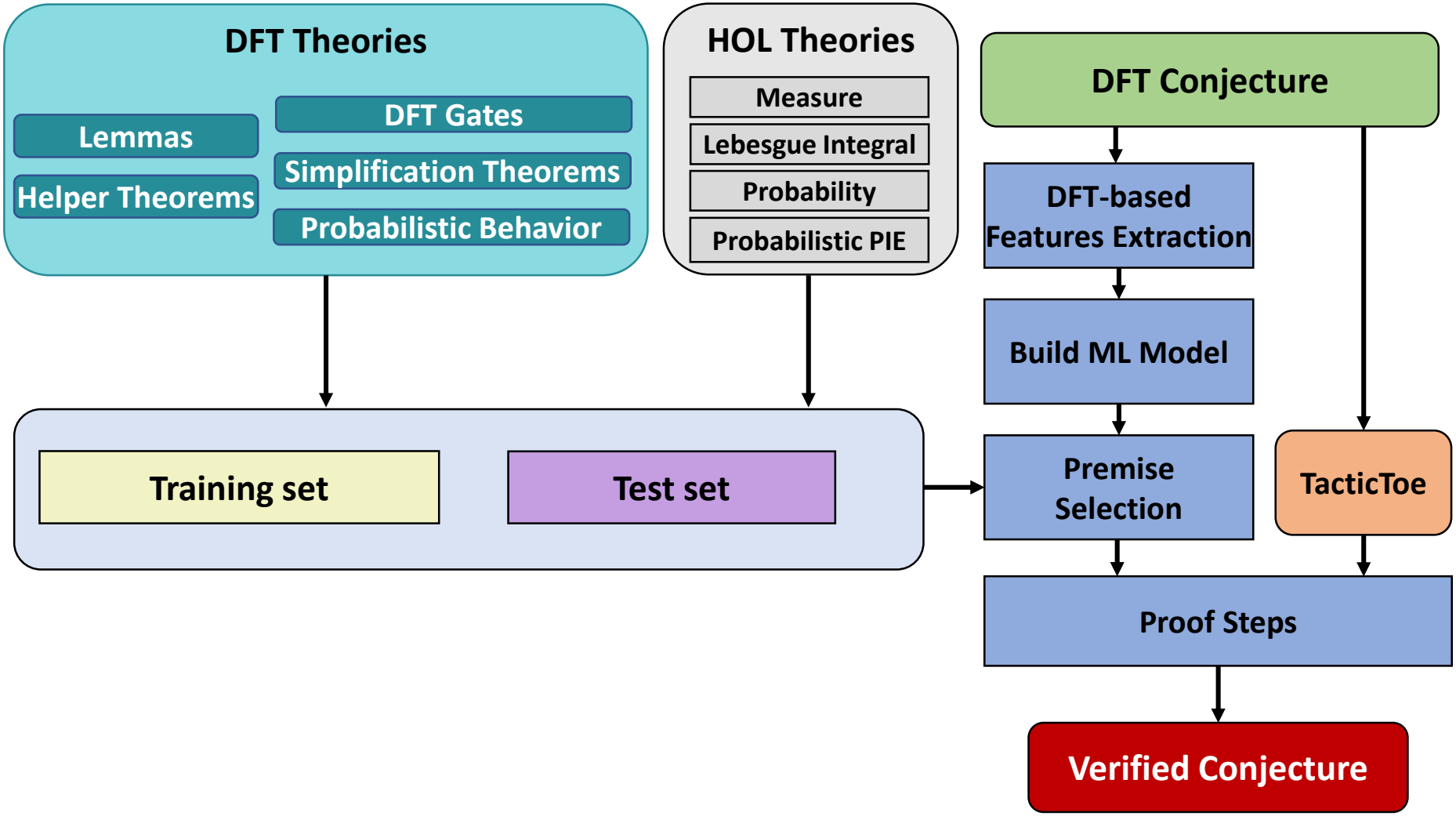
Conclusion

- Verified DFT algebraic analysis using interactive theorem proving
- A methodology to reduce user intervention in the analysis using machine learning techniques
- TacticToe is used with a small subset of DFT theorems

Future Work

- Divide DFT theories into training and testing sets
- Create ML models and use them with the testing set
- Use TacticToe to extract the proof steps that are useful in the proof steps
- Combine both ML models and TacticToe to generate the proof steps required to verify a given conjecture

Future Work


DFT Theories
Lemmas
DFT Gates
Helper Theorems
Simplification Theorems
Probabilistic Behavior
HOL Theories
Measure
Lebesgue Integral
Probability
Probabilistic PIE
DFT Conjecture
DFT-based Features Extraction
Build ML Model
Training set
Test set
Premise Selection
TacticToe
Proof Steps
Verified Conjecture

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