Tactic Learning for Coq

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Tactic-level automation:

Idea: Learn from human-written proof scripts

Try to match proof states with the right tactic

Advantage: We can make use of custom made, domain specific tactics written by clever humans

System Components

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- ▷ Proof Recording

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- Proof Recording
- Tactic Prediction

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- Proof Search

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- ▷ Proof Search
- ▷ Proof Reconstruction

Design goals

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- ▷ User Friendly
- ▷ Installation Friendly
- ▷ Integration Friendly
- ▷ Maintenance Friendly

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Downside: We potentially sacrifice the use of some awesome Machine Learning algorithms

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All components function!

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Demo time

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Proof Recording:

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Ltac, Mtac, SSreflect, ML-tactics, Ltac 2.0?

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Backtracking proof monad

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Seems impossible: How do we introspect the monad?

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Backtracking proof monad

Seems impossible: How do we introspect the monad?

For now, only Ltac recording

```
Definition left_pad_spec c s n :
  {s’ | (forall i, i < n - length s -> get i s’ = Some c)
     /\ (forall i, get i s = get (i + (n - length s)) s’)}.
Proof.
exists (left_pad c s n). unfold left_pad.
split.
- intros; rewrite <- append_correct1;
  [ rewrite cycle_get
  | rewrite cycle_length]; auto.
- intros; etransitivity;
  [ apply append_correct2
  | rewrite cycle_length; auto].
Qed.
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Proof.
record (exists (left_pad c s n)). record (unfold left_pad).
record (split).
- record (intros); record (rewrite <- append_correct1);
  [ record (rewrite cycle_get)
  | record (rewrite cycle_length)]; record (auto).
- record (intros); record (etransitivity);
  [ record (apply append_correct2)
  | record (rewrite cycle_length); record (auto)].
Qed.
```

```
c : ascii
s : string
n, i : nat
H : i < n - length s
______________________________________(1/1)
get i (cycle (n - length s) c ++ s) = Some c
```

```
rewrite <- append_correct1.
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```
c : ascii
s : string
n, i : nat
H : i < n - length s
______________________________________(1/2)
get i (cycle (n - length s) c) = Some c
______________________________________(2/2)
i < length (cycle (n - length s) c)
```

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[ascii]
[string]
[nat]
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```
[ascii]
[string]
[nat]
[le-i, le-minus, minus-n, minus-length, length-s]
______________________________________(1/1)
[eq-append, eq-Some, append-get, append-s, get-i, get-cycle, get-c, ...]
rewrite <- append_correct1.

[ascii]
[string]
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______________________________________(1/2)
[eq-get, eq-Some, get-i, get-cycle, cycle-minus, minus-n, minus-length, ...]
______________________________________(2/2)
[le-i, le-length, length-cycle, length-c, cycle-minus, cycle-n, ...]
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[ascii, string, nat,
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eq-get, eq-Some, get-i, get-cycle, cycle-minus, minus-n, minus-length, ...,
le-i, le-minus, minus-n, minus-length, length-s]
```

intros	[eq-minus, minus-length, minus-max, max-x, max-y, length-t, ...]
intros	[nat, bool, plus-n, plus-length, length-k, eq-plus, eq-n]
⋮	⋮
rewrite sub_diag	[nat, eq-minus, eq-zero, minus-n, minus-n]
rewrite sub_diag	[nat, list, eq-minus, eq-zero, minus-length, minus-length, ...]
⋮	⋮
rewrite append_correct1	[ascii, string, nat, le-i, le-minus, minus-n, minus-length, ...]
⋮	⋮

? | [string, nat, eq-plus, eq-n, plus-length, plus-get, ...]

⇕

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Machine Learning goal: Order database in order of relevance

k-nearest neighbor

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$$\text{Metric}: d(v_1, v_2) = \sum_{f \in v_1 \cap v_2} \log \frac{|D|}{|\{v \in D \mid f \in v\}|}$$

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$$\text{Euclid} : d(v_1, v_2) = |v_1 \cup v_2 - v_1 \cap v_2|$$

Evaluation on Coq Standard Library: 144115 recorded pairs

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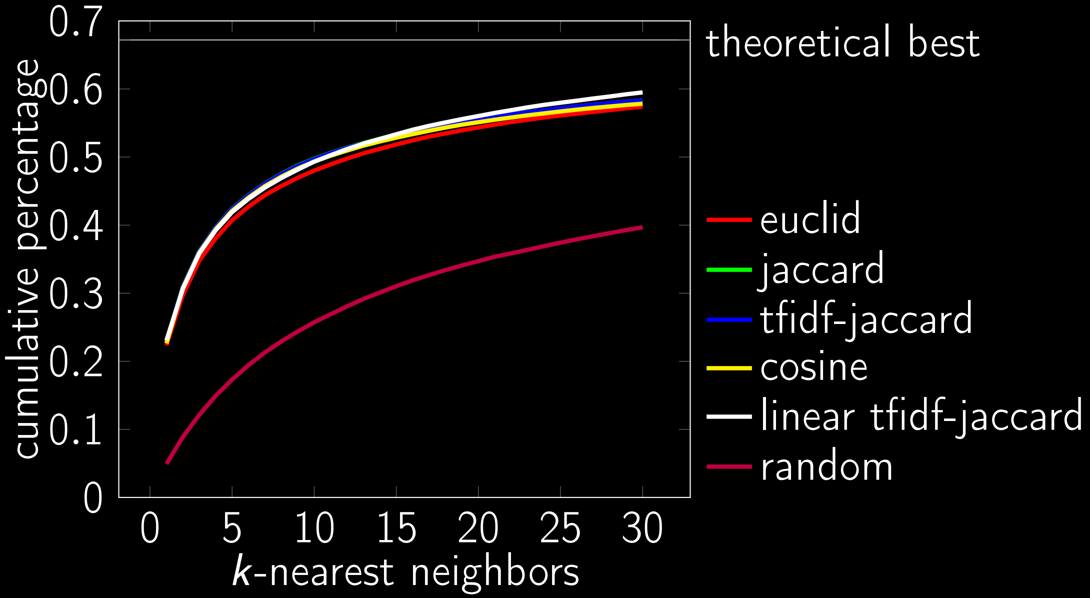
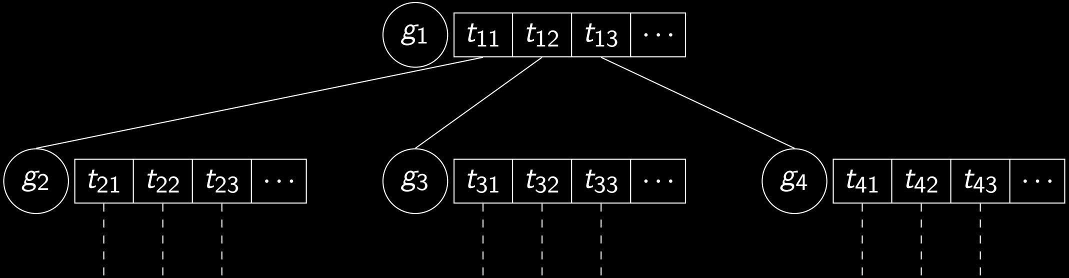


Proof Search



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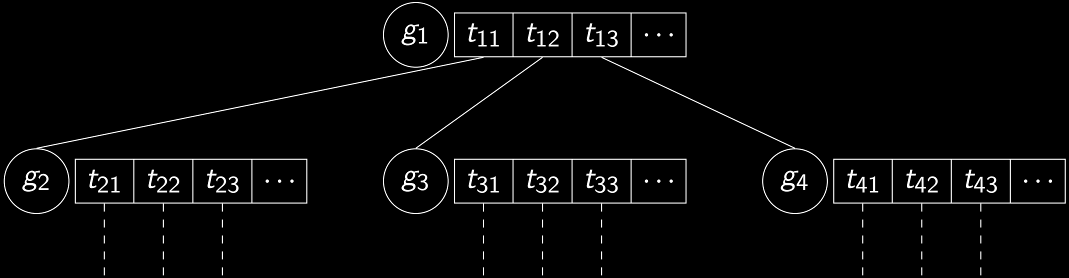

Skewed Breadth First Search

Let $t_1, \ldots, t_n$ be an ordered list of predicted tactics for goal g.
Subtree t_i is always explored one step deeper than subtree t_{i+1}.

Evaluation on Coq Standard Library

10778 lemmas, 2099 proved

19.5% Proved

Possible improvements

- Monte Carlo Tree Search
- Better Tactic Decomposition
- Better Feature Engineering
- Tactic Argument Prediction
- ...

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- What did I do wrong
- What can I improve
- Innovative ideas?