1 Abstract

Imagine a voice-enabled household robot that can pick up objects and transfer them from room to room. The owner might say "Pick up my red cart and put it in the garage." but the robot might hear either "Pick up my red car and put it in the garage." or the original sentence. A reasoning system might disambiguate the utterance or correct it by reasoning that a typical car weights 1.5 tons and that’s 3000 pounds and the carrying capacity of the robot is 100 pounds and the alternate text must be what was said. A system must construct an answer to a question that has never been asked before ("Can the robot carry a car?") perform some simple computation involving unit conversions, to understand that 3000lbs >100lbs and possibly explain its answer if required ("Why didn’t you move my car like I asked you to?")

General purpose first-order theorem provers historically haven’t done proofs with arithmetic. A relatively new language that addresses sound arithmetic calculation in a first order logic is called Typed First-order Form (TFF) [7]. It is implemented in several of the best modern, first-order provers, including Vampire [2].

To have useful and non-trivial reasoning about, for example, a robot’s capabilities, or to do question answering, we need not only a language capable of arithmetic calculation (as well as first order logical reasoning) but also a non-trivial body of axioms that has the information about the real world needed to form answers to such questions. For that reason, we need to use the Suggested Upper Merged Ontology (SUMO) [3, 4], which is a comprehensive and diverse set of logical statements about the world. At approximately 20,000 concepts and 80,000 logical statements (as well as including the large factbase of YAGO [1] and other such resources), it is large enough to answer interesting questions about a wide range of topics. Writing a translator from SUMO’s native formalization into TFF should open up many new opportunities for doing practical automated reasoning involving numbers and arithmetic.

In earlier work, we described [5] how to translate SUMO to the strictly first order language of TPTP [6]. SUMO has an extensive type structure and all relations have type restrictions on their arguments. Translation to TPTP involved (among many other steps) implementing a sorted (typed) logic axiomatically in TPTP by altering all implications in SUMO to contain type restrictions on any variables that appear. Note also that all the strictly higher-order content in SUMO is lost in translation to first-order, whether TPTP or TFF.

Like TPTP, TFF forms are valid Prolog syntax (although obviously not the same semantics!) TFF has five disjoint sorts: integers, real numbers, rational numbers, booleans and everything else. These are respectively called $int, $real, $rat, $o and $i in TFF syntax. Each variable that is used in a logical statement must be declared to be one of these sorts, or by default it will be assumed to be type $i.

TFF has built in to the language the basic arithmetic functions and arithmetic comparison operators. Each function and operator is polymorphic - it is actually a set of three different operators that can handle integers, rationals and reals. Equality is also defined for $o and $i.

TFF’s creators have planned to include the ability to define subtypes (subsorts) but this is not yet defined in specification or implemented in any prover. An additional issue is that since
all types are disjoint, and SUMO allows multiple inheritance, there is a mismatch between
the two type systems. So we have to continue to implement much of SUMO’s sort system
axiomatically in TFF as in TPTP, but have a special treatment of integers, rationals and reals
that does use the TFF type system, so we can use its arithmetic and comparison operators.

We also need to handle SUMO’s subsorts of Integer, RealNumber and RationalNumber.
This entails adding the constraints that specify these types to any axiom that uses them. For
e.g., PositiveInteger has a constraint that it’s simply a TFF $int that must be greater
than 0.

Because $int and $real are disjoint sorts in TFF, but Integer is a subtype of RealNumber
in SUMO we have to commit to one or the other when there is ambiguity, as in the case of a
number appearing without a decimal.

Lastly, all of these types can interact, requiring constraint propagation within an axiom. For
example, if we have addition between an Integer and a variable that is otherwise constrained
only to Number, the Number will have to be constrained to an Integer. An example translation
with such an issue is shown in Listing 1.

In the talk for this paper, I present examples of the required transformations and discussion
and examples about inferences with the resulting TFF theory translated from SUMO.

Listing 1: SUO-KIF to TFF with sorts as conditions, built-in TFF types and comparison
operator

References

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