Playing with Autoformalization over Mizar and ProofWiki

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Two Obstacles to Strong Computer Support for Math

1. Low reasoning power of automated reasoning methods, particularly over large complex theories
2. Lack of computer understanding of current human-level (math and exact science) knowledge

- The two are related: human-level math may require nontrivial reasoning to become fully explained. Fully explained math gives us a lot of data for training AITP systems.
- And we want to train AITP on human-level proofs too. Thus getting interesting formalization/ATP/learning feedback loops.
- In 2014 we have decided that the AITP/hammer systems are getting strong enough to try this. And we started to combine them with statistical translation of informal-to-formal math.
- One point was existence of “intermediate” informal corpora like ProofWiki that have a lot of regularity
- 2014: the first 100 proof sentence patterns cover about 50% of ProofWiki

Betting Slide from IHP'14, Paris

- In 25 years, 50% of the toplevel statements in LaTeX-written Msc-level math curriculum textbooks will be parsed automatically and with correct formal semantics
- Hurry up: I will only accept bets up to 10k EUR total (negotiable)
- More at `http://ai4reason.org/aichallenges.html`

Formal, Informal and Semiformal Corpora

- HOL Light and Flyspeck: some 25,000 theorems
- The Mizar Mathematical Library: some 60,000 theorems (most of them rather small lemmas), 10,000 definitions
- Coq: several large projects (Feit-Thompson theorem, ...)
- Isabelle, seL4 and the Archive of Formal Proofs
- Arxiv.org: 1M articles collected over some 20 years (not just math)
- Wikipedia: 25,000 articles in 2010 - collected over 10 years only
- Proofwiki - LaTeX but very semantic, re-invented the Mizar proof style

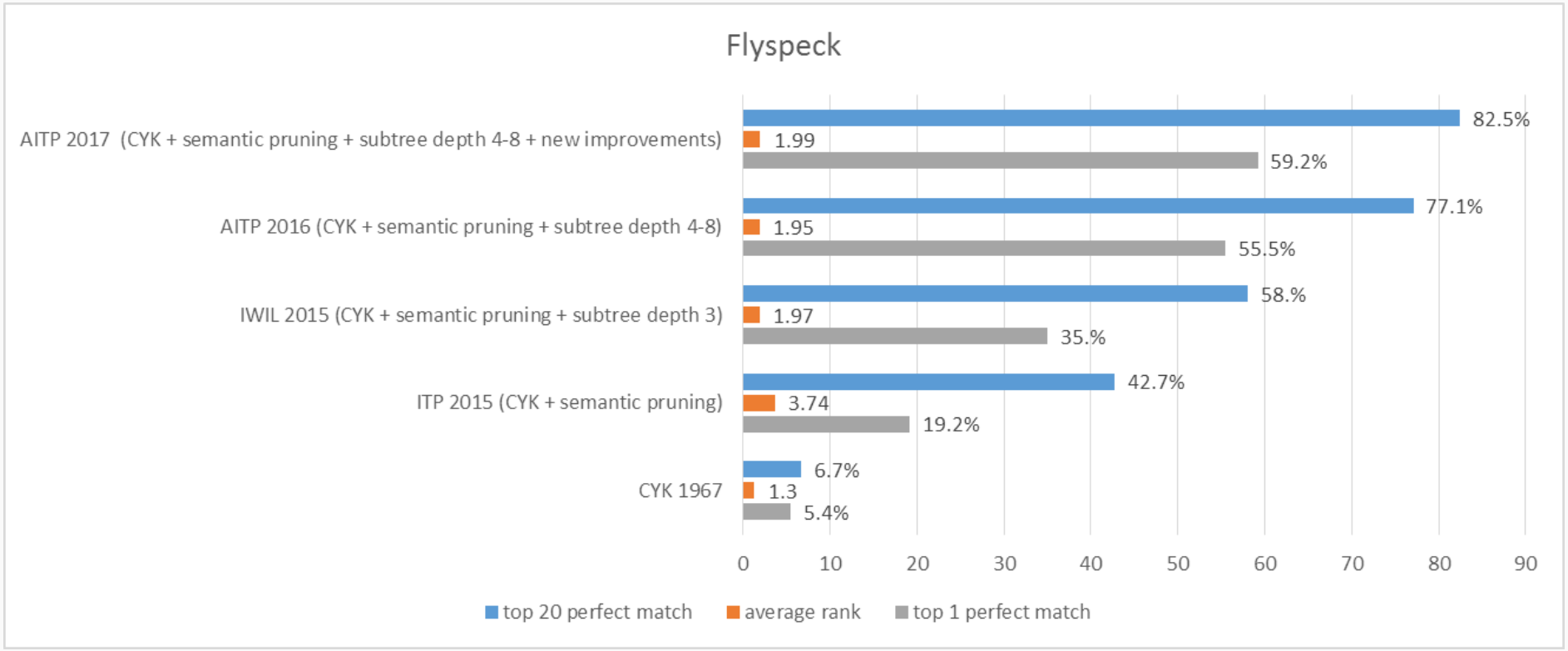
Our Approach/Plan So Far

- There is not yet much aligned informal/formal data
- So try first with “ambiguated” (informalized) formal corpora
- Try first with non black-box architectures such as probabilistic grammars
- Which can be easily enhanced internally by semantic pruning (e.g. type constraints)
- Develop feedback loops between training statistical parsing and theorem proving
- Start employing more sophisticated ML methods
- Progress to more complicated informal corpora/phenomena
- Both directly: ML/ATP with only cruder alignments (theorems, chapters, etc)
- And indirectly: train statistical/precise alignments across informal and formal corpora, use them to enhance our coverage
- Example: word2vec/Glove/neural learning of synonyms over Arxiv

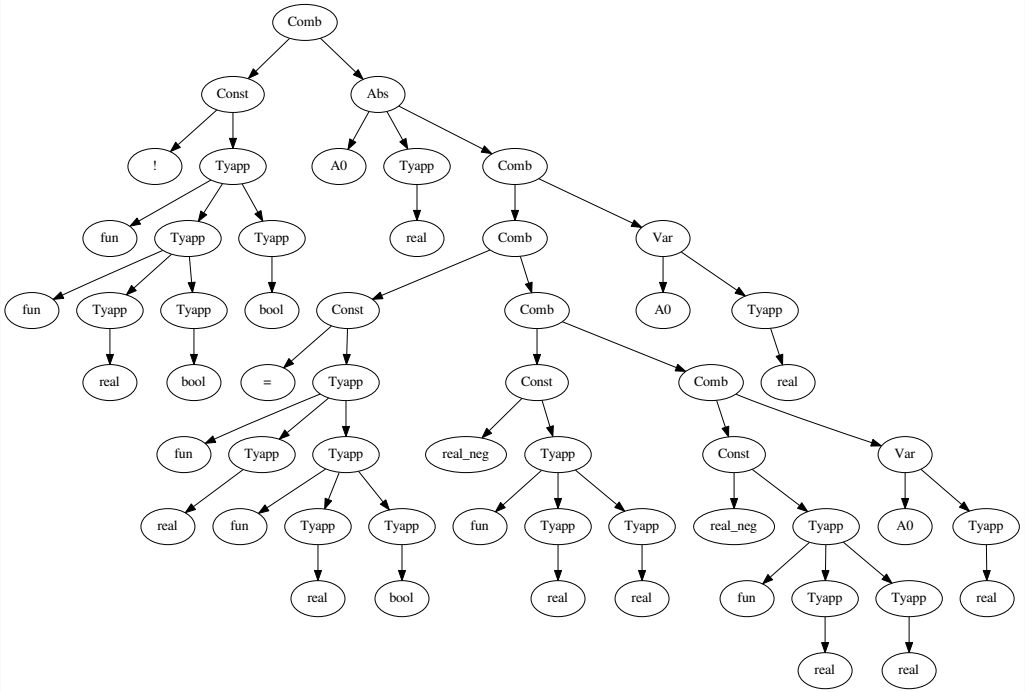
Work Done So Far: Informalized Flyspeck

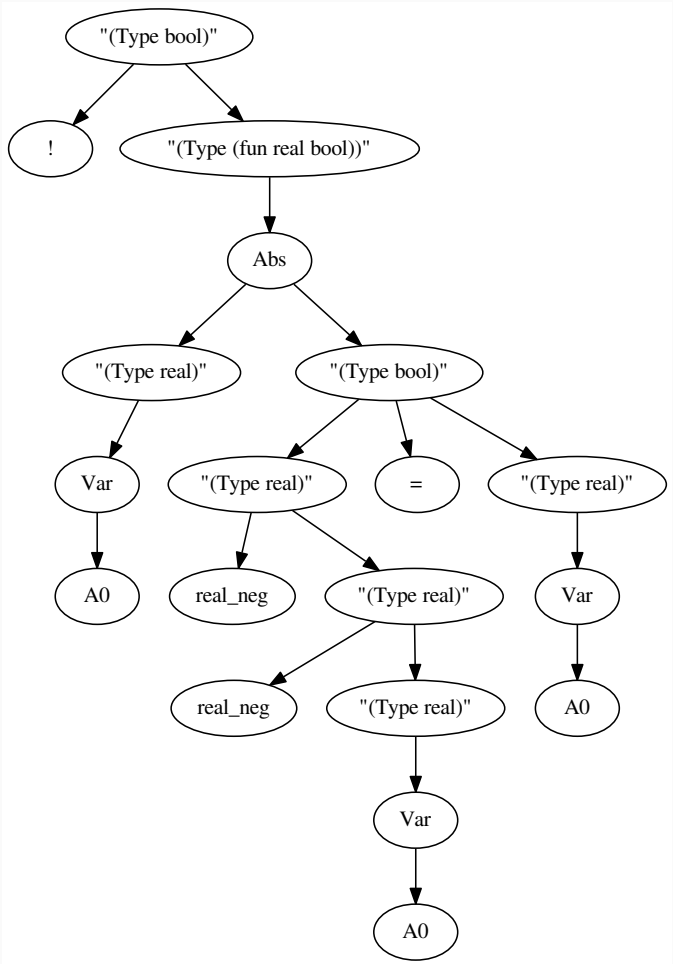
- 22000 Flyspeck theorem statements informalized
  - 72 overloaded instances like “+” for `vector_add`
  - 108 infix operators
  - forget “prefixes” `real_, int_, vector_, matrix_, complex_`, etc.
- Training a probabilistic grammar (context-free, later with deeper context)
- CYK chart parser with semantic pruning (compatible types of variables)
- Using HOL Light and HolyHammer to typecheck and prove the results

Flyspeck Progress



Example grammars





Informalized Mizar

- More natural-language features than HOL (designed by a linguist)
- Pervasisve overloading
- Declarative natural-deduction proof style (re-invented in ProofWiki)
- Adjectives, dependent types, hidden arguments, synonyms
- Addressed by using two layers:
  - user (pattern) layer - resolves overloading, but no hidden arguments completed, etc.
  - semantic (constructor) layer - hidden arguments computed, types resolved, ATP-ready

Examples of Mizar's Linguistic Mechanisms

```
definition
  let P,R be set;
  func P(#)R -> Relation means
  [x,y] in it iff ex z st [x,z] in P & [z,y] in R;
end;
notation synonym P*R for P(#)R; end;
definition
  let X,Y1,Y2,Z be set;
  let P be Relation of X,Y1;
  let R be Relation of Y2,Z;
  redefine func P*R -> Relation of X,Z;
end;
notation
  let f,g be Function;
  synonym g*f for f*g;
end;
```

Old ATP-based Approach

- AITP'17: be lazy and use ATP to connect the layers
- About 13000 Prolog-style formulas encoding the relation between user-level syntax and the semantic (MPTP) encoding
- Also the full set of Mizar typing rules needed for this - ca 30000 background knowledge rules
- Quite bad: Vampire proves about 40% in 60s, E with our mutant strategies about 50%
- Improved to about 60% in May 2017 by JU, however this also showed that our ATP encoding is unsound
- Making it sound would end up even heavier, hence our new approach

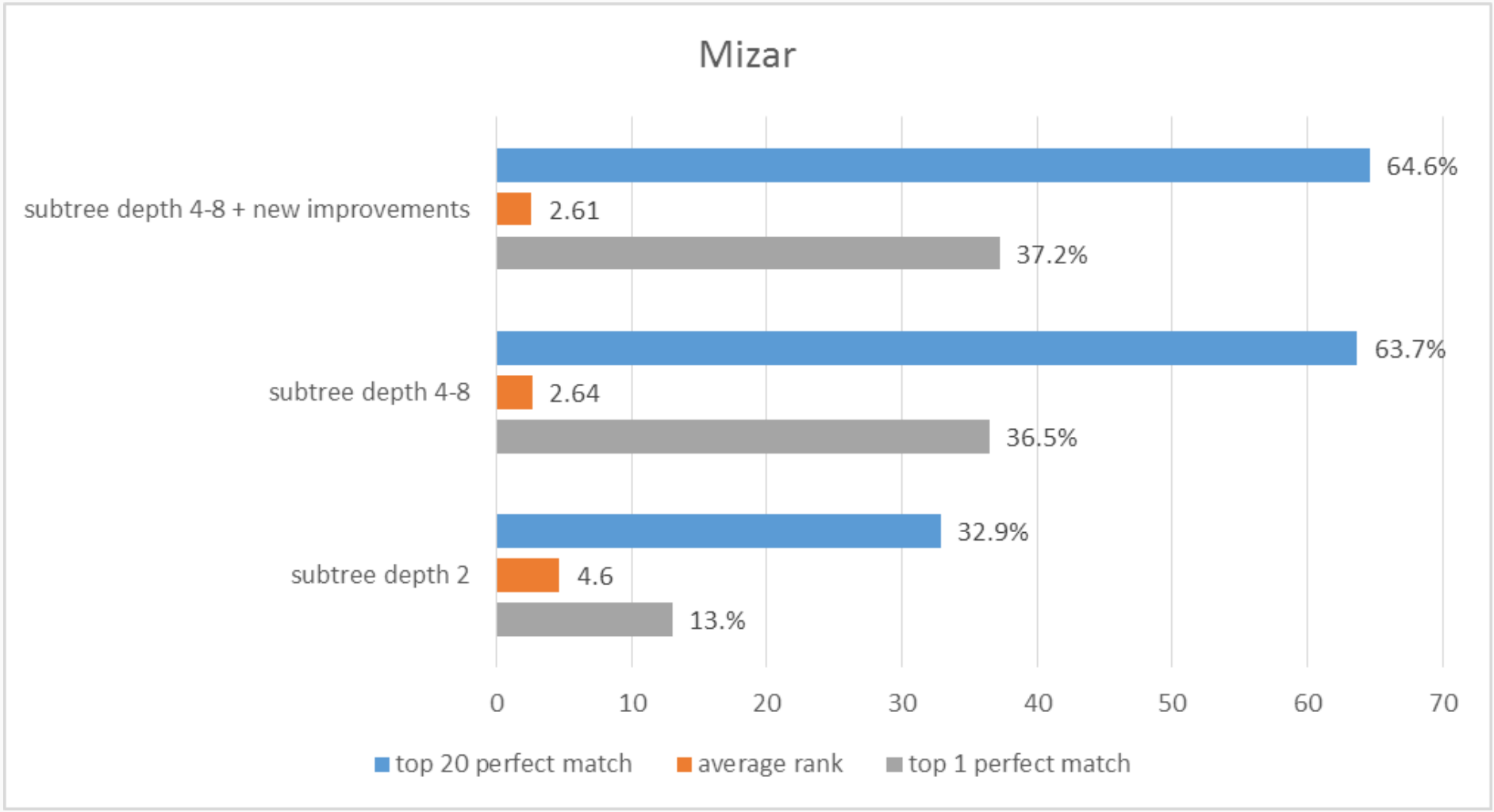
Enhancing our Parser with Mizar-style Algorithms

- Chad Brown: Mizar-style typing and elaboration inside the chart parser
- The typing and elaboration (of patterns to constructors) proceed in a mutual recursion
- Sometimes (in an incomplete parse) type guards need to be assumed
- They can be discharged or the parse may be pruned out at a later stage (when a bigger part of the formula is parse)

Elaboration results for toplevel statements

- Input: About 60K Mizar Theorems in Pattern Representation
- Output: Constructor Based Version or Failure or Timeout (20s)
- About 95% elaborated with no assumed pattern guards
- Another 2% elaborated with some assumed pattern guards
- Roughly 2% fail to fill in some implicit arguments
- Roughly 1% time out
- Things are more tricky when elaborating incomplete parses
- However we finally have a reasonable toolchain to go from ambiguated Mizar to ATP
- Not ATP-evaluated yet

First Mizar Results (100-fold Cross-validation)



ProofWiki vs Mizar – our CICM'14 Example

Example: ProofWiki vs Mizar vs Mizar-style automated proof

== Theorem ==
Let $(S, \circ)$ be an [[Definition:Algebraic Structure|algebraic structure]] that has a [[Definition:Zero Element|zero element]] $z \in S$. Then z is unique.
== Proof ==
Suppose z_1 and z_2 are both zeroes of $(S, \circ)$.
Then by the definition of [[Definition:Zero Element|zero element]]:
$z_2 \circ z_1 = z_1$ by dint of z_1 being a zero;
$z_2 \circ z_1 = z_2$ by dint of z_2 being a zero.
So $z_1 = z_2 \circ z_1 = z_2$.
So $z_1 = z_2$ and there is only one zero after all.
{{qed}}
// *NB: Informal proofs are buggy!*

```
Th9:  e1 is_a_left_unity_wrt o &
e2 is_a_right_unity_wrt o implies e1 = e2
proof
assume that A1:  e1 is_a_left_unity_wrt o and
A2:  e2 is_a_right_unity_wrt o;
thus e1 = o.(e1,e2) by A2,Def6 .= e2 by A1,Def5;
end;

z1 is_a_unity_wrt o & z2 is_a_unity_wrt o
implies z1 = z2 proof
assume that A1:  z1 is_a_unity_wrt o and
A2:  z2 is_a_unity_wrt o;
A3:  o.(z2,z1) = z1 by Th3,A2; ::[ATP]
A4:  o.(z2,z1) = z2 by Def 6,Def 7,A1,A3; ::[ATP]
hence z1 = z2 by Th9,A1,Def 7,A2; ::[ATP]
end;
```

Can We Align Proofwiki with Mizar and Parse It?

- Since 2015: Grzegorz Bancerek aligning Mizar and Proofwiki
- Over 500 ProofWiki pages
- Example: `https://proofwiki.org/wiki/Arithmetic_iff_Way_Below_Relation_is_Multiplicative_in_Algebraic_Lattice`
- Not just automated translation, but made to fit the math already developed in ProofWiki
- How do we use it?

ProofWiki vs Mizar Again

PW code

```
Let $\left(S, \preceq\right)$ be an ordered set.  Let $x \in S$.  Then
$\left\ x\right\$ is a chain of $\left(S, \preceq\right)$.
```

PW display

Let $(S, \preceq)$ be an ordered set. Let $x \in S$. Then $\{x\}$ is a chain of $(S, \preceq)$.

Mizar

```
for A being non empty reflexive RelStr for a being Element of A holds {a}
is Chain of A
```

Mizar parse

```
(Bool for (Varlist (Set (Var A))) being (Type (@ListOfAdjectives
(Adjective ($#~nv2_struct_0 non (Attribute ($#nv2_struct_0 empty))))
(Adjective (Attribute ($#nv3_orders_2 reflexive)))) ($#nl1_-
orders_2 RelStr)) (Bool for (Varlist (Set (Var a))) being (Type
(@ListOfAdjectives) ($#nm1_struct_0 Element) of (Set (Var A))) holds
(Bool (Set ($#nk6_domain_1 {) (Set (Var a)) ($#nk6_domain_1 })) is (Type
(@ListOfAdjectives) ($#nm2_orders_2 Chain) of (Set (Var A))))))
```

ProofWiki vs Mizar Issues

1. The Pr∞fWiki `chain` can map directly to the Mizar-style subtree `($#nm2_orders_2 chain)`, possibly additionally aligning `chain` with `Chain` as synonyms.
2. The Pr∞fWiki TEX text `"\left\{ {x}\right\}"` needs to be mapped to the Mizar-style subtree `(Set ($#nk6_domain_1 {) (Set (Var x)) ($#nk6_domain_1_part_1 }))`.
3. `"ordered set"` needs to be mapped to Mizar `"non empty reflexive RelStr"`.
4. `"Let...Then..."` needs to be mapped to Mizar as `"for...holds..."`. Etc.

ProofWiki vs Mizar Issues - Proposed Solutions

- (1) is just a new grammar rule that can be learned from the treebank.
- The other examples however require more complex tree transformations
- So we added grammar extension that allows evaluation of arbitrary Lisp-like programs at nonterminal positions
- Following is an example of a subtree (and code) that performs the mapping (2):

```
(Set  ("PW_TeX_Singleton@@@
      (lambda (L LSB LB X RB R RSB)
            (list (gtree '$#nk6_domain_1 '{) X (gtree
'$#nk6_domain_1_part_1 '})))"
     "\left" "\{" "{" (Set (Var "x")) "}" "\right" "\}" ))
```

Learning Lisp Programs

- We plan to learn Lisp-like programs by the following bootstrapping procedure:
  1. The parser run on the corpus of Pr∞fWiki texts will identify the parts of input that cannot be parsed yet.
  2. This can be done by using a special low-probability nonterminal `"UNKNOWN"` that propagates through most of the grammar rules, marking the failed fragments.
  3. The failed fragments will be aligned with the corresponding Mizar subtrees.
  4. This yields a corpus of Pr∞fWiki - Mizar pairs where the parsing fails so far.
  5. This corpus can be mined for common frequent patterns.
  6. Use symbolic learning methods (ILP, Genetic Programming, etc.) to gradually create a corpus of more and more advanced Lisp-like functions that build on each other.
  7. Sometimes we'll add a difficult Lisp function manually.
  8. As usual the most probable parses will be subjected to typechecking and large-theory ATP, using the whole Mizar library as a background knowledge and the internal Pr∞fWiki steps as lemmas

Thanks for listening!

- Questions?