Building an Auto-formalization Infrastructure from Mathematical Literature through Deep Learning – Project Description

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Overview

- Why Auto-formalization?
- Machine Learning in Auto-formalization
- Deep Learning
- Deep Learning in Theorem Proving
- An Initial Experiment
- Discussion

A mathematical paper published in 2001 in *Annals of Mathematics*:

Invariant differential operators and eigenspace representations on an affine symmetric space

By JING-SONG HUANG*

Abstract

Let G/H be an affine symmetric space of split rank r. Let $\mathbf{D}$ be a preferred polynomial algebra of G-invariant differential operators on G/H generated by r elements. We show that the space of K-finite joint eigenfunctions of $\mathbf{D}$ on G/H form an admissible $(\mathfrak{g}, K)$-module which is called an eigenspace representation. The main content of this paper is description of the algebras of invariant differential operators and determination of the eigenspace representations on G/H. We also obtain a Poisson transform for τ-spherical eigenfunctions on G/H by Eisenstein integrals.

Gaps were found in 2008. It took 7 years for the author to fixed the proof.

Erratum and Addendum to: Invariant Differential Operators and Eigenspace Representations on an Affine Symmetric Space

Jing-Song Huang

(Submitted on 15 Jul 2017)

The purpose of this erratum and addendum is to correct the errors in [1]. It consists of five components:
1. Lemma 7.1 and Proposition 7.2 are wrong and discarded;
2. A new proof of existence $\lambda(\xi)$ in (7.1) without Proposition 7.2;
3. Definition of a new bijection in Theorem 5.2 and a proof by a new technique;
4. A new proof of Theorem 5.5 based on the new bijection in Theorem 5.2;
5. Correction to the list of exceptional simple pairs in Proposition 3.1.
The main results of [1] remain true as stated. We also add a final remark on generalization.

In 2017, the 16-year old paper was withdrawn:

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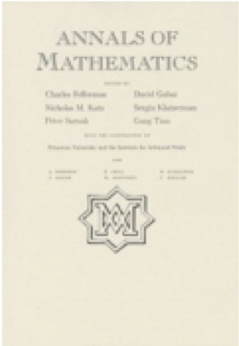
Author "shocked" after top math journal retracts paper

One of the world's most prestigious mathematics journals has issued what appears to be its first retraction.



The *Annals of Mathematics* recently withdrew a 2001 paper exploring the properties of certain symmetrical spaces.

Why Auto-formalization

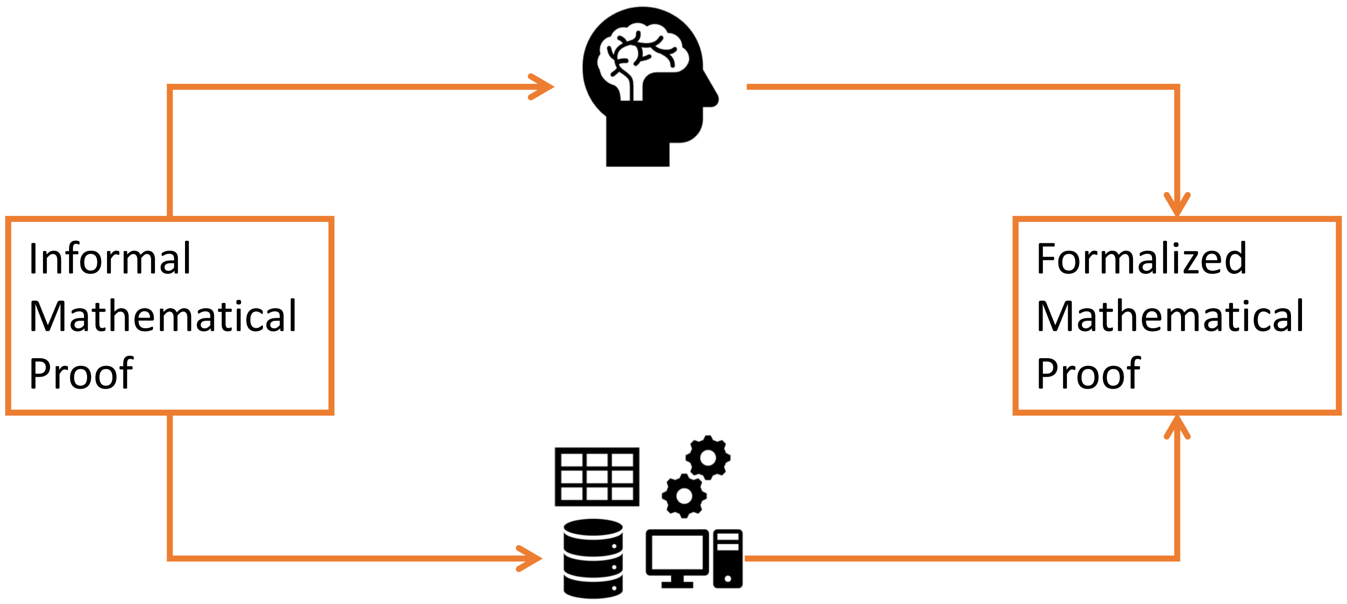
- Formalized libraries.

Coq Mizar HOL Metamath Lean Isabelle

- Mizar contains over 10k definitions and over 50k proofs, yet…

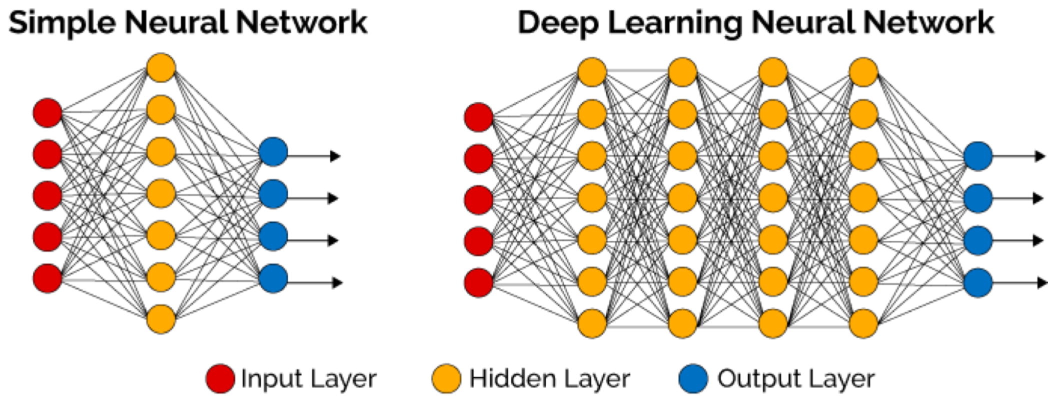
Machine Learning in Auto-formalization

- Function approximation view toward formalization and the prospect of machine learning approach to formalization.



Deep Learning

- Some theoretical results
  - Universal approximation theorem (Cybenko, Hornik), Depth separation theorem (Telgarsky, Shamir), etc
- Algorithmic techniques and novel architecture
  - Backpropagation, SGD, CNN, RNN, etc
- Advance in hardware and software
  - GPU, Tensorflow, etc
- Availability of large dataset
  - ImageNet, IWSLT, etc



Deep Learning in Theorem Proving

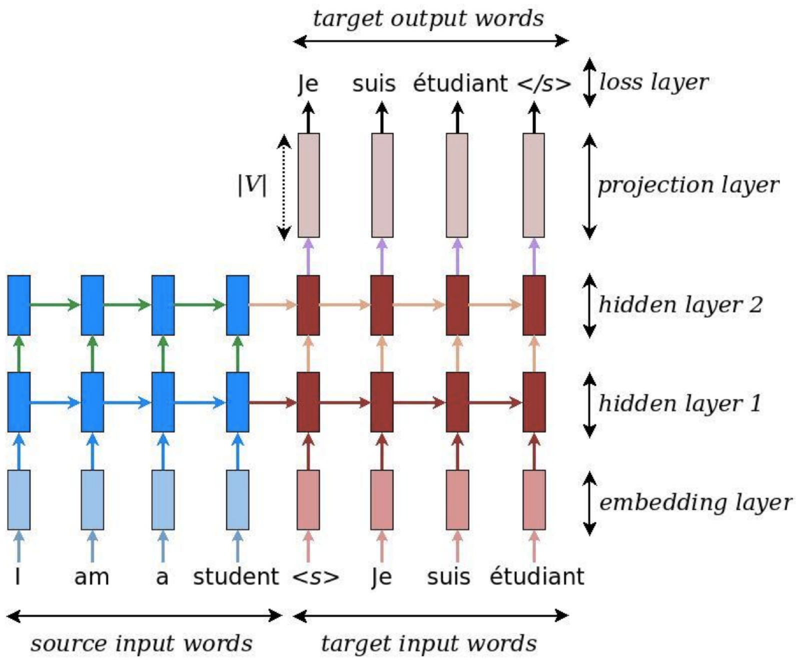
- Applications focus on doing ATP on existing libraries.

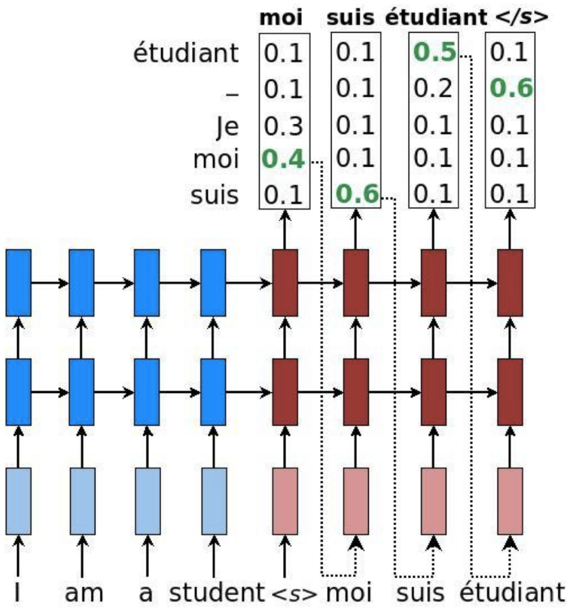
Year	Authors	Architecture	Dataset	Performance
Jun, 2016	Alemi et al.	CNN, LSTM/GRU	MMLFOF (Mizar)	80.9%
Aug, 2016	Whalen	RL, GRU	Metamath	14%
Jan, 2017	Loos et al.	CNN, WaveNet, RecursiveNN	MMLFOF (Mizar)	81.5%
Mar, 2017	Kaliszyk et al.	CNN, LSTM	HolStep (HOL-Light)	83%
Sep, 2017	Wang et al.	FormulaNet	HolStep (HOL-Light)	90.3%

- Opportunities of deep learning in formalization.

An Initial Experiment

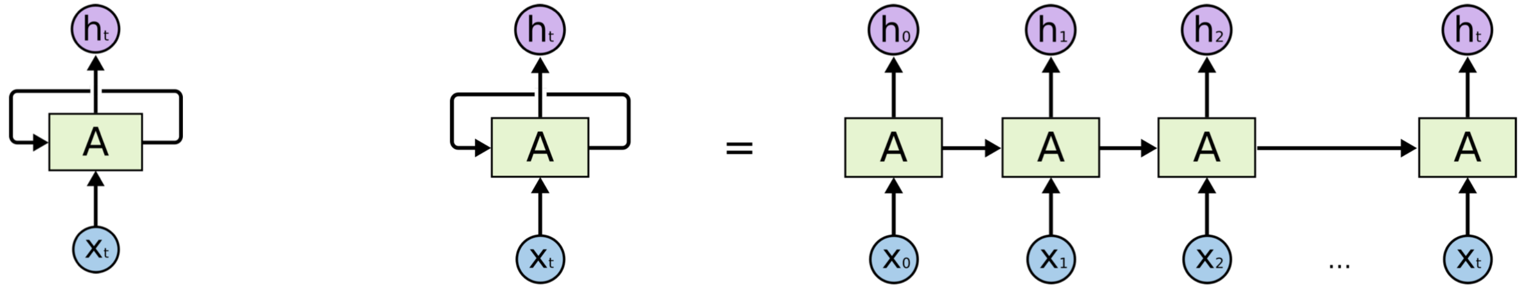
- Visit to Prague in January.
- Neural machine translation (Seq2seq model, Luong 2017).
- Can be considered as a complicated differentiable function.

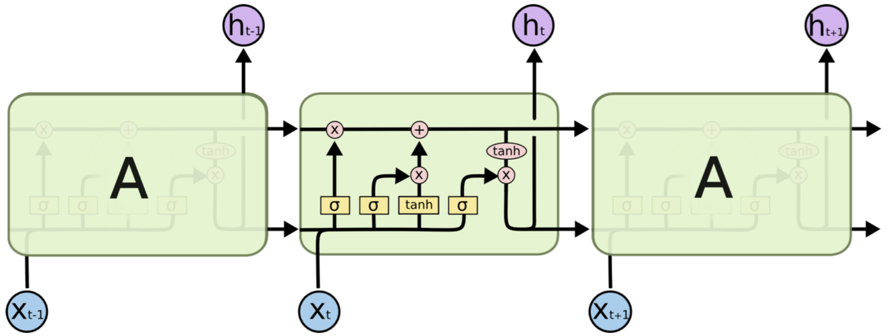


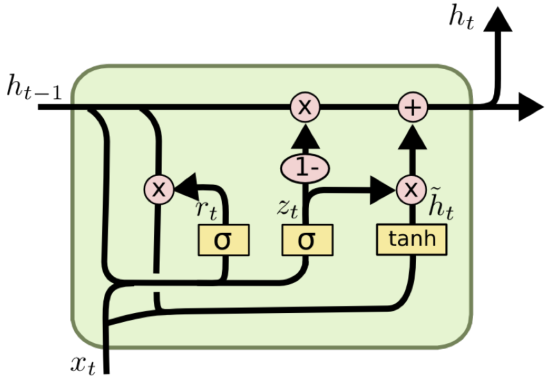


An Initial Experiment

- Recurrent neural network (RNN) and Long short-term memory cell (LSTM)







$$z_t = \sigma\left(W_z \cdot [h_{t-1}, x_t]\right)$$

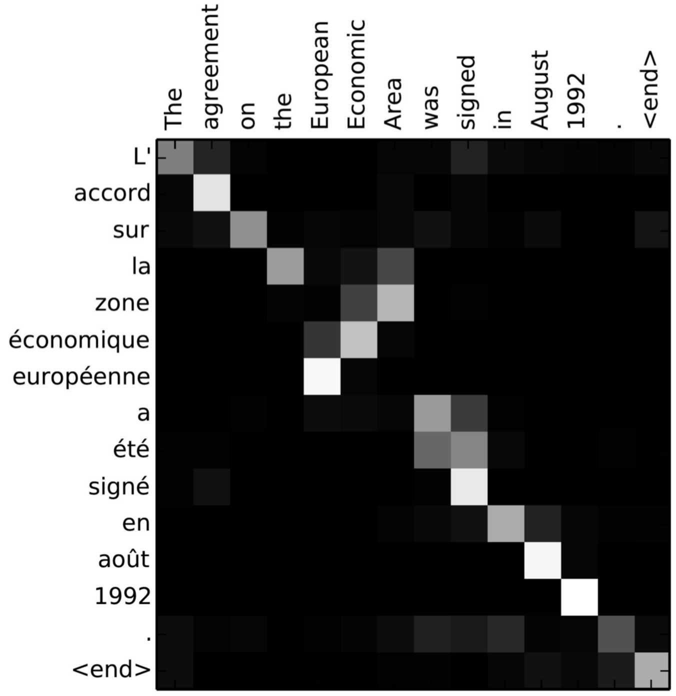
$$r_t = \sigma\left(W_r \cdot [h_{t-1}, x_t]\right)$$

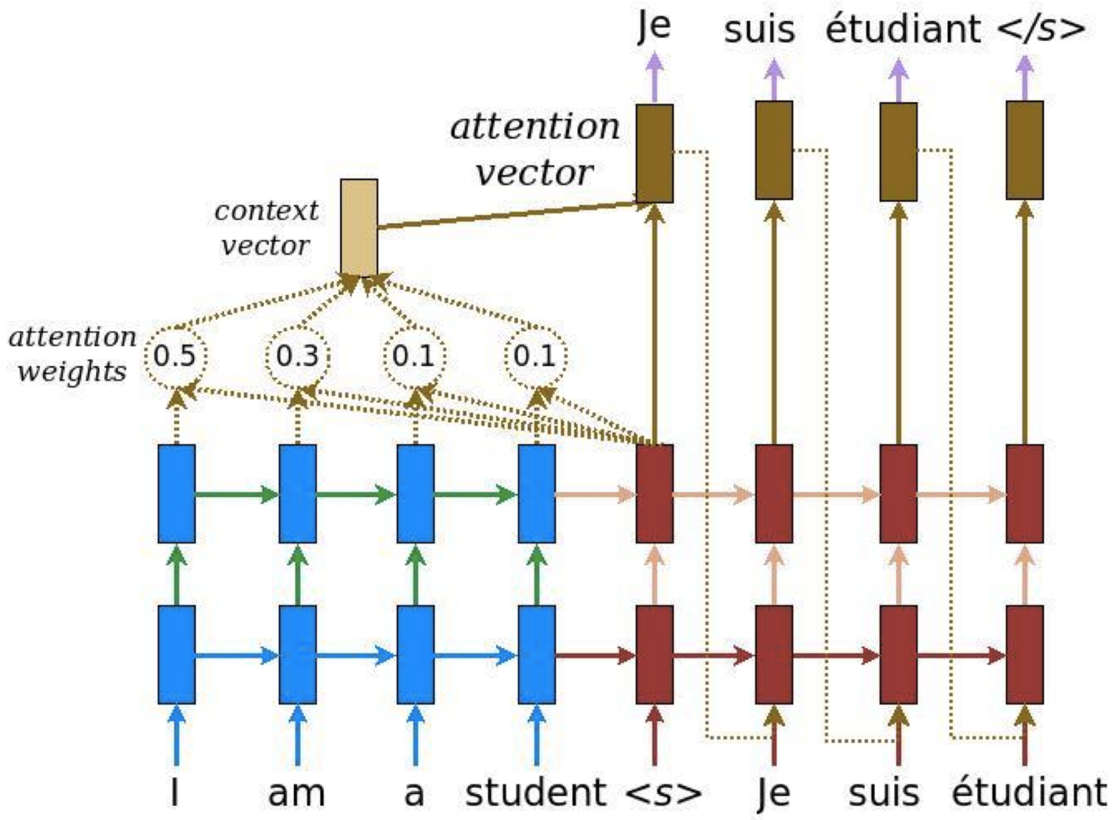
$$\tilde{h}_t = \tanh\left(W \cdot [r_t * h_{t-1}, x_t]\right)$$

$$h_t = (1 - z_t) * h_{t-1} + z_t * \tilde{h}_t$$

An Initial Experiment

- Attention mechanism





$$\alpha_{ts} = \frac{\exp\left(\operatorname{score}(\boldsymbol{h}_t, \bar{\boldsymbol{h}}_s)\right)}{\sum_{s'=1}^{S} \exp\left(\operatorname{score}(\boldsymbol{h}_t, \bar{\boldsymbol{h}}_{s'})\right)} \qquad \text{[Attention weights]} \qquad (1)$$

$$\boldsymbol{c}_t = \sum_s \alpha_{ts} \bar{\boldsymbol{h}}_s \qquad \text{[Context vector]} \qquad (2)$$

$$\boldsymbol{a}_t = f(\boldsymbol{c}_t, \boldsymbol{h}_t) = \tanh(\boldsymbol{W_c}[\boldsymbol{c}_t; \boldsymbol{h}_t]) \qquad \text{[Attention vector]} \qquad (3)$$

$$\operatorname{score}(\boldsymbol{h}_t, \bar{\boldsymbol{h}}_s) = \begin{cases} \boldsymbol{h}_t^\top \boldsymbol{W} \bar{\boldsymbol{h}}_s & \text{[Luong's multiplicative style]} \\ \boldsymbol{v}_a^\top \tanh\left(\boldsymbol{W_1} \boldsymbol{h}_t + \boldsymbol{W_2} \bar{\boldsymbol{h}}_s\right) & \text{[Bahdanau's additive style]} \end{cases} \qquad (4)$$

An Initial Experiment

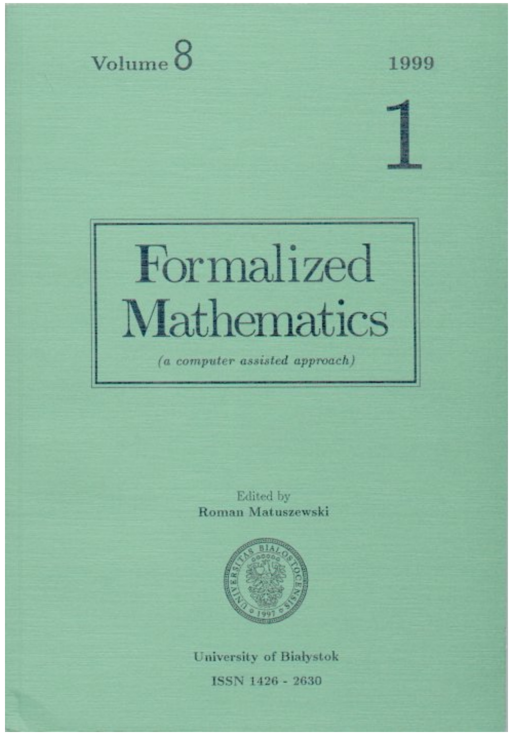
- Raw data from Grzegorz Bancerek (2017†).
- Formal abstracts of *Formalized mathematics*, which are generated latex from Mizar (v8.0.01_5.6.1169)
- Extract Latex-Mizar statement pairs as training data. Use Latex as source and Mizar as target.

```
theorem P0:
for L being right_zeroed non empty addLoopStr,
    S,T being Subset of L
st 0.L in T holds S c= S + T
```

Formalized Mathematics ↓ ↑ Seq2Seq

Now we state the propositions:

(1) Let us consider a right zeroed, non empty additive loop structure L, and subsets S, T of L. If $0_L \in T$, then $S \subseteq S + T$.



An Initial Experiment

- In total, 53368 theorems (schema) statements were divided by 10:1 into:
  - Training set: 48517 statements
  - Test set: 4851 statements
- Both Latex and Mizar tokenized to accommodate the framework.

Latex	`If $ X \mathrel { = } { \rm the ~ } { { { \rm carrier } ~ { \rm of } ~ { \rm } } } { A _ { 9 } } $ and $ X $ is plane , then $ { A _ { 9 } } $ is an affine plane .`
Mizar	`X = the carrier of AS & X is being_plane implies AS is AffinPlane ;`

Latex	`If $ { s _ { 9 } } $ is convergent and $ { s _ { 8 } } $ is a subsequence of $ { s _ { 9 } } $ , then $ { s _ { 8 } } $ is convergent .`
Mizar	`seq is convergent & seq1 is subsequence of seq implies seq1 is convergent ;`

An Initial Experiment

- Preliminary result (among the 4851 test statements)

Attention mechanism	Number of identical statements generated	Percentage
No attention	120	2.5%
Bahdanau	165	3.4%
Normed Bahdanau	1267	26.12%
Luong	1375	28.34%
Scaled Luong	1270	26.18%
Any	1782	36.73%

- A good correspondence between Latex and Mizar, probably easy to learn.

An Initial Experiment

- Sample unmatched statements

Attention mechanism	Mizar statement
Correct statement	`for T being Noetherian sup-Semilattice for I being Ideal of T holds ex_sup_of I , T & sup I in I ;`
No attention	`for T being lower-bounded sup-Semilattice for I being Ideal of T holds I is upper-bounded & I is upper-bounded ;`
Bahdanau	`for T being T , T being Ideal of T , I being Element of T holds height T in I ;`
Normed Bahdanau	`for T being Noetherian adj-structured sup-Semilattice for I being Ideal of T holds ex_sup_of I , T & sup I in I ;`
Luong	`for T being Noetherian adj-structured sup-Semilattice for I being Ideal of T holds ex_sup_of I , T & sup I in I ;`
Scaled Luong	`for T being Noetherian sup-Semilattice , I being Ideal of T ex I , sup I st ex_sup_of I , T & sup I in I ;`

- Further exploration in finding parsable statement, or hopefully generating syntactically correct statement.

Discussion

- Formalization using deep learning is a promising direction.
- Deep learning and AI, open to further development.
- Understanding mathematical statements versus general natural language understanding.
- Implication of achieving auto-formalization.
- Lots of challenges await us.

Thanks

...Ta mathemata [sic] are the things in so far as we take cognizance of them as what we already know them to be in advance, the body of the bodily, the plant-like of the plants, the animal-like of the animals, the thing-ness of the things, and so on. This genuine learning is therefore an extremely peculiar taking, a taking where one who takes only takes what one basically already gets...

Martin Heidegger, Modern Science, Metaphysics and Mathematics