Project Proposal: Prediction by Compression

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Compressor C such that $C(s)$ is the length of the compression of s

[Cilibrasi and Vitanyi 2003], [Li et al. 2004]

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$$s \text{ and } t \text{ share all information} \implies C(st) \approx C(s) + b$$
$$s \text{ and } t \text{ share no information} \implies C(st) \approx C(s) + C(t)$$

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$$NCD_C(s,t) = \frac{C(st) - \min(C(s), C(t))}{\max(C(s), C(t))}$$

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Under reasonable conditions for C, NCD_c approximates a metric

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Kolmogorov complexity K:

$$K(s) = \underset{p \in P \wedge L(p) = s}{\arg\min} |p|$$

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Kolmogorov complexity K:

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$$NCD_K(s,t) = \frac{K(st) - \min(K(s), K(t))}{\max(K(s), K(t))}$$

NCD_K is the distance metric:

$$\forall_{d,s,t} \ \ \text{computable}(d) \Rightarrow NCD_K(s,t) \leq d(s,t)$$

[Cilibrasi and Vitanyi 2003], [Li et al. 2004]

No domain-specific knowledge necessary!

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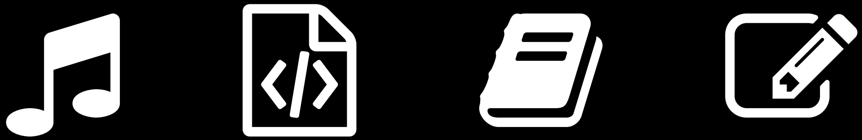

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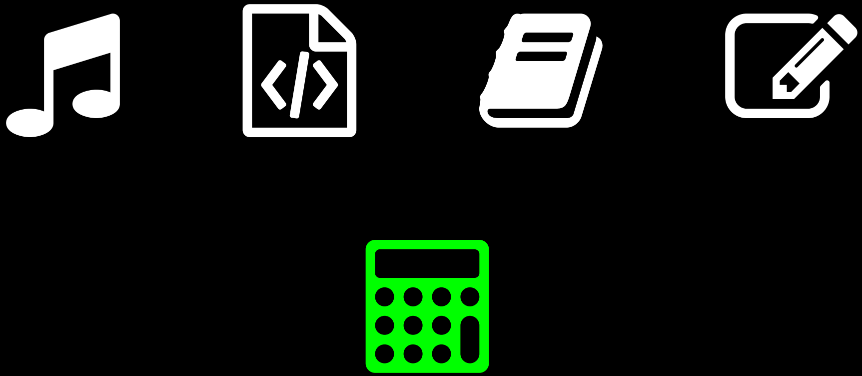

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Compression: Prediction by Partial Matching

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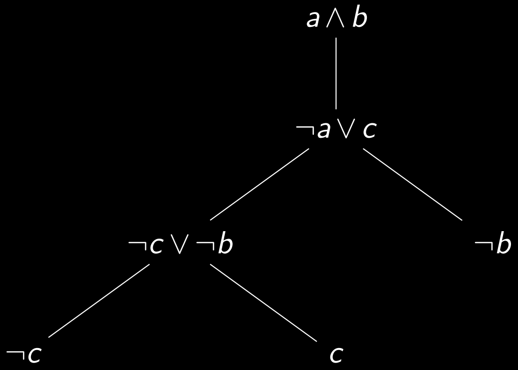
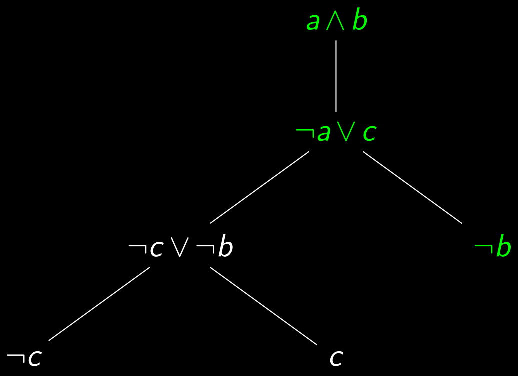
Compress entire proof states



[CKaliszyk, Urban and Vyskoci 2015]



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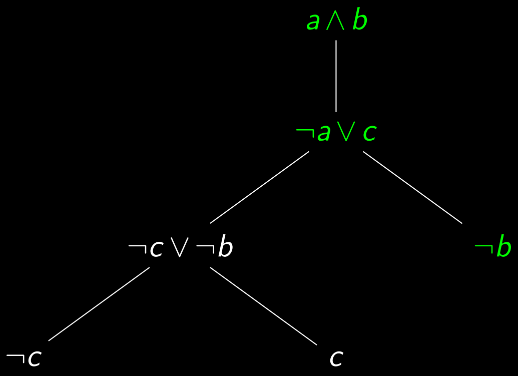

$a \quad b \quad \neg a \quad c$

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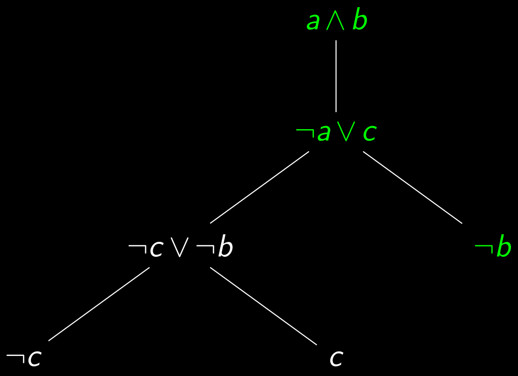

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"$a \wedge b \neg a \vee c \neg b a b \neg a c$"

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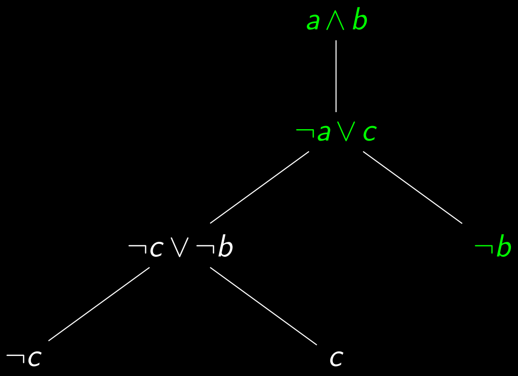
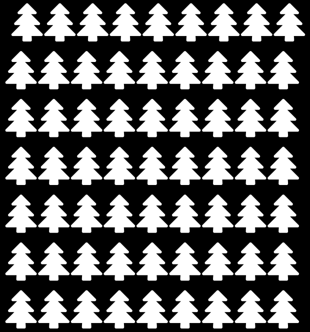


$a \quad b \quad \neg a \quad c$

"$a \wedge b \neg a \vee c \neg bab \neg ac$"

$\Leftrightarrow$

Database



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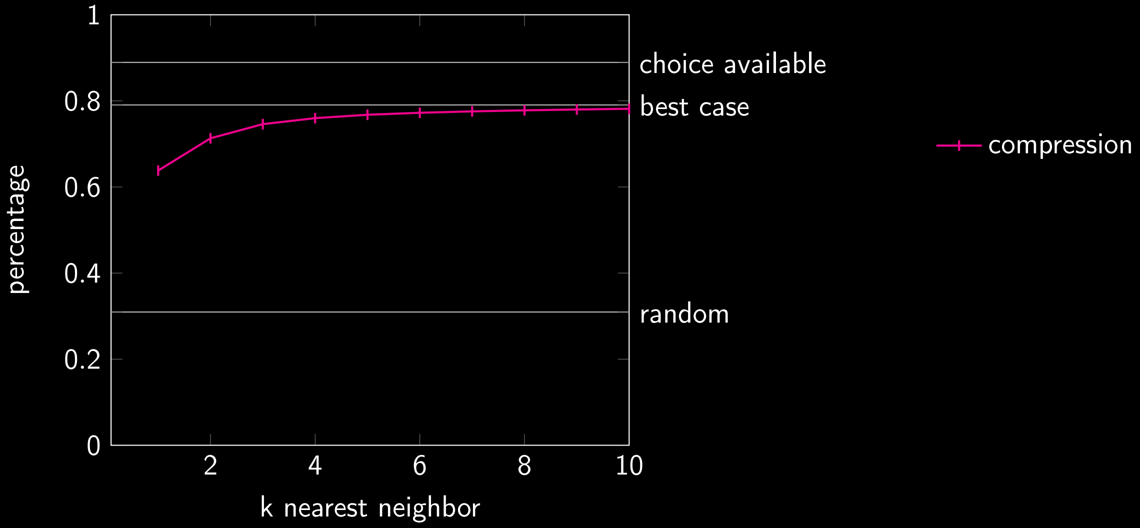
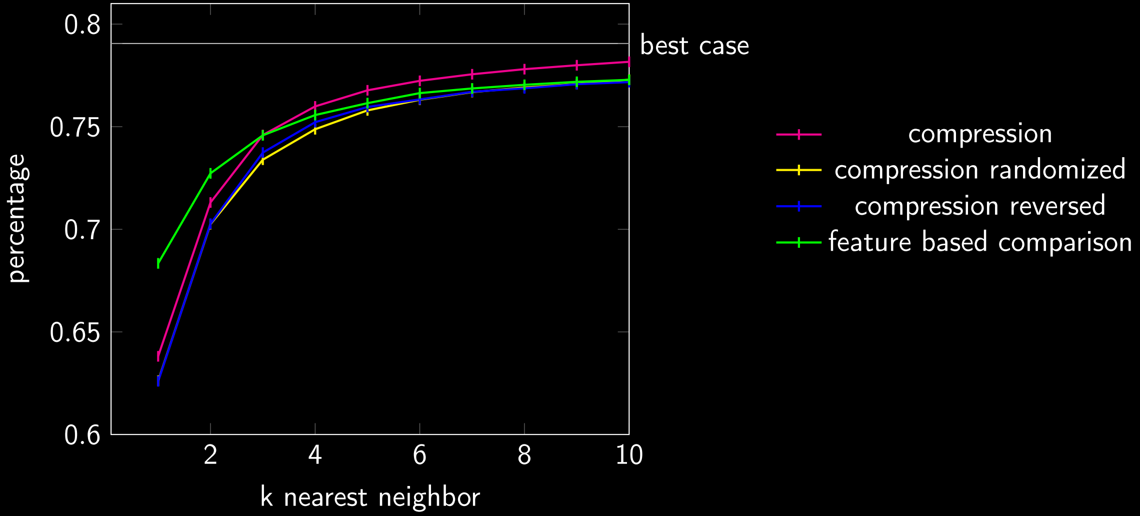

1
0.8
0.6
0.4
0.2
0
percentage
2
4
6
8
10
k nearest neighbor
choice available
best case
random
compression


0.8
0.75
0.7
0.65
0.6
2
4
6
8
10
percentage
k nearest neighbor
best case
compression
compression randomized
compression reversed
feature based comparison

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Idea: Impose structure through an n-dimensional lattice

$$S_n = \{X \subseteq S \mid |X| = n\}$$

$$\text{out}(s) = \arg\max_{X \in S_n} \frac{\sum_{t,u \in X} NCD(t,u)}{\sum_{t \in X} NCD(s,t)}$$

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- Predictions are competitive
- Robust against different representations of proof states

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Cons

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- ▷ No vector space

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