Automation by Analogy, in Coq

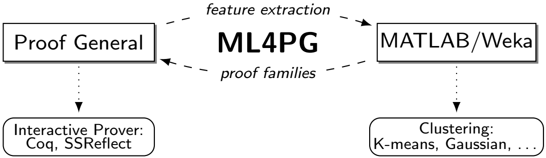
Alasdair Hill, Katya Komendantskaya

Heriot-Watt University, Scotland

20 March 2018

Machine Learning for Proof General (ML4PG)

ML4PG interfaces with proof general to extract features of lemmas from an ITP and uses a machine learning tool such as weka to cluster them.



Feature Extraction

Feature extraction is performed to cluster lemmas on both proof terms and types

[1]Komendantskaya, E., Heras, J. and Grov, G., 2012. Machine learning in proof general: Interfacing interfaces. EPTCS 118 (User Interfaces for Theorem Provers), 15-41.

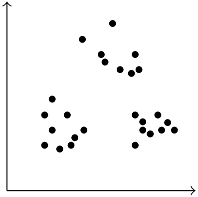
ML4PG approach to proof-clustering

We have integrated Proof General with a variety of clustering algorithms:

ML4PG approach to proof-clustering

We have integrated Proof General with a variety of clustering algorithms:

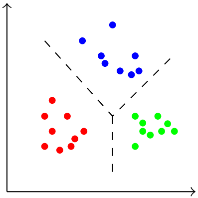
- Unsupervised machine learning technique:



ML4PG approach to proof-clustering

We have integrated Proof General with a variety of clustering algorithms:

- Unsupervised machine learning technique:

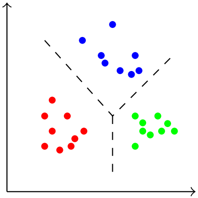


- Engines: Matlab, Weka, Octave, R, ...

ML4PG approach to proof-clustering

We have integrated Proof General with a variety of clustering algorithms:

- Unsupervised machine learning technique:

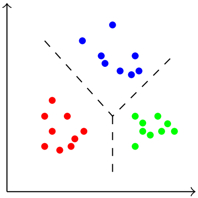


- Engines: Matlab, Weka, Octave, R, . . .

ML4PG approach to proof-clustering

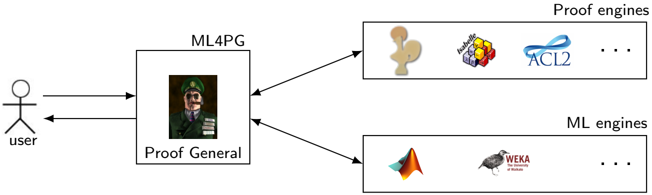
We have integrated Proof General with a variety of clustering algorithms:

- Unsupervised machine learning technique:



- Engines: Matlab, Weka, Octave, R, . . .
- Algorithms: K-means, Gaussian Mixture models, simple Expectation Maximisation, . . .

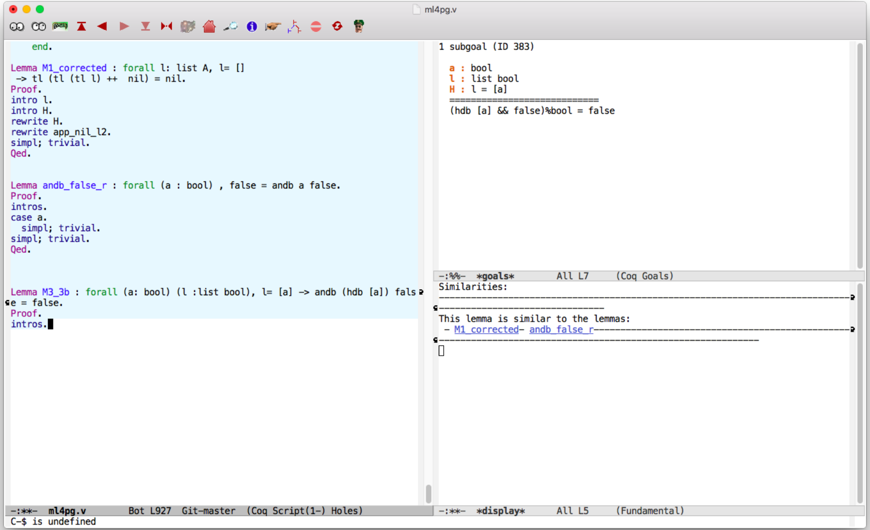
Overall architecture of ML4PG



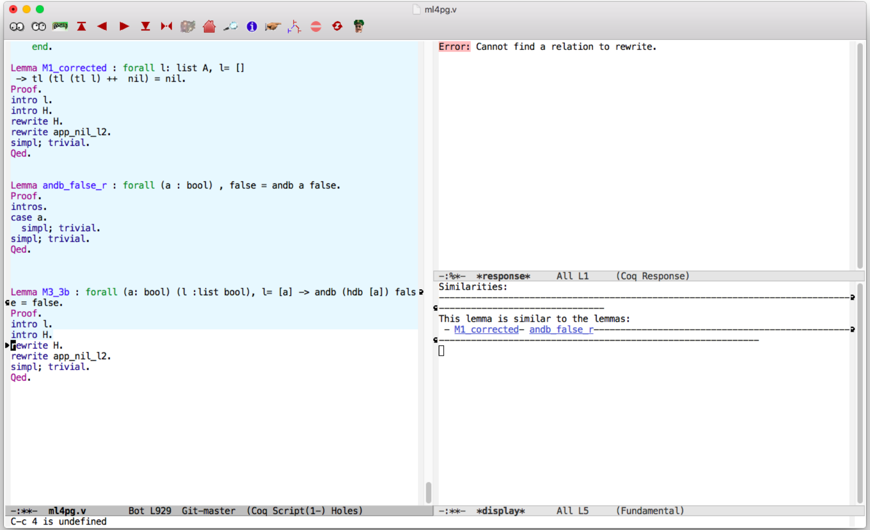
Interaction with ML4PG:

- One interacts with Proof General as usual,
- when one cannot proceed with a proof,
- he calls ML4PG (command line or editor button),
- ML4PG informs the user of similar existing proofs/definitions.

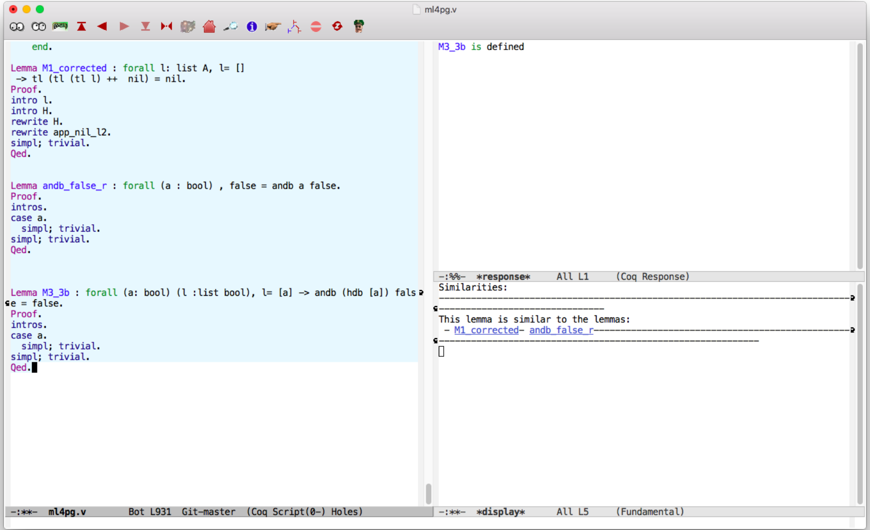
A proof in Coq with ML4PG help



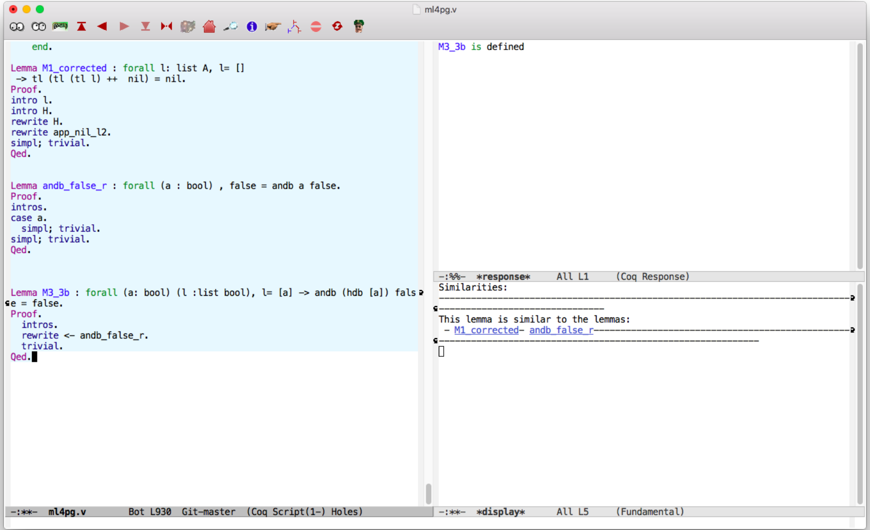
A proof in Coq with ML4PG help



A proof in Coq with ML4PG help



A proof in Coq with ML4PG help



Research Problem

Can clusters help with proof discovery?

Three methods have been created to automatically analogize proofs from these clusters.

These methods look to show that:

- Clusters created by ML4PG constain similar lemmas.
- New proofs can be analogized from these clusters that brute force would be unable to find.

Simple Search

Method:

- For each lemma in cluster copy entire proof and see if it is valid in current lemma.

Example:
Prove lemma:

```
Lemma plus_Sn_m : forall n m:nat, S n + m = S (n + m).
```

With Cluster: aux7_bis, mulnS, mult_n_O, aux10.

Simple Search Example

```
Lemma aux7_bis : forall a:nat, a-a = 0.
Proof.
induction a.
  simpl; trivial.
simpl; trivial.
Qed.

Lemma plus_Sn_m : forall n m:nat, S n + m = S (n + m).
Proof.
induction a.
  simpl; trivial.
simpl; trivial.
Qed.
```

Error.
Searching mulnS, mult_n_O, aux10.

Simple Search Example

```
Lemma mulnS : forall n m, n * S m = n + n * m.
Proof.
induction n.
   trivial. intro m.
rewrite mulSn. rewrite mulSn. rewrite addSn. rewrite addSn. rewrite addnCA.
rewrite IHn. trivial.
Qed.
```

```
Lemma plus_Sn_m : forall n m:nat, S n + m = S (n + m).
induction n.
   trivial. intro m.
rewrite mulSn. rewrite mulSn. rewrite addSn. rewrite addSn. rewrite addnCA.
rewrite IHn. trivial.
Qed.
```

Error.

Searching mult_n_O, aux10.

Simple Search Example

```
Lemma mult_n_O : forall n:nat, O = n * O.
Proof.
induction n.
simpl; trivial.
simpl; trivial.
Qed.

Lemma plus_Sn_m : forall n m:nat, S n + m = S (n + m).
Proof.
induction n.
simpl; trivial.
simpl; trivial.
Qed.
```

Proof Solved.

Simple Search

Success of simple search shows evidence towards the clusters being correct.

For Example:

Library	Size	Simple	SimpleBrute
Experimental	50	$31 \approx 62\%$	$40 \approx 80\%$
Paths (in Coq HoTT library)	41	$38 \approx 93\%$	$39 \approx 95\%$

Depth First Search

Method:

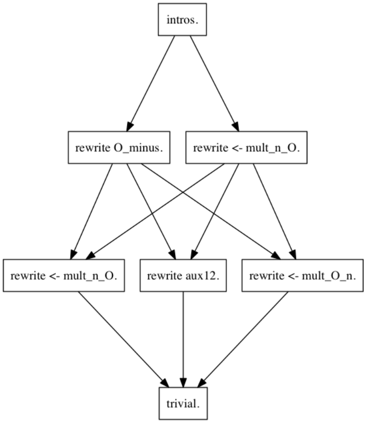
1. Create list of lists of all tactics used in proofs of other lemmas in clusters.
2. Depth first search the list of tactics until proof is found or no tactics remaining.

Example:
Prove lemma:

```
Lemma M26 : forall a b: nat, (0 - a) * S b = 0.
```

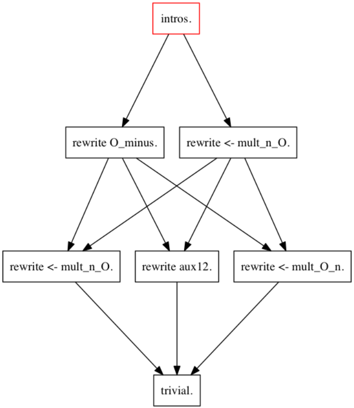
With Cluster: M41, M37, M32, M31, M22

Depth First Search Proof Tree



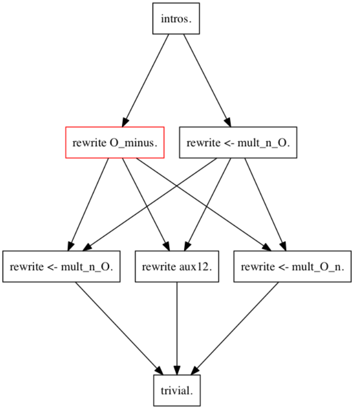
Depth First Search Example

```
Lemma M26 : forall a b: nat, (0 - a)
      * S b = 0.
Proof.
  intros.
```



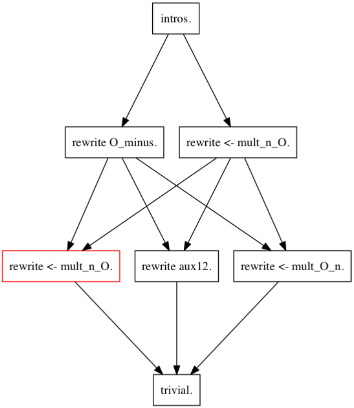
Depth First Search Example

```
Lemma M26 : forall a b: nat, (0 - a)
      * S b = 0.
Proof.
  intros.
  rewrite O_minus.
```



Depth First Search Example

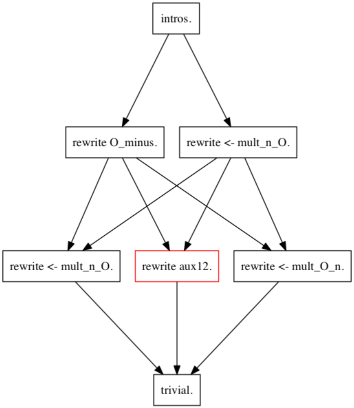
```
Lemma M26 : forall a b: nat, (0 - a)
      * S b = 0.
Proof.
  intros.
  rewrite O_minus.
  rewrite <- mult_n_O.
```



Error.

Depth First Search Example

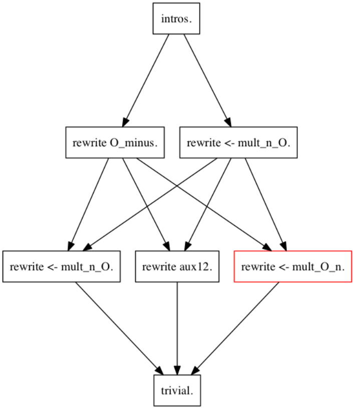
```
Lemma M26 : forall a b: nat, (0 - a)
      * S b = 0.
Proof.
  intros.
  rewrite O_minus.
  rewrite <- aux12.
```



Error.

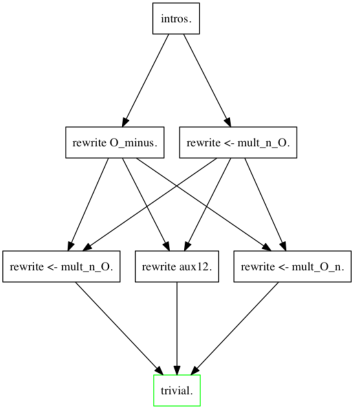
Depth First Search Example

```
Lemma M26 : forall a b: nat, (0 - a)
      * S b = 0.
Proof.
  intros.
  rewrite O_minus.
  rewrite <- mult_O_n.
```



Depth First Search Example

```
Lemma M26 : forall a b: nat, (O - a)
      * S b = O.
Proof.
  intros.
  rewrite O_minus.
  rewrite <- mult_O_n.
  trivial.
Qed.
```



Proof Solved.

Context Mining Search

Method:

1. Extract each lemma removing internal variable references.
2. Perform a depth first search on the extracted lemmas using variables from the context instead of the internal ones.
3. If there is a reference to an external lemma all other lemmas in its cluster are also tried.

Context Mining Search Example

Example:

Prove lemma:

```
Lemma M23 : forall a: nat, (a + 0) * S 0 = a.
```

With Cluster: andb_false_r, aux11, M1_corrected, aux12, mulSn, addSn, plus_0_n, app_nil_l2b, app_nil_l, mulnS, aux7, addnCA, addnS

Context Mining Search Example

How context mining search represents the proof found:

```
(1 . "induction")
(semi (0 . "simpl") (0 . "trivial"))
(semi (0 . "simpl") (0 . "trivial"))
(ext "rewrite" . "addSn")
(ext "rewrite" . "addnCA")
(1 . "rewrite")
(0 . "trivial")
```

Context Mining Search Example

(1 . "induction")

One variable used in tactic. Possible variables from context: a

```
Lemma M23 : forall a: nat, (a + 0) * S 0 = a.
Proof.
induction a.
```

Context Mining Search Example

(semi (0 . "simpl") (0 . "trivial"))
No variables used in tactics and tactics are seperated by a semi colon.

```
Lemma M23 : forall a: nat, (a + 0) * S 0 = a.
Proof.
induction a.
simpl; trivial.
```

Context Mining Search Example

(semi (0 . "simpl") (0 . "trivial"))
No variables used in tactics and tactics are seperated by a semi colon.

```
Lemma M23 : forall a: nat, (a + 0) * S 0 = a.
Proof.
induction a.
simpl; trivial.
simpl; trivial.
```

Context Mining Search Example

(ext "rewrite" . "addSn")

External rewrite with no arrows referenced.

Perform rewrite on variables in addSn clusters: addSn, andb_false_r, M23, aux11, M1_corrected, aux12, mulSn, plus_0_n, app_nil_l2b, app_nil_l

```
Lemma M23 : forall a: nat, (a + O) * S O = a.
Proof.
induction a.
simpl; trivial.
simpl; trivial.
rewrite addSn.
```

Error.

Context Mining Search Example

Remaining lemmas: andb_false_r, M23, aux11, M1_corrected, aux12, mulSn, plus_0_n, app_nil_l2b, app_nil_l

```
Lemma M23 : forall a: nat, (a + 0) * S 0 = a.
Proof.
induction a.
simpl; trivial.
simpl; trivial.
rewrite andb_false_r.
```

Error.

Context Mining Search Example

Remaining lemmas: M23, aux11, M1_corrected, aux12, mulSn, plus_0_n, app_nil_l2b, app_nil_l

```
Lemma M23 : forall a: nat, (a + 0) * S 0 = a.
Proof.
induction a.
simpl; trivial.
simpl; trivial.
rewrite M23.
```

Error.

Context Mining Search Example

Remaining lemmas: aux11, M1_corrected, aux12, mulSn, plus_0_n, app_nil_l2b, app_nil_l

```
Lemma M23 : forall a: nat, (a + 0) * S 0 = a.
Proof.
induction a.
simpl; trivial.
simpl; trivial.
rewrite aux11.
```

Error.

Context Mining Search Example

Remaining lemmas: M1_corrected, aux12, mulSn, plus_0_n, app_nil_l2b, app_nil_l

```
Lemma M23 : forall a: nat, (a + O) * S O = a.
Proof.
induction a.
simpl; trivial.
simpl; trivial.
rewrite M1_corrected.
```

Error.

Context Mining Search Example

Remaining lemmas: aux12, mulSn, plus_0_n, app_nil_l2b, app_nil_l

```
Lemma M23 : forall a: nat, (a + 0) * S 0 = a.
Proof.
induction a.
simpl; trivial.
simpl; trivial.
rewrite aux12.
```

Error.

Context Mining Search Example

Remaining lemmas: mulSn, plus_0_n, app_nil_l2b, app_nil_l

```
Lemma M23 : forall a: nat, (a + O) * S O = a.
Proof.
induction a.
simpl; trivial.
simpl; trivial.
rewrite mulSn.
```

Error.

Context Mining Search Example

Remaining lemmas: plus_0_n, app_nil_l2b, app_nil_l

```
Lemma M23 : forall a: nat, (a + O) * S O = a.
Proof.
induction a.
simpl; trivial.
simpl; trivial.
rewrite plus_0_n.
```

Context Mining Search Example

(ext "rewrite" . "addnS")
External rewrite with no arrows referenced. Perform rewrite on variables in addnCA, M23, mulnS, aux7, addnS

```
Lemma M23 : forall a: nat, (a + O) * S O = a.
Proof.
induction a.
simpl; trivial.
simpl; trivial.
rewrite plus_0_n.
rewrite addnCA.
```

Error.

Context Mining Search Example

Remaining lemmas: M23, mulnS, aux7, addnS

```
Lemma M23 : forall a: nat, (a + 0) * S 0 = a.
Proof.
induction a.
simpl; trivial.
simpl; trivial.
rewrite plus_0_n.
rewrite M23.
```

Error.

Context Mining Search Example

Remaining lemmas: mulnS, aux7, addnS

```
Lemma M23 : forall a: nat, (a + 0) * S 0 = a.
Proof.
induction a.
simpl; trivial.
simpl; trivial.
rewrite plus_0_n.
rewrite mulnS.
```

Error.

Context Mining Search Example

Remaining lemmas: aux7, addnS

```
Lemma M23 : forall a: nat, (a + O) * S O = a.
Proof.
induction a.
simpl; trivial.
simpl; trivial.
rewrite plus_0_n.
rewrite aux7.
```

Error.

Context Mining Search Example

Remaining lemmas: addnS

```
Lemma M23 : forall a: nat, (a + 0) * S 0 = a.
Proof.
induction a.
simpl; trivial.
simpl; trivial.
rewrite plus_0_n.
rewrite addnS.
```

Context Mining Search Example

(1 . "rewrite")
Two variables available IHa and a. Trying rewrite on both.

```
Lemma M23 : forall a: nat, (a + 0) * S 0 = a.
Proof.
induction a.
simpl; trivial.
simpl; trivial.
rewrite plus_0_n.
rewrite addnS.
rewrite IHa.
```

Context Mining Search Example

(0 . "trivial")
No variables used in tactic

```
Lemma M23 : forall a: nat, (a + 0) * S 0 = a.
Proof.
induction a.
simpl; trivial.
simpl; trivial.
rewrite plus_0_n.
rewrite addnS.
rewrite IHa.
trivial.
Qed.
```

Proof Solved.

Another Context Mining Search Example

CompCert Proof

```
Lemma ireg_of_eq :
  forall r r', ireg_of r = OK r' -> preg_of r = IR r'.
Proof.
  unfold ireg_of; intros. destruct (preg_of r); inv H; auto.
Qed.
```

Another Context Mining Search Example

CompCert Proof

```
Lemma ireg_of_eq :
  forall r r', ireg_of r = OK r' -> preg_of r = IR r'.
Proof.
  unfold ireg_of; intros. destruct (preg_of r); inv H; auto.
Qed.
```

Context Mining Proof

```
Lemma ireg_of_eq :
  forall r r', ireg_of r = OK r' -> preg_of r = IR r'.
Proof.
intros.
destruct r, r'; inv H; auto .
Qed.
```

Context Mining Advantages

- Makes use of clustering to find additional lemmas to rewrite and apply.
- Stops errors due to using incorrect variable name.
- Finds brand new proof which cannot be found by brute force.

Method Results

This table only counts lemmas that are in a cluster.

Library	Size	Simple	DFS	CMS	Total
Experimental	50	$\approx 62\%$	$\approx 66\%$	$\approx 76\%$	$\approx 80\%$
Paths (in Coq HoTT library)	41	$\approx 93\%$	$\approx 93\%$	$\approx 80\%$	$\approx 93\%$

Pending: CompCert

Conclusion

- An add on for Proof General has been created for automatic analogizing of Coq Proofs.
- Three methods for analogizing Coq proofs from ML4PG clusters in proof general have been created.
- Clustering performed by ML4PG has been shown to find similar lemmas.
- More complex searching algorithms can be run on these clusters to find new proofs.

Conclusion

- An add on for Proof General has been created for automatic analogizing of Coq Proofs.
- Three methods for analogizing Coq proofs from ML4PG clusters in proof general have been created.
- Clustering performed by ML4PG has been shown to find similar lemmas.
- More complex searching algorithms can be run on these clusters to find new proofs.

Further Work?