Measuring progress to predict success: Can a good proof strategy be evolved?

Giles Reger[1], Martin Suda[2]

[1]School of Computer Science, University of Manchester, UK
[2]TU Wien, Vienna, Austria

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Vampire advertising

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- a “reasonably well-performing” first-order ATP
- unfortunately not open source
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Things are actually not so dark:

- email me, I can send you an executable
- find one at `https://www.starexec.org/`
- (don't) look for the source at:
  `http://www.cs.miami.edu/~tptp/CASC/J8/Entrants.html`

Outline

1. The role of strategies in modern ATPs
2. Proving with orderings
3. How to evolve a precedence?
4. Conclusion

The role of strategies in modern ATPs

Strategy:

- there are many-many options to setup the proving process
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If a strategy solves a problem then it typically solves it within a short amount of time (say, 5 seconds).

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From the ATP lore

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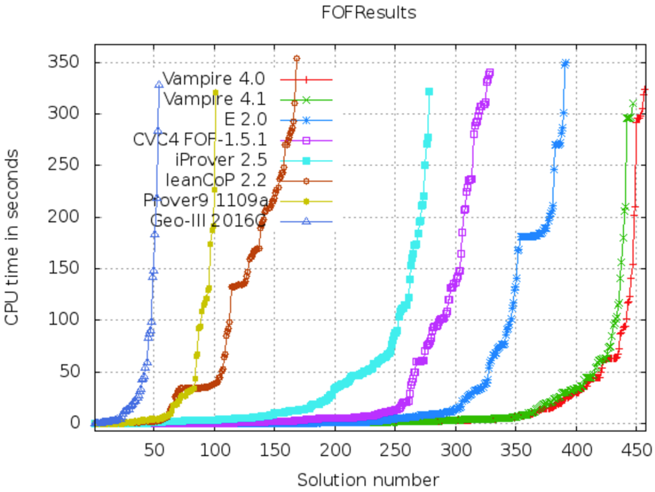
What does this mean?

- There is no single best strategy
- It's usually better to start something else than to wait
- Strategy Scheduling (portfolio approach)

CASC-mode: a conditional schedule of strategies

```
case Property::FNE:
  if (atoms > 2000) {
    quick.push("dis+1011_40_bs=on:cond=on:gs=on:gsaa=from_current:nwc=1:sfr=on:ssfp=1000:ssfq=2.0:smm=sco
    quick.push("lrs+1011_3_nwc=1:stl=90:sos=on:spl=off:sp=reverse_arity_133");
    quick.push("dis-10_5_cond=fast:gsp=input_only:gs=on:gsem=off:nwc=1:sas=minisat:sos=all:spl=off:sp=occu
    quick.push("lrs+1011_5_cond=fast:gs=on:nwc=2.5:stl=30:sd=3:ss=axioms:sdd=off:sfr=on:ssfp=100000:ssfq=
    quick.push("lrs-3_5:4_bs=on:bsr=on:cond=on:fsr=off:gsp=input_only:gs=on:gsaa=from_current:gsem=on:lcm=
  }
  else if (atoms > 1200) {
    quick.push("lrs+1011_5_cond=fast:gs=on:nwc=2.5:stl=30:sd=3:ss=axioms:sdd=off:sfr=on:ssfp=100000:ssfq=
    quick.push("dis+1011_8_bsr=unit_only:cond=fast:fsr=off:gs=on:gsaa=full_model:nm=0:nwc=1:sas=minisat:so
    quick.push("dis+11_7_gs=on:gsaa=full_model:lcm=predicate:nwc=1.1:sas=minisat:ssac=none:ssfp=1000:ssfq=
    quick.push("ins+11_5_br=off:gs=on:gsem=off:igbrr=0.9:igrr=1/64:igrp=1400:igrpq=1.1:igs=1003:igwr=on:lc
  }
  else {
    quick.push("dis+11_7_16");
    quick.push("dis+1011_5:4_gs=on:gsssp=full:nwc=1.5:sas=minisat:ssac=none:sdd=off:sfr=on:ssfp=40000:ssfq
    quick.push("dis+1011_40_bs=on:cond=on:gs=on:gsaa=from_current:nwc=1:sfr=on:ssfp=1000:ssfq=2.0:smm=sco
    ...
```

Results for FOF division of CASC 2016[1]



[1] www.cs.miami.edu/~tptp/CASC/J8/WWWFiles/ResultsPlots.html

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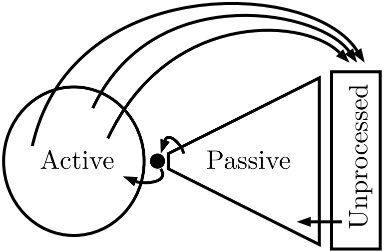
The Saturation Loop

Saturate a set of clauses with respect to an inference system



- Initially: the input clauses start in passive, active is empty
- Given clause: selected from passive as the next to be processed
- Move the give clause from active to passive and perform all inferences between clauses in active and the given clause

Resolution

$$\frac{A \vee C_1 \quad \neg A' \vee C_2}{(C_1 \vee C_2)\theta},$$

Factoring

$$\frac{A \vee A' \vee C}{(A \vee C)\theta},$$

where, for both inferences, $\theta = \mathrm{mgu}(A, A')$ *and* A *is not an equality literal, and* A *and* $\neg A'$ *are (strictly) maximal in their respective clauses*

Superposition

$$\frac{l \simeq r \vee C_1 \quad L[s]_p \vee C_2}{(L[r]_p \vee C_1 \vee C_2)\theta} \quad \text{or} \quad \frac{l \simeq r \vee C_1 \quad t[s]_p \otimes t' \vee C_2}{(t[r]_p \otimes t' \vee C_1 \vee C_2)\theta},$$

where $\theta = \mathrm{mgu}(l, s)$ *and* $r\theta \not\succeq l\theta$ *and, for the left rule* $L[s]$ *is not an equality literal, and for the right rule* $\otimes$ *stands either for* $\simeq$ *or* $\not\simeq$ *and* $t'\theta \not\succeq t[s]\theta$

EqualityResolution

$$\frac{s \not\simeq t \vee C}{C\theta},$$

where $\theta = \mathrm{mgu}(s, t)$

EqualityFactoring

$$\frac{s \simeq t \vee s' \simeq t' \vee C}{(t \not\simeq t' \vee s' \simeq t' \vee C)\theta},$$

where $\theta = \mathrm{mgu}(s, s')$, $t\theta \not\succeq s\theta$, *and* $t'\theta \not\succeq s'\theta$

How important could an ordering be?

Consider proving a formula

$$\psi = \bigwedge_{i=1,\ldots,n} (a_i \vee b_i) \to \bigwedge_{i=1,\ldots,n} (a_i \vee b_i)$$

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$$(a_i \vee b_i), \quad (\neg m_i \vee \neg a_i), (\neg m_i \vee \neg b_i), \quad (m_1 \vee \ldots \vee m_n),$$

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Question:

What will superposition derive under an ordering where

$$m_i \succ a_j \text{ and } m_i \succ b_j \text{ for every } i \text{ and } j?$$

Choosing an ordering

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ATPs typically provide a few schemes for fixing the precedence

Example

- Vampire: arity, reverse arity, occurrence
- E: frequency (`invfreq`), many more

Playing with precedence

Rules of the game

- Fix a single theorem proving strategy in Vampire:
  `-av off -sa discount -awr 10 -lcm predicate`
- Then by varying only the precedence
- try to solve as many TPTP problems as possible

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- 9457 solved by "frequency" in 300s (Thank you, Stephan!)
- ∼12500 solved in 300s by either `casc` or `casc_sat` mode

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- $\sim$7100 solved with a random precedence (3s)
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Shuffle a few times:

- 9387 solved in a union of 9 independent random precedence 60s runs (1678 problems in the grey zone)

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3.0s (7093) 3.0s (330) 3.1s (192) 3.2s (111) 3.3s (101) 4.4s (163) 4.5s (87) 4.8s (79) 5.0s (64) 6.2s (108) 9.6s (156) 11.1s (104) 11.5s (64) 21.4s (169) 205.3s (736)

Solves 9557 problems (9566 on validation set)

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SGSS in a nutshell:

A strategy that leads to a slowly growing search space will likely be more successful at finding a proof (in reasonable time) than a strategy that leads to a rapidly growing one.

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Successfully applied in previous work on literal selection [RSV16]

Using SGSS to look for a good precedence

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Look for strategies which minimize the number of derived clauses after a certain (small) number of iterations of the saturation loop.

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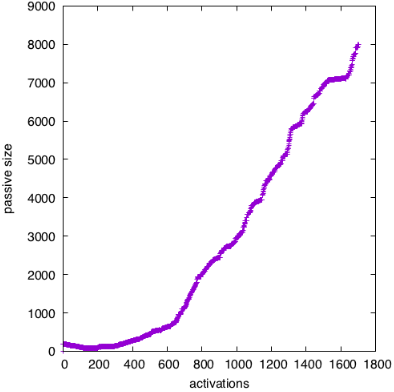
Can this work in practice?

- Probably not under tight time constraints.
- In any case:
  Are there actually any good precedences out there?
- Possible application:
  solve hard previously unsolved problems

A(n a)typical development of the passive set's size



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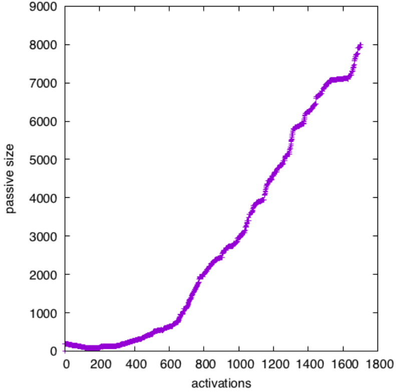

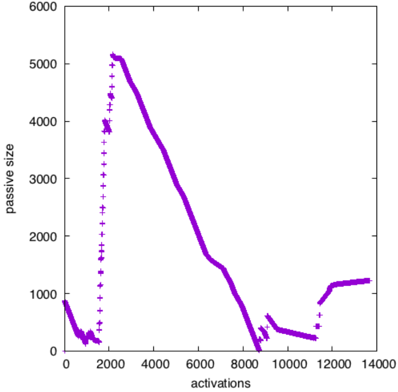

Can it possibly work?

Using the 9 independent random-precedence 60 second runs

On the set P of 1678 problems from the "grey zone"

Record size of passive every 100 activations

Compute nine respective sums s_i until the first stream stops:
$S_1(p) = s_1(p, 0) + s_1(p, 100) + s_1(p, 200) + \ldots$
$\ldots$
$S_9(p) = s_9(p, 0) + s_9(p, 100) + s_1(p, 200) + \ldots$

Denote the average $S_i(p)$ over (un)successful runs i as $\bar{S}_{(un)succ}(p)$

For how many $p \in P$ is $\bar{S}_{succ}(p) < \bar{S}_{unsucc}(p)$?

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Answer: 1130 (out of 1669)

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Optimize_precedence(p, t_1, t_2)

- run "frequency" for 1s to establish *act_cnt*
- spawn a population Π of n random precedences
- the fitness of $\pi \in \Pi$ is $S_\pi(p)$:
  the sum of the passive set sizes during a run on p
  summing every step from 0 to *act_cnt* activations
- loop for t_1 seconds:
  - pick a $\pi \in \Pi$
  - randomly (adaptively) perturb π to obtain π'
  - evaluate π' as above
  - keep the better of π and π'
- Finally, run with π_{best} for t_2 seconds

Results

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How many have solved in total?

- "frequency" 300s: 9457 (40 uniques)
- all the "harmonic" runs: 10030 (202 uniques)
- the long optimizing run: 9604 (87 uniques)
- In total: 10176

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Thank you for your attention!