Measuring progress to predict success: Can a good proof strategy be evolved?

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## Vampire advertising

<table>
<thead>
<tr>
<th>Vampire</th>
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<tr>
<td>- a “reasonably well-performing” first-order ATP</td>
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Things are actually not so dark: email me, I can send you an executable. Find one at [https://www.starexec.org/](https://www.starexec.org/) (don’t) look for the source at: [http://www.cs.miami.edu/~tptp/CASC/J8/Entrants.html](http://www.cs.miami.edu/~tptp/CASC/J8/Entrants.html)
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- email me, I can send you an executable
- find one at https://www.starexec.org/
- (don't) look for the source at:
1. The role of strategies in modern ATPs
2. Proving with orderings
3. How to evolve a precedence?
4. Conclusion
Strategy:

- there are many-many options to setup the proving process
- a strategy is a concrete way to do this setup
The role of strategies in modern ATPs

Strategy:
- there are many-many options to setup the proving process
- a **strategy** is a concrete way to do this setup

From the ATP lore
If a strategy solves a problem then it typically solves it within a short amount of time (say, 5 seconds).
The role of strategies in modern ATPs

Strategy:

- there are many-many options to setup the proving process
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From the ATP lore

If a strategy solves a problem then it typically solves it within a short amount of time (say, 5 seconds).

What does this mean?

- There is no single best strategy
- It’s usually better to start something else than to wait
- Strategy Scheduling (portfolio approach)
CASC-mode: a conditional schedule of strategies

case Property::FNE:
    if (atoms > 2000) {
        quick.push("lrs+1011_3_nwc=1:stl=90:sos=on:spl=off:sp=reverse_arity_133");
        quick.push("dis-10_5_cond=fast:gsp=input_only:gs=on:gsem=off:nwc=1:sas=minisat:sos=all:spl=off:sp=occurrence_190");
    }
    else if (atoms > 1200) {
        quick.push("dis+11_7_16");
    }
    else {
        quick.push("dis+11_7_16");
    }
Results for FOF division of CASC 2016

![FOFResults](image)

1[www.cs.miami.edu/~tptp/CASC/J8/WWWFiles/ResultsPlots.html](http://www.cs.miami.edu/~tptp/CASC/J8/WWWFiles/ResultsPlots.html)
Outline

1. The role of strategies in modern ATPs
2. Proving with orderings
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The Saturation Loop

Saturate a set of clauses with respect to an inference system

- Initially: the input clauses start in passive, active is empty
- Given clause: selected from passive as the next to be processed
- Move the given clause from active to passive and perform all inferences between clauses in active and the given clause
The superposition calculus (≻)

**Resolution**

\[
\frac{A \lor C_1 \quad \neg A' \lor C_2}{(C_1 \lor C_2)\theta}, \quad \frac{A \lor A' \lor C}{(A \lor C)\theta},
\]

where, for both inferences, \(\theta = \text{mgu}(A, A')\) and \(A\) is not an equality literal, and \(A\) and \(\neg A'\) are (strictly) maximal in their respective clauses.

**Factoring**

\[
\frac{A \lor C_1}{(A \lor C)\theta},
\]

**Superposition**

\[
\frac{l \cong r \lor C_1 \quad L[s]_p \lor C_2}{(L[r]_p \lor C_1 \lor C_2)\theta}, \quad \frac{l \cong r \lor C_1 \quad t[s]_p \otimes t' \lor C_2}{(t[r]_p \otimes t' \lor C_1 \lor C_2)\theta},
\]

where \(\theta = \text{mgu}(l, s)\) and \(r\theta \not\succeq l\theta\) and, for the left rule \(L[s]\) is not an equality literal, and for the right rule \(\otimes\) stands either for \(\cong\) or \(\not\simeq\) and \(t'\theta \not\succeq t[s]\theta\).

**EqualityResolution**

\[
\frac{s \not\simeq t \lor C}{C\theta},
\]

where \(\theta = \text{mgu}(s, t)\)

**EqualityFactoring**

\[
\frac{s \cong t \lor s' \cong t' \lor C}{(t \not\simeq t' \lor s' \cong t' \lor C)\theta},
\]

where \(\theta = \text{mgu}(s, s')\), \(t\theta \not\simeq s\theta\), and \(t'\theta \not\simeq s'\theta\).
How important could an ordering be?

Consider proving a formula

$$\psi = \bigwedge_{i=1,...,n} (a_i \lor b_i) \rightarrow \bigwedge_{i=1,...,n} (a_i \lor b_i)$$
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- goes down to $3n + 1$ with Tseitin encoding:
  
  $$(a_i \lor b_i), \quad (\neg m_i \lor \neg a_i), (\neg m_i \lor \neg b_i), \quad (m_1 \lor \ldots \lor m_n),$$

  where $m_i$ is a name for $\neg a_i \land \neg b_i$
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Question:

What will superposition derive under an ordering where

$$m_i \succ a_j \text{ and } m_i \succ b_j \text{ for every } i \text{ and } j$$
Orderings typically used in ATPs:
- Knuth-Bendix Ordering (KBO),
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We have $n!$ possibilities for choosing the ordering
Choosing an ordering

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ATPs typically provide a few schemes for fixing the precedence

Example
- Vampire: arity, reverse arity, occurrence
- E: frequency ($\text{invfreq}$), many more
Playing with precedence

Rules of the game

- Fix a single theorem proving strategy in Vampire:
  -av off -sa discount -awr 10 -lcm predicate
- Then by varying only the precedence
- try to solve as many TPTP problems as possible
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- ~12500 solved in 300s by either casc or casc_sat mode
How good is a random precedence?

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Shuffle once:

- \(~7100\) solved with a random precedence (3s)
- \(~8450\) solved with a random precedence (60s)
- \(~9100\) solved with a random precedence (300s)
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Shuffle once:
- $\sim 7100$ solved with a random precedence (3s)
- $\sim 8450$ solved with a random precedence (60s)
- $\sim 9100$ solved with a random precedence (300s)

Shuffle a few times:
- 9387 solved in a union of 9 independent random precedence 60s runs (1678 problems in the grey zone)
Question:
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The setup:
- for $i = 1$ to 100
  - run over TPTP with a $seed = i$ and time limit $300.0/i$ s
- $17280 \cdot H_{100} \cdot 300s \approx 311$ days of computation
Scheduler with Dice and Harmonic numbers

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<tr>
<td>3.0s</td>
<td>(7093)</td>
</tr>
<tr>
<td>3.0s</td>
<td>(330)</td>
</tr>
<tr>
<td>3.1s</td>
<td>(192)</td>
</tr>
<tr>
<td>3.2s</td>
<td>(111)</td>
</tr>
<tr>
<td>3.3s</td>
<td>(101)</td>
</tr>
<tr>
<td>4.4s</td>
<td>(163)</td>
</tr>
<tr>
<td>4.5s</td>
<td>(87)</td>
</tr>
<tr>
<td>4.8s</td>
<td>(79)</td>
</tr>
<tr>
<td>5.0s</td>
<td>(64)</td>
</tr>
<tr>
<td>6.2s</td>
<td>(108)</td>
</tr>
<tr>
<td>9.6s</td>
<td>(156)</td>
</tr>
<tr>
<td>11.1s</td>
<td>(104)</td>
</tr>
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<td>(64)</td>
</tr>
<tr>
<td>21.4s</td>
<td>(169)</td>
</tr>
<tr>
<td>205.3s</td>
<td>(736)</td>
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Solves 9557 problems (9566 on validation set)
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The Slowly-Growing-Search-Space heuristic

**SGSS in a nutshell:**

A strategy that leads to a slowly growing search space will likely be more successful at finding a proof (in reasonable time) than a strategy that leads to a rapidly growing one.

Intuition:
Can we find the proof before it chokes?
Since it's hard to predict if we are getting close ...
... try to postpone the choking until we (hopefully) get there.
Successfully applied in previous work on literal selection [RSV16]
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Idea

Look for strategies which minimize the number of derived clauses after a certain (small) number of iterations of the saturation loop.
Using SGSS to look for a good precedence

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Can this work in practice?

- Probably not under tight time constraints.
- In any case:
  - Are there actually any good precedences out there?
- Possible application:
  - solve hard previously unsolved problems
A(n a)typical development of the passive set’s size
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Can it possibly work?

Using the 9 independent random-precedence 60 second runs

On the set $P$ of 1678 problems from the “grey zone”

Record size of passive every 100 activations

Compute nine respective sums $s_i$ until the first stream stops:

$S_1(p) = s_1(p, 0) + s_1(p, 100) + s_1(p, 200) + \ldots$

$\ldots$

$S_9(p) = s_9(p, 0) + s_9(p, 100) + s_1(p, 200) + \ldots$

Denote the average $S_i(p)$ over (un)succesful runs $i$ as $\bar{S}_{(un)succ}(p)$.

For how many $p \in P$ is $\bar{S}_{succ}(p) < \bar{S}_{unsucc}(p)$?

Answer: 1130 (out of 1669)
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How did we evolve, then?
How did we evolve, then?

**Optimize_precedence**(\(p, t_1, t_2\))

- run “frequency” for 1s to establish \(act\_cnt\)
- spawn a population \(\Pi\) of \(n\) random precedences
- the fitness of \(\pi \in \Pi\) is \(S_\pi(p)\):
  - the sum of the passive set sizes during a run on \(p\)
  - summing every step from 0 to \(act\_cnt\) activations
- loop for \(t_1\) seconds:
  - pick a \(\pi \in \Pi\)
  - randomly (adaptively) perturb \(\pi\) to obtain \(\pi'\)
  - evaluate \(\pi'\) as above
  - keep the better of \(\pi\) and \(\pi'\)
- Finally, run with \(\pi_{best}\) for \(t_2\) seconds
Results

First a test run:

- optimizing for 300s and final run for 60s: 8965
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How many have solved in total?
- “frequency” 300s: 9457 (40 uniques)
- all the “harmonic” runs: 10030 (202 uniques)
- the long optimizing run: 9604 (87 uniques)
- In total: 10176
Lessons learned:

- A good ordering can make a difference
- If out of ideas, check out what E does
- The slowly-growing-search-space heuristic works!
Conclusion

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Thank you for your attention!