Applying Formal Verification to Reflective Reasoning

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Artificial Intelligence for Theorem Proving, Obergurgl 2017

Who am I?

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PhD, University of Cambridge

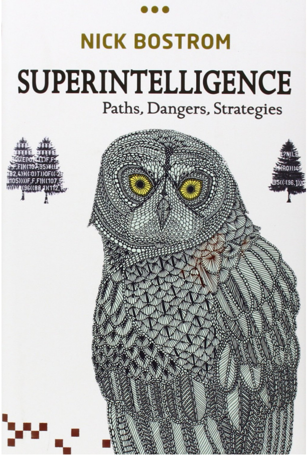
Researcher, Data61, CSIRO

Theorem Proving in HOL

CAKEML
A Verified Implementation of ML

MIRI MACHINE INTELLIGENCE RESEARCH INSTITUTE DATA 61 CSIRO

Context: Beneficial AI



Source: Future of Humanity Institute, Oxford.

See also: `https://intelligence.org/why-ai-safety/`

Context: Beneficial AI

MIRI
MACHINE INTELLIGENCE
RESEARCH INSTITUTE

Technical Agenda

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Highly Reliable Agent Design

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Highly Reliable Agent Design

- Foundations
- Basic problems lacking in-principle solutions

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(Note: This is not MIRI's only research agenda.)

One problem within MIRI's 2014 agenda happened to seem to align with my expertise, theorem proving and self-verification

Problem Statement

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Design a system that

- *always satisfies some safety property,*
- *but is otherwise capable of arbitrary self-improvement.*

Problem of Self Trust

Too little self-trust

Cannot make simple self-modifications

Too much self-trust

Unsound reasoning about successors

Overview

Reflective Reasoning

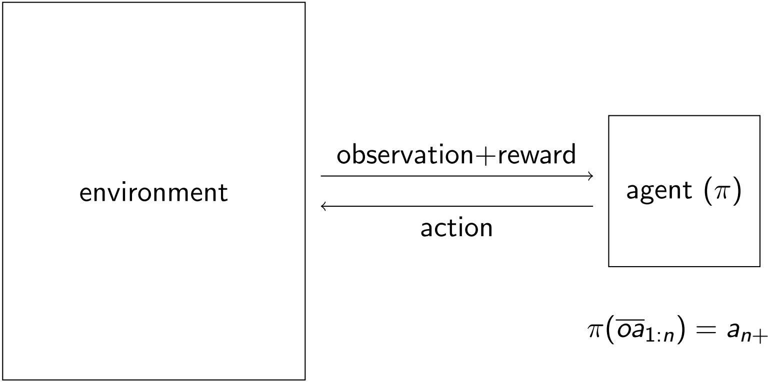
- Self-Modifying Agents
- Vingean Reflection
- Suggester-Verifier Architecture
- Problem and Partial Solutions

Implementation

- Botworld
- Formalisation in HOL

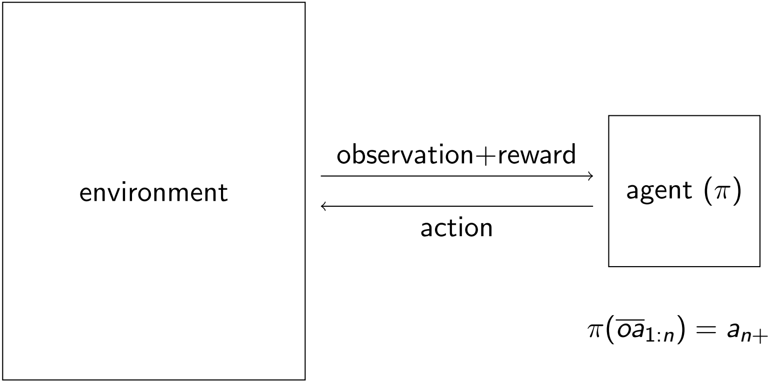
Reflective Reasoning

The Agent Framework



$$\pi(\overline{oa}_{1:n}) = a_{n+1}$$

The Agent Framework



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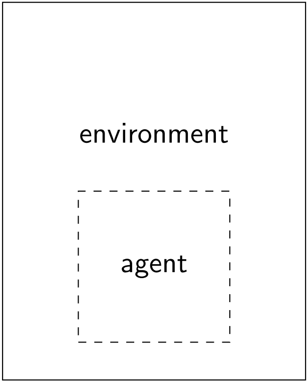
Cartesian boundary

- agent computed outside environment

Reality is not Cartesian



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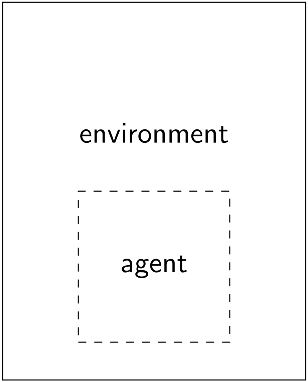

$$\pi_n(o_n) = (a_{n+1}, \ulcorner \pi_{n+1} \urcorner)$$

Vingean Principle

One can reason only abstractly about a stronger reasoner

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Relevance

Self-improving system must reason about programs it cannot run: its successors

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Approach

Formal logic as a model of abstract reasoning

Suggester-Verifier Architecture

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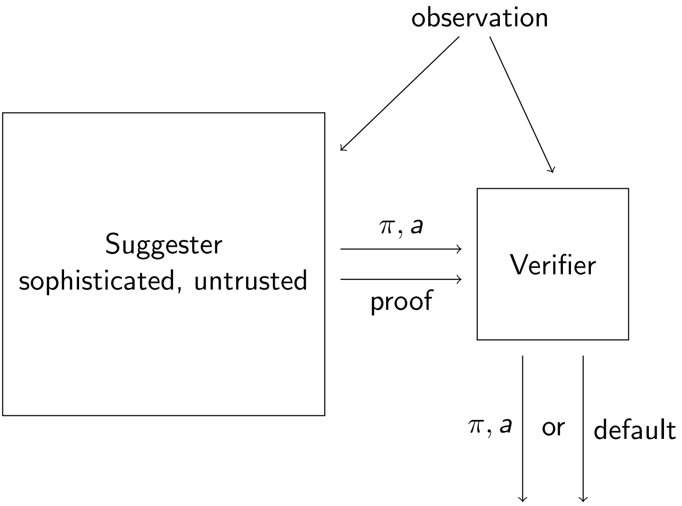
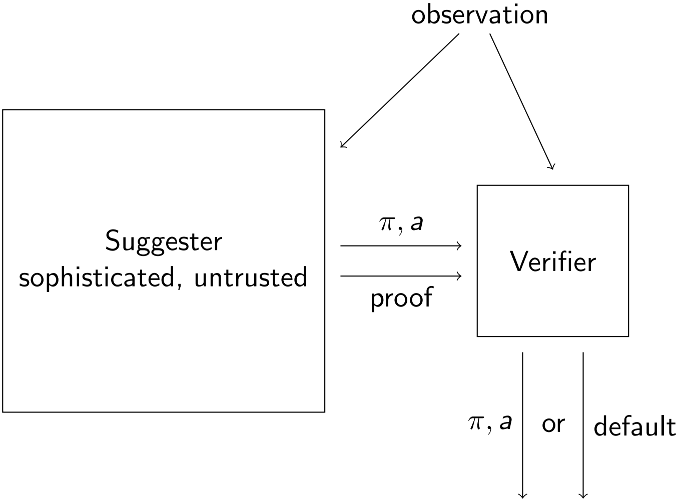
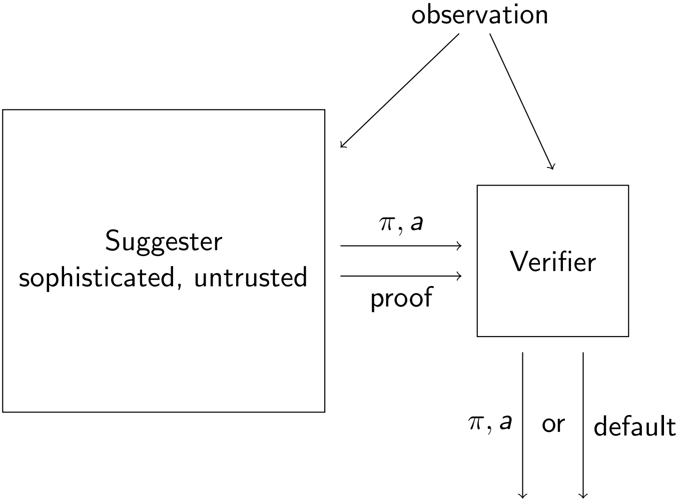


Suggester-Verifier Architecture



Verify: $\vdash u(h(\pi, a)) \geq u(h(\text{default}))$

Suggester-Verifier Architecture



Verify: $\vdash u(h(\pi, a)) \geq u(h(\text{default}))$ ($\approx \text{Safe}(a)$)

Problem with Self-Modification

Argument for Safety of Successor

- To create a successor, must prove that its actions will be safe
- If successor follows s-v architecture, it will only take actions it has proven to be safe
- However, to conclude that an action is *actually* safe from a *proof* is problematic.

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- However, to conclude that an action is *actually* safe from a *proof* is problematic.
  This principle, $T \vdash \Box_T \ulcorner \varphi \urcorner \implies \varphi$, is inconsistent.
  (Gödel/Löb)

Partial Solutions

Descending Trust

$T_{100} \vdash \Box_{T_{99}} \ulcorner \varphi \urcorner \implies \varphi, \; T_{99} \vdash \Box_{T_{98}} \ulcorner \varphi \urcorner \implies \varphi,$

$\ldots$

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Model Polymorphism

$$0 < \kappa, T \vdash \forall n. \Box_T \ulcorner \varphi(\bar{n}) \urcorner \implies \varphi[\kappa - 1/\kappa](n)$$

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If $\mathrm{Safe}(a) \equiv \forall n.\, \mathrm{Safe}(a, n)$
Take $\varphi(n) \equiv n \leq \kappa \implies \mathrm{Safe}(a, n)$

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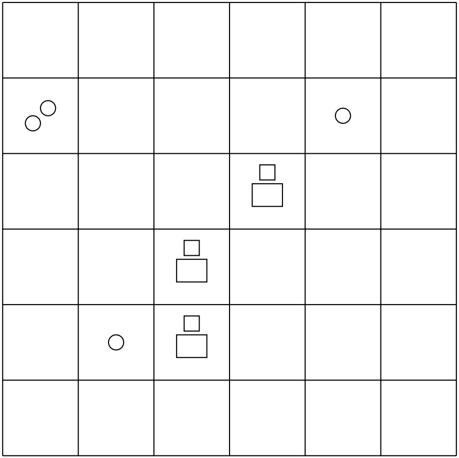
$$\forall a.\, \Box \ulcorner \forall n \leq \overline{t+1} + \kappa.\, \mathrm{Safe}(\bar{a}, n) \urcorner \implies$$
$$\forall n \leq t + \kappa.\, \mathrm{Safe}(a, n)$$

Implementation

Botworld: Concrete Framework for Embedded Agents



Robots can construct/inspect/destroy/program other robots

Botworld Formalisation

Semantics

- step : state $\longrightarrow$ state

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- steph : s-w-h $\rightarrow$ a $\rightarrow$ (obs, state) option

Suggester-Verifier Implementation

sv(π_{default},σ,obs):

1. $(\pi, a) =$ run π_{default}
2. $(\pi', a', \text{thm}) =$ run $\sigma(\text{obs}, \pi, a)$
3. Check thm has correct form
4. Write (π, a) or (π', a') accordingly

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Reflection Library

Automation for: $\Box\ulcorner \text{LCA } \bar{k} \implies P \urcorner$ implies $\text{LCA } (k+1) \implies P$

Implementation Challenge

Project Proposal

Build a Botworld agent that self-modifies into a *provably safe* agent of the same architecture.

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Eventual Project

Discover how far theorem proving technology is from implementing the above...

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- Looks possible, but with more effort than anticipated
- I would estimate 4 person-years.

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Theorem Proving for AI

- Specifications Needed!
- Novel Architectures for AI Systems, e.g., improve on Suggester-Verifier to support logical induction and non-proof-based reasoning
- Reducing Problems to Functional Correctness