Progress in Automating Formalization

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Two Obstacles to Strong Computer Support for Math

1. Low reasoning power of automated reasoning methods, particularly over large complex theories
2. Lack of computer understanding of current human-level (math and exact science) knowledge

- The two are related: human-level math may require nontrivial reasoning to become fully explained. Fully explained math gives us a lot of data for training AITP systems.
- And we want to train AITP on human-level proofs too. Thus getting interesting formalization/ATP/learning feedback loops.
- In 2014 we have decided that the AITP/hammer systems are getting strong enough to try this. And we started to combine them with statistical translation of informal-to-formal math.
- We are pretty cautious, but this really seems possible.

Favorable developments in the last decade

- Reasonably big formal corpora of common math are coming
- Reasonably strong proving methods over them are developed
- Large part of the latter was thanks to learning methods (40–50% of Mizar theorems automatically provable today)
- We are even getting some aligned informal/formal corpora:
- Flyspeck, Compendium of Continuous Lattices, Feit-Thompson
- So let's use what works:
- Statistical machine translation combined with strong learning-assisted automated reasoning over large libraries providing the common reasoning background!

Formal, Informal and Semiformal Corpora

- HOL Light and Flyspeck: some 25,000 theorems
- The Mizar Mathematical Library: some 60,000 theorems (most of them rather small lemmas), 10,000 definitions
- Coq: several large projects (Feit-Thompson theorem, ...)
- Isabelle, seL4 and the Archive of Formal Proofs
- Arxiv.org: 1M articles collected over some 20 years (not just math)
- Wikipedia: 25,000 articles in 2010 - collected over 10 years only
- Proofwiki - LaTeX but very semantic, re-invented the Mizar proof style

Experiments with Informalized Flyspeck

- 22000 Flyspeck theorem statements informalized
  - 72 overloaded instances like "+" for `vector_add`
  - 108 infix operators
  - forget all "prefixes"
    - `real_, int_, vector_, nadd_, hreal_, matrix_, complex_`
    - `ccos, cexp, clog, csin, ...`
    - `vsum, rpow, nsum, list_sum, ...`
  - Deleting all brackets, type annotations, and casting functors
    - `Cx` and `real_of_num` (which alone is used 17152 times).

Statistical Parsing of Informalized HOL

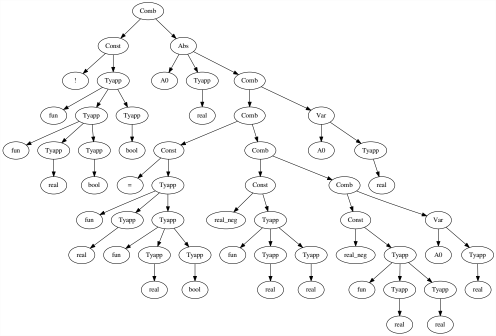
- Experiments with Stanford parser and CYK chart parser
- Examples (treebank) exported from Flyspeck formulas
  - Along with their informalized versions
- Grammar parse trees
  - Annotate each (nonterminal) symbol with its HOL type
  - Also “semantic (formal)” nonterminals annotate overloaded terminals
  - guiding analogy: word-sense disambiguation using CYK is common
- Terminals exactly compose the textual form, for example:
- REAL_NEGNEG: $\forall x. - -x = x$

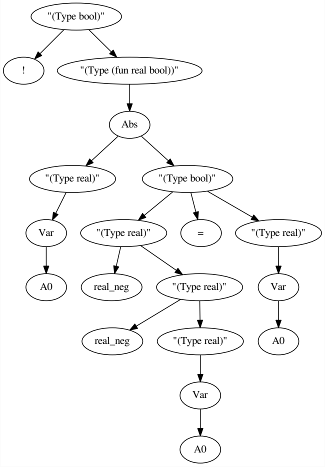
```
(Comb (Const "!" (Tyapp "fun" (Tyapp "fun" (Tyapp "real") (Tyapp "bool"))
(Tyapp "bool"))) (Abs "A0" (Tyapp "real") (Comb (Comb (Const "=" (Tyapp "fun"
(Tyapp "real") (Tyapp "fun" (Tyapp "real") (Tyapp "bool")))) (Comb (Const
"real_neg" (Tyapp "fun" (Tyapp "real") (Tyapp "real"))) (Comb (Const
"real_neg" (Tyapp "fun" (Tyapp "real") (Tyapp "real"))) (Var "A0" (Tyapp
"real"))))) (Var "A0" (Tyapp "real")))))
```

- becomes

```
("(Type bool)" ! ("(Type (fun real bool))" (Abs ("(Type real)"
(Var A0)) ("(Type bool)" ("(Type real)" real_neg ("(Type real)"
real_neg ("(Type real)" (Var A0)))) = ("(Type real)" (Var A0))))))
```

Example grammars



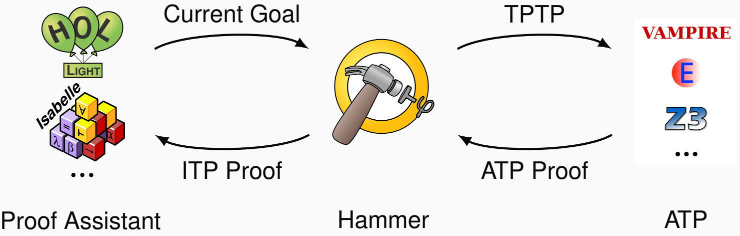


CYK Learning and Parsing

- Induce PCFG (probabilistic context-free grammar) from the trees
  - Grammar rules obtained from the inner nodes of each grammar tree
  - Probabilities are computed from the frequencies
- The PCFG grammar is binarized for efficiency
  - New nonterminals as shortcuts for multiple nonterminals
- CYK: dynamic-programming algorithm for parsing ambiguous sentences
  - input: sentence – a sequence of words and a binarized PCFG
  - output: N most probable parse trees
- Additional semantic pruning
  - Compatible types for free variables in subtrees
- Allow small probability for each symbol to be a variable
- Top parse trees are de-binarized to the original CFG
  - Transformed to HOL parse trees (preterms, Hindley-Milner)

Things that type-check are still not too good

Why not use today's AI/ATP ("hammers")?



Online parsing system

- "`sin ( 0 * x ) = cos pi / 2`"
- produces 16 parses
- of which 11 get type-checked by HOL Light as follows
- with all but three being proved by HOL(y)Hammer

```
sin (&0 * A0) = cos (pi / &2) where A0:real
sin (&0 * A0) = cos pi / &2 where A0:real
sin (&0 * &A0) = cos (pi / &2) where A0:num
sin (&0 * &A0) = cos pi / &2 where A0:num
sin (&(0 * A0)) = cos (pi / &2) where A0:num
sin (&(0 * A0)) = cos pi / &2 where A0:num
csin (Cx (&0 * A0)) = ccos (Cx (pi / &2)) where A0:real
csin (Cx (&0) * A0) = ccos (Cx (pi / &2)) where A0:real^2
Cx (sin (&0 * A0)) = ccos (Cx (pi / &2)) where A0:real
csin (Cx (&0 * A0)) = Cx (cos (pi / &2)) where A0:real
csin (Cx (&0) * A0) = Cx (cos (pi / &2)) where A0:real^2
```

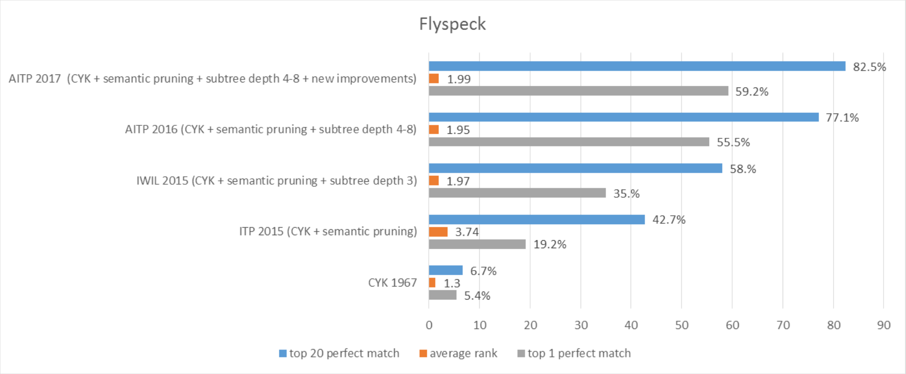
What we can correctly parse

```
! A0 ! A1 ! A2 ! A3 ! A4 FAN vec 0 , V_SY vecmats A4 ,
E_SY vecmats A4 /\ 1 < dimindex UNIV /\ 1 <= A0
/\ A0 <= dimindex UNIV /\ row A0 vecmats A4 = A3
/\ row SUC A0 MOD dimindex UNIV vecmats A4 = A1
/\ row SUC SUC A0 MOD dimindex
   UNIV MOD dimindex UNIV vecmats A4 = A2
/\ ! A5 ! A6 1 <= A5 /\ A5 <= dimindex UNIV /\ 1 <= A6
/\ A6 <= dimindex UNIV
/\ row A5 vecmats A4 = row A6 vecmats A4
==> A5 = A6 ==>
ivs_azim_cycle EE A1 E_SY vecmats A4 vec 0 A1 A2 = A3
```

Typechecking and proving over Flyspeck

- 698,549 of the parse trees typecheck (221,145 do not)
- 302,329 distinct (modulo alpha) HOL formulas
- For each HOL formula we try to prove it with a single AI-ATP method
- 70,957 (23%) can be automatically proved
  - A significant part of them are not interesting because of wrong parenthesizing
- In 39.4% of the 22,000 Flyspeck sentences the correct (training) HOL parse tree is among the best 20 parses
- its average rank: 9.34

Recent Progress on Flyspeck



Betting Slide from IHP'14, Paris

- In 25 years, 50% of the toplevel statements in LaTeX-written Msc-level math curriculum textbooks will be parsed automatically and with correct formal semantics
- Hurry up: I will only accept bets up to 10k EUR total (negotiable)
- More at `http://ai4reason.org/aichallenges.html`

Parsing Mizar – New Features

- More natural-language features than HOL (Andrzej was a linguist too)
- Arbitrary symbols, heavily overloaded
- Declarative natural-deduction style (re-invented in ProofWiki)
- Adjectives and their Prolog-style propagation (registrations)
- Dependent types
- Hidden arguments (derived from the context)
- Syntactic macros (synonyms, antonyms, expandable modes)
- This is all closer to LaTeX, but also a big challenge

Parsing Mizar – Phase0: Treebank Creation

- New transformation of the Mizar internal XML based on the HTML-izer
- The main trick: instead of hyperlinking, use the links as disambiguating nonterminals
- This is followed by using symbolic AI (ATP in our case) for mapping the syntax to the semantic layer
- Example: `RCOMP_1:5` in Mizar, Lisp, “semantic” TPTP and “syntactic” TPTP
- `for s, g being real number holds [.s,g.]  is closed`
- `(Bool "for" (Varlist (Set (Var "s")) "," (Varlist (Set (Var "g")))) "being" (Type ($#nv1_xreal_0 "real" ) ($#nm1_ordinal1 "number" ) ) "holds" (Bool (Set ($#nk1_rcomp_1 "[." ) (Set (Var "s")) "," (Set (Var "g")) ($#nk1_rcomp_1 ".]" ) ) "is" ($#nv2_rcomp_1 "closed" ) ))`
- `![A]: v1_xreal_0(A) => !  [B]: (v1_xreal_0(B) => v2_rcomp_1(k1_rcomp_1(A, B)))`
- `![A]: ![B]: ( ( nm1_ordinal1(A) & nv1_xreal_0(A) & nm1_ordinal1(B) & nv1_xreal_0(B) ) => nv2_rcomp_1(nk1_rcomp_1(A,B)) ) )).`

Examples of Mizar's Advanced Syntactic Mechanisms

```
definition
  let P,R be set;
  func P(#)R -> Relation means
  [x,y] in it iff ex z st [x,z] in P & [z,y] in R;
end;
notation synonym P*R for P(#)R; end;
definition
  let X,Y1,Y2,Z be set;
  let P be Relation of X,Y1;
  let R be Relation of Y2,Z;
  redefine func P*R -> Relation of X,Z;
end;
notation
  let f,g be Function;
  synonym g*f for f*g;
end;
```

Examples of How This Is Translated

```
::   synonym g*f for f*g;
fof(dt_nk3_funct_1, axiom,(![A,B]:(((v1_relat_1(A) &
v1_funct_1(A))  & (v1_relat_1(B) & v1_funct_1(B)))
=> nk3_funct_1(B, A)=nk6_relat_1(B,A)))).

:: synonym P*R for P(#)R;
fof(dt_nk6_relat_1, axiom,(![A,B]:((v1_relat_1(A)
& v1_relat_1(B))  => nk6_relat_1(A, B)=nk5_relat_1(A, B)))).
```

Parsing Mizar – Phase1: Statistical Parsing and Translation to Prolog/TPTP

- the most probable parses for an ambiguous Mizar-like sentence
- `for s, g being real number holds [.s,g.]  is closed`
- becomes
- `(Bool "for" (Varlist (Set (Var "s")) "," (Varlist (Set (Var "g")))) "being" (Type ($#nv1_xreal_0 "real" ) ($#nm1_ordinal1 "number" ) ) "holds" (Bool (Set ($#nk1_rcomp_1 "[." ) (Set (Var "s")) "," (Set (Var "g")) ($#nk1_rcomp_1 ".]" ) ) "is" ($#nv2_rcomp_1 "closed" ) ))`
- which is postprocessed (Lisp-to-TPTP) into the “syntactic TPTP”:
- `![A]: ![B]: ( ( nm1_ordinal1(A) & nv1_xreal_0(A) & nm1_ordinal1(B) & nv1_xreal_0(B) ) => nv2_rcomp_1(nk1_rcomp_1(A,B)) ) )).`

Parsing Mizar – Phase2: ATP connects the layers

- About 13000 Prolog-style formulas encoding the relation between user-level syntax and the semantic (MPTP) encoding
- Also the full set of Mizar typing rules needed for this!
- Altogether about 30000 background knowledge rules used for the mapping - pretty bad
- We try to prove that the syntactic form is implied by the semantic form
- Relatively non-trivial task for ATPs, requires premise selection and good ATP strategies
- Vampire: about 40% proved in 60s
- Targeted E strategies invented automatically on the corpus by our BliStrTune system: about 50% proved (by 14 strategies)

Parsing Mizar – Phase2: Example ATP problem

```
include('../stdincl').
fof(t5_rcomp_1,axiom,( ! [A] : ( v1_xreal_0(A) => ! [B] :
( v1_xreal_0(B) => v2_rcomp_1(k1_rcomp_1(A,B)) ) ) )).
fof(c_t5_rcomp_1,conjecture,( ! [A] : ! [B] : (
( nm1_ordinal1(A) & nv1_xreal_0(A) & nm1_ordinal1(B) &
nv1_xreal_0(B) ) => nv2_rcomp_1(nk1_rcomp_1(A,B)) ) )).
```

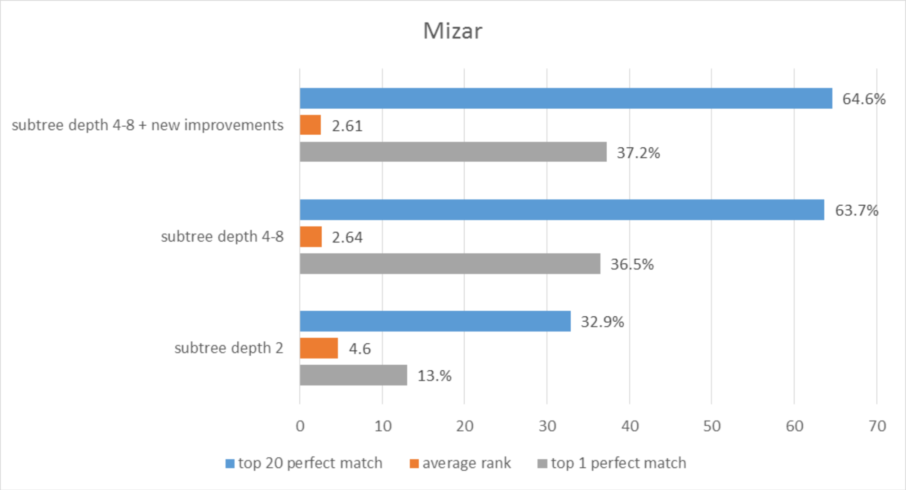
Further Improvements of the Parsing – Three Pass Algorithm

- bottom-up pass, we use CYK to compute only the set of reachable nonterminals for every cell in the parsing chart (instead of computing all partial parses)
- top-down pass we prune from the chart all nonterminals that cannot be reached from the top
- (bottom-up) parse we run the standard (full) CYK, however avoiding the unreachable nonterminals detected before
- => about 30% speedup on Mizar dataset

Further Improvements of the Parsing – Occam's Razor

- Occam's Razor to prefer simpler parses, where simpler means that the parse was constructed using fewer parsing rules
- this discourages e.g. from formulas that parse the very overloaded symbol + in many different ways
- $\frac{\texttt{probability of a standard partial parse}}{\texttt{num of all parsing rules of a partial parse}}$

First Mizar Results (100-fold Cross-validation)



Future Work

- Starting to look at full Mizar proofs and their alignment to ProofWiki
- Tighter integration of probabilistic parsing with semantic pruning (simple congruence closure already in)
- More corpora $\rightarrow$ more alignments $\rightarrow$ more knowledge $\rightarrow$ ...
- Smarter parsing methods
- Looping self-teaching systems:
- train on some data $\rightarrow$ parse $\rightarrow$ typecheck/prove the parses ...
- ... and thus get more data to train on $\rightarrow$ loop ...
- merge with other AI/ATP self-improving systems (MaLARea, concept alignment)

Thanks for listening!

- Questions?