Logic-Independent Premise Selection for Automated Theorem Proving

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Obergurgl
Outline

1. Introduction

2. Theoretical Foundation: Entailment Relations

3. Case Study


5. Conclusion
Introduction
Automated Theorem Proving

- A theorem proving problem consists of Axioms and a conjecture.
- An automated theorem prover (ATP) runs an algorithm to find a proof.
- A typical ATP is efficient on small problems.
- Large problems lead to combinatorial explosion.
  - ATP reach their time or memory limit.
  - Return with no result.
Example

- Suggested Upper Merged Ontology (SUMO)
- Formalised in FOF and THF in the TPTP library
- Problems with (tens of) thousands of axioms
- Pick CSR119^3 (THF): \( \approx 5000 \) Axioms
- Higher-Order Prover Leo-II runs into a timeout (60 seconds)
Solution: Premise Selection

- Reduce the set of axioms for the proving task
- Proving time decreases or proving even becomes possible at all
- The SUMO example $CSR119^3$ passes in less than a second
- The axiom set was reduced to 390 out of over 5000 axioms by SInE
Logic Dependence

Problem:
- There are many premise selection algorithms
- Implemented only for FOF or some higher order logics
- ...even though some are described logic-independently

Solution:
- Lift the algorithms to logic-independence
- Run them in an abstract notion of ‘logic’
- Transfer results to the concrete logic
Tool Support

Problem:
Many provers operate on one logic/syntax only
- FaCT, Pellet: Description logic with OWL
- Darwin, E-Prover, Geo-III, SPASS, Vampire: First-order logic with TPTP/FOF
- Leo-II, Satallax, Isabelle: Higher-order logic with TPTP/THF
- Isabelle/HOL’s own logic
- ...

Solution:
- Lift the algorithms to logic-independence
- Run them in an abstract notion of ‘logic’
- Transfer results to the concrete logic
Theoretical Foundation: Entailment Relations
An entailment relation with symbols \((\text{Sen}, \text{Sym} \vdash, \text{symbols})\) consists of

- A set of sentences \(\text{Sen}\)
- A set of symbols \(\text{Sym}\)
- A relation \(\vdash \subseteq \mathcal{P}(\text{Sen}) \times \text{Sen}\) which is
  - reflexive (Axioms are theorems)
  - transitive (We may use lemmas)
  - monotonic (We may use premise selection)
- A function \(\text{symbols} : \text{Sen} \rightarrow \mathcal{P}(\text{Sym})\) giving the symbols that occur in a sentence
Case Study
Implementation: Ontohub

- Web application: https://ontohub.org
- Version controlled repository for ontologies/specifications/theories
- Version control (git)
- Integrated editor for small files
- Analyses theories
- Has interfaces with ATPs
- Back-end: Hets
Implementation: Ontohub cont’d

- Supports different logics
  - Propositional Logic
  - OWL
  - FOL / TPTP-FOF
  - FOL + Induction
  - CASL
  - Modal Logic
  - Common Logic
  - HOL / TPTP-THF
  - Isabelle/HOL
  - …

- Brings tool support
  - FaCT, Pellet
  - CVC4, Darwin, E-Prover, Geo-III, SPASS, Vampire
  - Leo-II, Satallax, Isabelle
  - …
Premise Selection: The Algorithm ‘SInE’

- Developed by Kryštof Hoder
- Fully automatic with a few user-defined parameters
- Operates on syntax
- Selects recursively the axioms that share a symbol with the conjecture or an already selected axiom
- The shared symbol that allows to select an axiom must hold more conditions
- Selection stops after $n$ recursion steps
- We implemented SInE in Ontohub
Data Flow

User

Ontohub/Hets

Prover

Theory

Axioms, Conjectures, Symbols

Premise Selection

Modified Theory

Proof

Proof
Repositories
107 repositories currently available
There is also a complete list of repository clone-urls

Featured Repositories

- BFO: This repository contains an axiomatization of Basic Formal Ontology (BFO) 2.0. This axiomatization is the result of a collaboration between Berry Smith and Fabian Neuenhaus. NOTE: At this stage the axiomatization is incomplete and under change.
- Bioportal: Mirror of http://bioportal.bioontology.org/ with all ontologies below 5 megabytes.
- FOIS Ontology Competition: The aim of the FOIS 2014 ontology competition is (1) to encourage ontology authors to make their ontologies publicly available and (2) to subject them to evaluation according to a set of pre-determined criteria. See http://fois2014.inf.uibk.ac.at/call-for-ontology-competition.html

Common Repositories

- AdHoc: Domain-specific ontology to detect messages on key personnel turnover in the ad hoc publicity of public companies.
- AlgebraBlending
- Amalgams: Music: Initial theories for testing chord blending in CASIL.
- Amalgams: OWL examples: owl ontologies for amalgams
- AppliedOntologyOntohubPaper: These are examples that are used in the paper “Ontohub -- A semantic repository for heterogeneous ontologies”
- bfo-test
- Bioportal_mappings: Mappings from Bioportal
- BOC2016: Ontology Building Competition 2016 at Technical University of Cluj-Napoca, Romania Intelligent Systems Group
- Eugen's Repositories:
  - bfo-test
  - Eugens Test 1
  - zzz-push-test
  - zzz-issue-960-test
  - eugens-test-5
  - Hets-Lib
  - eugens-test-3
  - eugens-test-4
  - eugens-test-6
  - AppliedOntologyOntohubPaper
  - OntohubADpaperExamples
  - eugens-test2-tbtp
  - eugens-test3-tbtp
  - eugens-test-tbtp
  - eugens-test-2
### grundlagen

#### File browser

In this page, you have direct access to the file system where the repository is stored. Below is the list of files in the current directory, independent of whether they contain ontologies or not. In order to find a particular file, you can navigate through the file system.

<table>
<thead>
<tr>
<th>grundleben</th>
<th>bushy-by</th>
<th>Create Subdirectory</th>
<th>Upload file</th>
<th>History of this directory</th>
</tr>
</thead>
<tbody>
<tr>
<td>Li_all_th1</td>
<td>p</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Li_all_thi</td>
<td>p</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Li_comilf</td>
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<tr>
<td>Li_changep</td>
<td>p</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Li_eq_t1</td>
<td>p</td>
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<td>Li_eq</td>
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<tr>
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</tr>
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<tr>
<td>Li_eq_eq_1_3</td>
<td>p</td>
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<td></td>
</tr>
</tbody>
</table>
29 March 2017 Logic-Independent Premise Selection

File browser

In this page, you have direct access to the file system where the repository is stored. Below are the contents of the file presented as text as well as its path relative to the repository home directory. If you want to see the contents of this file as ontologies, click on the ontology mappings above the text box.

---

AGT001+1.p

---

1 %-----------------------------------------------
2  File  : AGT001+1 : TPTP v4.4.0. BugFixed v3.1.0.
3  Domain : Agents
4  Problem : Problem for the CPlanT system
5  Version : [Bar03] axioms : Special.
6  English :
7  % Refs
8  : [Bar03] Barto, J. (2003), Email to G. Sutcliffe
9  : [BTP03] Barto et al. (2003), Meta-Reasoning in CPlanT Multi-Ag
10  : [TPR03] Tovicka et al. (2003), Meta-reasoning for Agents’ Priv
11  Source :
12  : [Bar03]
13  % Names :
14  % Status : Theorem
15  Rating : 0.03 v6.4.0, 0.08 v6.3.0, 0.12 v6.1.0, 0.10 v6.0.0, 0.09 v5.5.0, 0.07 v5.4.0, 0.0
16  Syntax : Number of formulae : 556 ( 524 unit)
17  % Number of atoms : 656 ( 2 equality)
18  Maximal formula depth : 8 ( 1 average)
19  Number of connectives : 143 ( 43 ; ; 1 ; 67 &)
20  % ( 16 <= ; 16 == ; 0 <=)
21  % ( 0 <= ; 0 <= ; 0 =)
22  % Number of predicate(s) : 17 ( 0 propositional; 1-4 arity)
23  Number of functions : 289 ( 286 constant; 0-2 arity)
24  Number of variables : 70 ( 0 singleton; 70 ; 0 ?)
25  Maximal term depth : 5 ( 1 average)
26  SPC :
27  %-- Include axioms of CPlanT
28  include: 'Axioms/AGT001+0.ax'.
29  %-- Include events of CPlanT
30  include: 'Axioms/AGT001+1.ax'.
31  %-- Include axioms for RDM and RDM less
32  include: 'Axioms/RDM05+0.ax'.
33  include: 'Axioms/RDM05+1.ax'.
34  %-- Include conjecture
35  fcf(query_1,conjecture,
36  [ andEq_team(countryYmedicalorganization,countryYnotforprofitorganization,towns,nb) ]).
37  %--
SUMO SInE Test

Overview | Ontologies | File browser | History | Settings

ontology defined in the file /sumo-sine-test/sumo_problem.p

functions | predicates | typeconstants | axioms | theorems

1 | 2 | 3 | 4 | 5 | Next » | Last »

25 per page

acquaintance_THFTYPE_i
angleOfFigure_THFTYPE_i
angularMeasure_THFTYPE_i
Amphibian_THFTYPE_i
brother_THFTYPE_i
citizen_THFTYPE_i
Amu_THFTYPE_i
div_THFTYPE_III
documentation_THFTYPE_i
editor_THFTYPE_i
equal_THFTYPE_i
### Defining query_1

**Definition of query_1**

\[ \text{for(query_1, conjecture, \{accept_team\|country\|medical\|organization, country\|civil\|organization, towna, nwij\})} \]

**Proof Attempts**

<table>
<thead>
<tr>
<th>Number</th>
<th>Date</th>
<th>Time taken</th>
<th>Prover</th>
<th>Proof Status</th>
<th>Evaluation State</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>a month ago</td>
<td>66 seconds</td>
<td>Isabelle</td>
<td>THM</td>
<td>done (a month ago)</td>
</tr>
<tr>
<td>8</td>
<td>a month ago</td>
<td>0 seconds</td>
<td>Leo-II</td>
<td>THM</td>
<td>done (a month ago)</td>
</tr>
<tr>
<td>7</td>
<td>a month ago</td>
<td>0 seconds</td>
<td>CVC4</td>
<td>THM</td>
<td>done (a month ago)</td>
</tr>
<tr>
<td>6</td>
<td>a month ago</td>
<td>1 seconds</td>
<td>Geo-III</td>
<td>UNK</td>
<td>done (a month ago)</td>
</tr>
<tr>
<td>5</td>
<td>a month ago</td>
<td>0 seconds</td>
<td>Darwin</td>
<td>THM</td>
<td>done (a month ago)</td>
</tr>
<tr>
<td>4</td>
<td>a month ago</td>
<td>0 seconds</td>
<td>EProver</td>
<td>THM</td>
<td>done (a month ago)</td>
</tr>
<tr>
<td>3</td>
<td>a month ago</td>
<td>0 seconds</td>
<td>Satallax</td>
<td>ERR</td>
<td>done (a month ago)</td>
</tr>
<tr>
<td>2</td>
<td>a month ago</td>
<td>0 seconds</td>
<td>SPASS</td>
<td>THM</td>
<td>done (a month ago)</td>
</tr>
<tr>
<td>1</td>
<td>a month ago</td>
<td>0 seconds</td>
<td>Vampire</td>
<td>THM</td>
<td>done (a month ago)</td>
</tr>
</tbody>
</table>
Attempt to prove theorem query_1

Definition

Name | Text
--- | ---
query_1 | fof(query_1, conjecture, (accept_team(country=medicalorganization, country=civilorganization, town, n6))).

Proving Configuration

Provers
- Vampire
- SPASS
- EProver
- Darwin
- Isabelle
- Geo-III

Select none will result in using the default prover.
Select all | Select none

Timeout
Default (10 seconds) | The timeout will be used for each proof obligation individually.

Axiom Selection Method
- Manual
- SnIE

SnIE Axiom Selection

Commonness threshold
Greater values result in selecting more axioms containing uncommon symbols.

Depth limit
Number of SnIE selection iterations. Greater values result in selecting more axioms. The value '-1' is interpreted as infinity.

Tolerance
Greater values result in selecting more axioms.
Experiments: Setup

We applied our implementation of SInE to

- All 2078 problems of the MPTP2078 (FOF)
- A subset (501 problems) of a formalisation into THF0 of the Automath formalization of Landau’s ‘Grundlagen der Analysis’
Results: FOF

Provability with and without Premise Selection

- blue: only without SInE
- green: both with and without SInE
- orange: only with SInE

Number of Solved Problems

- CVC4
- Darwin
- EProver
- Geo-III
- Isabelle
- Leo-II
- SPASS
- Vampire
- Any
- Any Except EProver
Results: THF0

Provability with and without Premise Selection
- only without SInE
- both with and without SInE
- only with SInE

Number of Solved Problems

Leo-II
Satallax
Any
There’s More: Signatures and Logic Translations.
Special handling of THF

Problem:
- THF (among other logics) is typed
- Symbols must be declared with a formula before their first use
- Such ‘signature-defining declarations’ must not be removed

Solution:
- Preserve the needed ‘signature-defining declarations’ after the premise selection
Theoretical Foundation: Entailment Systems

An entailment system with symbols \((\text{Sign}, \text{Sen}, \text{Sym}, \vdash, \text{symbols})\) consists of

- a category \(\text{Sign}\) of signatures and signature morphisms
- a functor \(\text{Sen}: \text{Sign} \to \text{Set}\) giving the set of sentences over a signature
- a faithful functor \(\text{Sym}: \text{Sign} \to \text{Set}\) giving the set of symbols of a signature
- for each \(\Sigma \in |\text{Sign}|\) a relation \(\vdash_\Sigma \subseteq \mathcal{P}(\text{Sen}(\Sigma)) \times \text{Sen}(\Sigma)\) which
  - is reflexive, transitive, monotonic
  - and satisfies \(\vdash\)-translation: Given a signature morphism \(\sigma: \Sigma_1 \to \Sigma_2\), we have \(\Gamma \vdash_{\Sigma_1} \varphi \Rightarrow \text{Sen}(\sigma)(\Gamma) \vdash_{\Sigma_2} \text{Sen}(\sigma)(\varphi)\)
- a natural transformation \(\text{symbols}: \text{Sen} \to \mathcal{P} \circ \text{Sym}\) giving the symbols of a sentence
Tool Support for Logics

Problem:
- Some logics don’t have direct tool support, e.g. CASL, Common Logic, modal logic
- People need to formalise the theory in tool-supported logics
- ...or cannot use ATP (with premise selection)

Solution:
- Run premise selection in the desired logic
- Translate the modified theory to logic with tool support
- Run the prover on the translation
Theoretical Foundation: Entailment relation morphism

An entailment relation morphism

\[ \alpha : (\text{Sen}_S, \text{Sym}_S, \vdash_S, \text{symbols}_S) \to (\text{Sen}_T, \text{Sym}_T, \vdash_T, \text{symbols}_T) \]

is a function \( \alpha : \text{Sen}_S \to \text{Sen}_T \) such that

for all \( \Gamma \subseteq \text{Sen}_S, \varphi \in \text{Sen}_S : \Gamma \vdash_S \varphi \) implies \( \alpha(\Gamma) \vdash_T \alpha(\varphi) \)

\( \alpha \) is called conservative if

for all \( \Gamma \subseteq \text{Sen}_S, \varphi \in \text{Sen}_S : \Gamma \vdash_S \varphi \) if and only if \( \alpha(\Gamma) \vdash_T \alpha(\varphi) \)
Theoretical Foundation: Theoroidal entailment relation morphism

A conservative theoroidal entailment relation morphism $(\alpha, \Delta)$ contains

- a function $\alpha : (\text{Sen}_S, \text{Sym}_S, \vdash_S, \text{symbols}_S) \rightarrow (\text{Sen}_T, \text{Sym}_T, \vdash_T, \text{symbols}_T)$
- a base set of sentences $\Delta \subseteq \text{Sen}_T$

that hold

for all $\Gamma \subseteq \text{Sen}_S, \varphi \in \text{Sen}_S : \Gamma \vdash_S \varphi$ if and only if $\Delta \cup \alpha(\Gamma) \vdash_T \alpha(\varphi)$
Hets

- Evaluation component of Ontohub
- Actually analyses theories
- Translates theories
- Interfaces with provers
Data Flow

User → Ontohub/Hets → Prover

- Theory
- Axioms, Conjectures, Symbols
- Premise Selection
- Translation
- Modified, Translated Theory

Proof
Conclusion
Conclusion and Future Work

Conclusion

• Premise selection improves proving performance significantly
• Entailment relation morphisms allow its use with different logics
• And different reasoning tools
• SInE in Ontohub is only a proof of concept

Future Work

• Develop more premise selection algorithms and deploy them to Ontohub
• Learn from found proofs and disproofs (after premise selection)
• Use modular structure (signature morphisms)
Thank you for listening!

Questions?