When Should We Add Theory Axioms And Which Ones?

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Outline

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Vampire

- Automated theorem prover for first-order logic (+)
- Regular winner of various divisions in the CACS competition
- Notoriously hard to obtain

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Machine learning

- How to select theory axioms
- Our current machine learning playground
- Work in progress report

Vampire and the CASC competition

CASC 2015 results[1]

Higher-order Theorems	Satallax 2.8	LEO-II 1.6.2	*Satallax-l 1.3*	Isabelle 2015
Solved/400	271/400	195/400	285/400	267/400
Av. CPU Time	14.96	12.25	21.67	61.02
Solutions	268/400	191/400	0/400	0/400

Higher-order Non-theorems	Nitpick 2015	Refute 2015	Satallax 2.8
Solved/200	200/200	74/200	49/200
Av. CPU Time	7.92	24.34	0.05

Typed First-order Theorems +*-/	VampireZ 1.0	CVC4 TFA-1.5	Vampire 4.0	Beagle 0.9.22	SPASS+T 2.2.22	ZenonAri 0.1.0	Princess 20150706	*CVC4 1.4-TFF*
Solved/200	172/200	163/200	160/200	131/200	108/200	60/200	143/200	131/200
Av. CPU Time	11.85	17.27	10.75	21.76	10.04	2.86	17.38	10.67
Solutions	172/200	163/200	160/200	131/200	108/200	60/200	0/200	0/200

Typed First-order Non-theorems +*-/	CVC4 TFN-1.5	Princess 20150706	Beagle 0.9.22
Solved/20	10/20	6/20	6/20
Av. CPU Time	0.00	0.97	1.33

First-order Theorems	Vampire 4.0	*Vampire 2.6*	E 1.9.1	ET 0.2	CVC4 FOF-1.5	iProver 2.0	leanCoP 2.2	iProverM 0.7-0.3	Prover9 1109a	ePrincess 1.0	Muscadet 4.5	Geo-III 2015E
Solved/400	380/400	371/400	316/400	303/400	257/400	222/400	159/400	127/400	111/400	113/400	37/400	37/400
Av. CPU Time	12.20	14.86	20.18	20.96	33.40	21.12	46.76	30.15	28.01	48.39	7.32	38.47
Solutions	374/400	368/400	316/400	303/400	256/400	217/400	159/400	127/400	111/400	88/400	37/400	37/400

First-order Non-theorems	Vampire SAT-4.0	iProver SAT-2.0	*iProver SAT-1.0*	CVC4 FNT-1.5	E FNT-1.9.1	Geo-III 2015E
Solved/200	195/200	163/200	134/200	71/200	51/200	38/200
Av. CPU Time	38.95	44.11	79.93	57.78	9.62	21.89
Solutions	195/200	163/200	134/200	71/200	51/200	38/200

Effectively Propositional CNF	Vampire 4.0	*iProver 0.9*	iProver 2.0	E 1.9.1	Geo-III 2015E
Solved/200	192/200	161/200	153/200	101/200	9/200
Av. CPU Time	27.61	27.91	36.57	11.09	86.71

Large Theory Batch Problems	Vampire 4.0-LTB	MaLARea 0.5	E 1.9.1-LTB	iProver 2.0-LTB
Solved/1600	1208/1600	837/1600	799/1600	352/1600
Av. WC Time	6.51	8.30	7.40	13.04
Solutions	1208/1600	837/1600	799/1600	352/1600

[1] http://www.cs.miami.edu/~tptp/CASC/25/WWWFiles/DivisionSummary1.html

Why I think Vampire is good

- State of the art calculi / techniques
  - superposition [BG94,NR01]
  - AVATAR [V14]
  - InstGen [GK03]
  - finite model finding [McC94,CS04]
  - SInE [HV11]

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- Heavy (optional) use of incomplete but useful procedures
  - Limited Resource Strategy [RV03]
  - Literal selection [HRSV16]
  - Set of Support
  - ...

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  [Andrei Voronkov]
- Database of problems and proofs and strategy scheduling based on it

The need for many strategies

- Theorem proving is hard
- Chaos reigns (butterfly effect)
- If a strategy solves, it usually does so very fast!
- We need to combine strategies
  - not only good ones overall
  - but also complementary / exotic ones

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CASC-mode

- Conditional schedule of strategies
- Optimized for a good coverage over the TPTP

A CASC-mode code excerpt

```
case Property::FNE:
  if (atoms > 2000) {
    quick.push("dis+1011_40_bs=on:cond=on:gs=on:gsaa=from_current:nwc=1:sfr
    quick.push("lrs+1011_3_nwc=1:stl=90:sos=on:spl=off:sp=reverse_arity_133
    quick.push("dis-10_5_cond=fast:gsp=input_only:gs=on:gsem=off:nwc=1:sas=
    quick.push("lrs+1011_5_cond=fast:gs=on:nwc=2.5:stl=30:sd=3:ss=axioms:sd
    quick.push("lrs-3_5:4_bs=on:bsr=on:cond=on:fsr=off:gsp=input_only:gs=on
  }
  else if (atoms > 1200) {
    quick.push("lrs+1011_5_cond=fast:gs=on:nwc=2.5:stl=30:sd=3:ss=axioms:sd
    quick.push("dis+1011_8_bsr=unit_only:cond=fast:fsr=off:gs=on:gsaa=full_
    quick.push("dis+11_7_gs=on:gsaa=full_model:lcm=predicate:nwc=1.1:sas=mi
    quick.push("ins+11_5_br=off:gs=on:gsem=off:igbrr=0.9:igrr=1/64:igrp=140
  }
  else {
    quick.push("dis+11_7_16");
    quick.push("dis+1011_5:4_gs=on:gsssp=full:nwc=1.5:sas=minisat:ssac=none
    quick.push("dis+1011_40_bs=on:cond=on:gs=on:gsaa=from_current:nwc=1:sfr
    ...
```

Vampire and arithmetic

The big next challenge

- Reasoning with quantifiers and theories

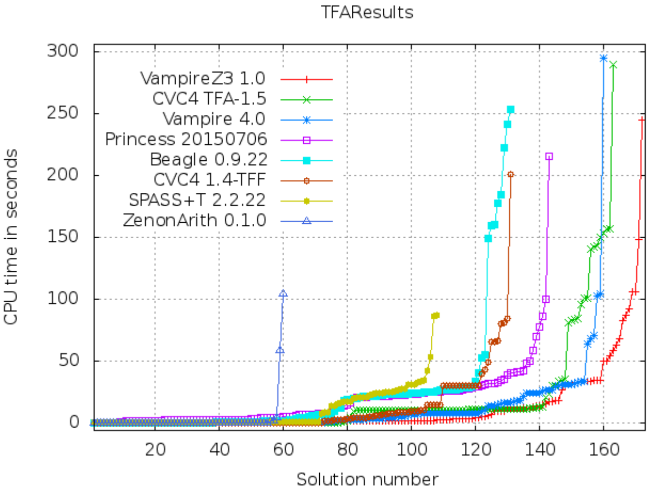
Vampire and arithmetic

The big next challenge

- Reasoning with quantifiers and theories

- Evaluation of ground interpreted terms ($1 + 1 \longrightarrow 2$)
- Interpreted operations treated specially by ordering
- Normalization of interpreted operations, i.e. only use $\leq$
- Theory axioms
  - hand-crafted set
  - either all added or none added (based on option)
- AVATAR with an SMT solver
  - current implementation for Z3
  - Idea: Vampire only explores theory-consistent ground sub-problems

Results for TFA (Typed First-order Theorems +*-/)[2]



[2] http://www.cs.miami.edu/~tptp/CASC/25/WWWFiles/ResultsPlots.html

Axiom selection experiment

```
tff(mix_quant_ineq_sys_solvable_2,conjecture,(
    ! [X: $int] : ( $less(5,X)
     => ? [Y: $int] : ( $less(Y,3)
          & $less(7,$sum(X,Y)) ) ) )).
```

Motivation

ARI581=1.p is a small problem which the default strategy solves instantly if we add all axioms except the commutativity of $+$, but does not solve in 60 seconds with commutativity.

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The experiment

Take the 15 pre-selected axioms for reasoning about linear integers, consider all 2^{15} strategies corresponding to each subset, evaluate them on a set of problems and see what can be (machine-) learned from that.

The 15 hand-crafted axioms (for linear integers)

1	$X + 0 = X$	2	$0 + X = X$
3	$X + Y = Y + X$	4	$X + (Y + Z) = (X + Y) + Z$
5	$0 = X + (-X)$	6	$(-X) + (-Y) = -(X + Y)$
7	$(X + (-Y)) + Y = X$		
8	$X \leq X$	9	$X \leq Y \vee Y \leq X$
10	$X \not\leq Y \vee Y \not\leq X \vee X = Y$	11	$X \not\leq Y \vee Y \not\leq Z \vee X \leq Z$
12	$X \leq Y \vee Y + 1 \leq X$	13	$X \not\leq Y \vee Y + 1 \not\leq X$
14	$X + 1 \not\leq X$	15	$X \not\leq Y \vee X + Z \leq Y + Z$

Preparation

Test problems selection

- Start with all TFA problems in TPTP (1128 problems)
- Focus on pure integer arithmetic with linear operators (+,-) (giving 515 problems)
- Drop those solvable by Vampire using the default strategy without theory axioms (and no Z3) in 30 seconds
- Giving us 282 problems in total

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Obtaining the data

- There are 15 theory axioms relevant to our set of problems
- This gives 32,768 combinations of theory axioms
- Given 282 problems this gives 9,273,344 experiments
- We ran each experiment for 5 seconds
- Almost 1.4 years of computation time... Thank you, StarExec!

"The cube" – basic info

Strategies

- min: 0 at (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)
- med: 63 at (0, 1, 0, 0, 1, 0, 1, 1, 0, 0, 0, 0, 0, 1, 1)
- max: 115 at (0, 1, 1, 0, 0, 0, 1, 1, 0, 1, 1, 1, 0, 1, 1)
- avg: 60.9
- $2^{15} - 4$ such that there exists a problem solved by it

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Problems

- min: 9 at `ARI182=1.p`
- med: 11869 at `DAT026=1.p`
- max: 32460 at `NUM893=1.p`
- avg: 14054.0
- 142 such that there exists a strategy solving it

Reducing the complexity without losing solutions

$*$-notations

$S(0,0,0,0,0,0,0,0,0,0,0,0,0,0,0) = 0$
$S(0,1,1,0,0,0,1,1,0,1,1,1,0,1,1) = 115$
$S(*,*,*,*,*,*,*,*,*,*,*,*,*,*,*) = 142$
$C(*,*,*,*,*,*,*,*,*,*,*,*,*,*,*) = 15$

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$S(*,*,*,*,*,*,*,*,*,*,*,*,*,*,*) = 142$
$C(*,*,*,*,*,*,*,*,*,*,*,*,*,*,*) = 15$

Hardcoding choices about particular axioms

- Start from $a = (*,*,*,*,*,*,*,*,*,*,*,*,*,*,*)$
- If there is an index $i = 1,\ldots,15$ s.t. $a[i] = *$ and a value $v \in \{0,1\}$ s.t. $S(a) = S(a[i \to v])$ then recurse on $a[i \to v]$
- otherwise report a and $C(a)$

Reducing the complexity – results

Four winners

$a_1 = (0, 0, *, *, 0, 0, *, 1, *, *, *, *, 0, *, *)$
$a_2 = (1, *, *, *, 0, 0, 1, 0, *, *, *, *, 0, *, *)$
$a_3 = (1, 0, *, *, 0, 0, *, 0, *, *, *, *, 0, *, *)$
$a_4 = (1, 0, *, 1, *, 0, *, 1, *, *, *, *, *, 0, *)$
$C(a_i) = 9, S(a_i) = 142$

Reducing the complexity – results

Four winners

$a_1 = (0, 0, *, *, 0, 0, *, 1, *, *, *, *, 0, *, *)$
$a_2 = (1, *, *, *, 0, 0, 1, 0, *, *, *, *, 0, *, *)$
$a_3 = (1, 0, *, *, 0, 0, *, 0, *, *, *, *, 0, *, *)$
$a_4 = (1, 0, *, 1, *, 0, *, 1, *, *, *, *, *, 0, *)$
$C(a_i) = 9, S(a_i) = 142$

Other "leaf" nodes

- $S(\_) = 142$, but $C(\_) > 9$ and cannot be minimized further
- 31 more with $C(\_) = 10$
  20 more with $C(\_) = 11$
  6 more with $C(\_) = 12$

“Greedy” CASC mode creation

Finding a good schedule

- pose as the set cover problem
- employ the obvious greedy algorithm

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Finding a good schedule

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- employ the obvious greedy algorithm

ord	contrib	choices	best	strategy
1	115	1	115	(0, 1, 1, 0, 0, 0, 1, 1, 0, 1, 1, 1, 0, 1, 1)
2	12	5	93	(0, 1, 0, 1, 1, 0, 0, 1, 1, 1, 0, 1, 0, 1, 1)
3	6	5	87	(0, 0, 0, 1, 0, 0, 1, 1, 0, 0, 1, 1, 0, 1, 1)
4	3	38	90	(1, 0, 1, 0, 0, 1, 1, 1, 0, 0, 1, 0, 1, 1, 1)
5	2	17	49	(1, 1, 0, 0, 0, 1, 1, 1, 1, 0, 1, 0, 0, 0, 0)
6	1	459	100	(1, 0, 0, 1, 1, 0, 0, 0, 1, 1, 1, 1, 0, 1, 1)
7	1	450	88	(0, 1, 0, 1, 1, 0, 0, 1, 0, 1, 0, 1, 0, 1, 1)
8	1	229	85	(0, 0, 1, 1, 1, 0, 1, 1, 0, 0, 1, 0, 1, 0, 1)
9	1	166	67	(1, 1, 1, 1, 1, 0, 0, 1, 1, 0, 0, 0, 1, 0, 0)
	142	$\ll 2^{15}$		

Machine Learning Experiments

- Tried to apply some out-of-the-box techniques
- More specifically
  - Split problems into training and testing
  - Extracted some features from problems
  - Used these to prepare some
  - Downloaded WEKA and tried running some of the algorithms
- Details next...
- Summary of lessons learned
  - Nothing truly 'out-of-the-box' as need to understand parameters
  - WEKA struggled with amount of data
  - Still not clear how best to harness machine learning

Problem Features

- Just considered static features initially. For example,
  - Standard syntactic features not related to theory reasoning
  - Frequency of each interpreted operation (generally and in goal)
  - Frequency of sorted variables and equalities (generally and in goal)
  - Usage of special numbers 0 and 1
- Ideas for dynamic features (i.e. after short run)

  Inspect descendants of each theory axiom and look for
  - Involvement with goal
  - Reductions (of and with)
  - Interaction with other theory axioms (pure descendants)
  - Groundness

Idea 1: Classification

- Want: function from problem feature vector to set of theory axioms
- Issue: 2^{15} different 'classes'
- Idea: train classifier per axiom, with other axioms as extra features
- i.e. *given problem and other axioms should I use this one?*
- New Issue: Unclear how to combine classifiers (search problem?)

- Tried a few algorithms on slightly different problem
  - Given problem features, axioms used and class (whether solved)
  - Build model for predicting class
  - Linear regression had 0.72 accuracy
  - Naive Bayes had 0.829 precision, 0.593 recall
  - SVM methods never finished

Idea 2: Association Rule Mining

- Idea: Mine rules that indicate associations between axioms
- Hopefully of the form *If adding A then I should probably (not) add B*
- Could be used to suggest which axiom sets are sensible

- Input is just the set of axioms used for each experiment
- Currently treat positive and negative data separately

- Use association rule mining
- Initial experiment failed to find rules with good confidence

Tentative conclusion

What have been done?

- No blood, sweat, nor tears, yet!
- Simplified “small” setup

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In real life . . .

- only limited number of samples from the strategy space
  - but can get as many as we want
- how to sample adaptively?

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What have been done?

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Other things to try

- mining proofs to see which axioms were used together in proofs, or more complex relations
- ...

More questions

How do we evaluate what we (will) have done?

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- With time reduced to 2.5s the best strategy still solves 112 problems and the largest union of two strategies has size 125.

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For theory axioms; what is important?

- Is it more important to be conservative, i.e., knowing what not to add to avoid explosion?
- Is there actually a problem to be solved via machine learning here, or can we just develop some hand-built heuristics that are good enough?

Thank you for attention!

Any answers?

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Any questions?

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Let's go skiing!